Asset Pricing with Price Levels

Thummim Cho and Christopher Polk*
London School of Economics

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*Cho: Department of Finance, London School of Economics, London WC2A 2AE, UK. Email: t.cho@lse.ac.uk.
Phone: +44 (0)20 7107 5017. Polk: Department of Finance, London School of Economics, London WC2A 2AE, UK and CEPR. Email: c.polk@lse.ac.uk. Phone: +44 (0)20 7849 4917. We thank Ben Hébert, Christian Julliard, Lukas Kremens, Dong Lou, Hanno Lustig, Ian Martin, Ran Shi, Dimitri Vayanos, and seminar participants at the LSE for useful comments and discussions. Amirabas Salarkia provided excellent research assistance.
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Abstract

We derive an exact identity linking future abnormal returns to current price-level deviations and estimate asset-pricing models using price levels rather than returns. Our identity highlights that abnormal returns occurring sooner, in more valuable states, or after relatively large capital gains are associated with larger price-level deviations. With our novel approach in hand, we measure the extent to which several well-known return anomalies are associated with price-level deviations from the CAPM. We are unable to reject the hypothesis that the CAPM explains the cross-section of prices of size-, book-to-market-, momentum-, and profitability-sorted portfolios well. However, the cross-section of prices of investment-sorted portfolios rejects the model.

JEL classification: G12, G14, G32
1 Introduction

The fundamental question of asset pricing is whether stock prices reflect intrinsic value, defined as the expectation of the present value of cash flows with respect to an asset pricing model. However, the vast majority of prior research instead uses abnormal returns as the metric with which to evaluate proposed models. While testing asset pricing models based on abnormal returns tells us whether a model is approximately right for investors who engage in dynamic trading, evaluating a model’s ability to explain price levels can tell us about the degree to which market price reflects the buy-and-hold intrinsic value implied by the asset pricing model, arguably a more important distortion for firm managers and policy makers with a long investment horizon.

Stock prices may differ from a model’s intrinsic value because the resulting stochastic discount factor (SDF) is an approximation (as is the case in many linear factor pricing models), because the empirical counterpart to the theoretical asset pricing model is subject to measurement error (Roll 1977), or because of fundamental mispricing that persists due to limits to arbitrage. However, as we carefully detail in the literature review and document in our empirical work, no extant method satisfactorily measures price-level errors. To fill this critical gap in the literature, we derive a novel exact identity relating subsequent abnormal returns to a price-level measure of model misspecification. With this identity in hand, we take a model’s implied stochastic discount factor proxy and precisely answer the question: How far off are model-implied valuations, i.e. intrinsic values, from observed price levels?

In particular, we define “delta” (δ) as a price-level measure of mispricing that under mild assumptions equals to the negative of the expected sum of future abnormal returns or “alpha” (α) on an asset, discounted by the cumulative SDF times the cumulative capital gains. In particular,

\[ \delta_t = \frac{(P_t - V_t)}{P_t} = -\sum_{j=1}^{\infty} E_t \left[ \phi_{t,t+j-1} \frac{\alpha_{t+j}}{1 + R_{f,t+j}} \right], \]

\[ \phi_{t,t+j} \equiv M_{t,t+j} D_{t+j} / P_t, \]

where \( V_t = E_t \left[ \sum_{j=1}^{\infty} M_{t,t+j} D_{t+j} \right] \) is the intrinsic value implied by the cumulative candidate SDF \( M_{t,t+j}, D \) is dividend, \( P \) is price, and \( R_{f,t+j} \) and \( \alpha_{t+j} \) are the risk-free rate of return and conditional
abnormal return from $t + j - 1$ to $t + j$ respectively. Thus, we provide an exact way to translate alphas into delta.

The term $\phi_{t,t+j}$ is simply a price-weighted version of the cumulative discount factor that conveniently converts a future rate of abnormal return into present values of future abnormal payoffs. As a consequence and as with any stochastic cashflow, abnormal payoffs that occur sooner or in more valuable states are more valuable today and affect the magnitude of mispricing more. Moreover, our explicit formulation of $\phi$ highlights that the correlation between alpha and prior capital gain is a critical ingredient when measuring delta, one of the many important implications of our identity that we explore in our empirical work. For one thing, $\phi$ can naturally be interpreted as the realized duration of the asset. Moreover, our analysis highlights how variation in price levels can be decomposed to variation in $\delta$ and variation in price-level risk, which in turn depends in part on how the asset’s risk factor loadings are expected to evolve in the future.

We exploit this price-level approach to estimating asset-pricing models to reveal important facts about the cross-section of prices. Our primary focus is how well the capital asset pricing model (CAPM) explains price levels; subsequent versions of this analysis will test other popular asset-pricing models.¹

Our initial analysis studies book-to-market-sorted portfolios. As Cohen, Polk, and Vuolteenaho (2009) point out, these portfolios should present a challenge to asset pricing tests based on price levels as not only are the average abnormal returns on value-minus-growth strategies highly statistically significant (Fama and French 1992), the book-to-market ratio is a persistent variable that forecasts the returns on the firm’s stock far in the future (see Cohen, Polk, and Vuolteenaho 2003). Thus, price-to-book-sorted portfolios have the potential of being significantly mispriced by our price-level measure of model misspecification. We find that the CAPM does a relatively good job describing the cross-section of average price levels of book-to-market-sorted portfolios; for example, we are unable to reject the null hypothesis that the difference in mispricing between

¹In particular, a natural next step is to consider the price-level patterns associated with specifications of the intertemporal CAPM. In particular, Campbell, Giglio, Polk, Turley (2018) document that incorporating stochastic volatility into the ICAPM framework of Campbell and Vuolteenaho (2004) significantly reduces the pricing errors relative to the CAPM in standard SDF return tests. We will also examine other influential factor models, including intermediary-based asset pricing models.
growth and value stocks is zero.\footnote{Though the estimated equity premium is relatively reasonable at 13.8%, if we go to the extreme of either forcing the CAPM SDF to match the in-sample Sharpe Ratio or requiring a zero intercept when fitting the cross-section of price levels, the fit deteriorates somewhat with price-level differences becoming statistically significant.}

We then apply our price-level tests to portfolios formed using other characteristics, including size, momentum, profitability, and investment. For at least some of these characteristics, we might expect the link between alpha and delta to be weaker. For example, though size is a relatively persistent variable, its ability to describe the cross-section of average returns is relatively poor. Similarly, though momentum strongly describe cross-sectional variation in average returns, the characteristic is quite transitory. Moreover, the extent to which momentum is an overreaction phenomenon may further attenuate any price-level consequence if momentum profits strongly revert (Lou and Polk 2019). Unlike momentum, both profitability and investment are strongly related to the cross-section of average returns and relatively persistent, though not as much as the book-to-market ratio.

Consistent with these facts, we find that both size and momentum have little association with cross-sectional variation in our price-level misspecification metric, \( \delta \). Estimates are an order of magnitude smaller than those we measure for book-to-market-sorted portfolios. Moreover, differences in delta between large and small stocks or winner and loser stocks are not only economically but also statistically insignificant. Somewhat surprisingly, though profitability may be relatively more persistent than the momentum characteristic, we find that this characteristic generates little variation in price-level pricing errors. Thus, though profitability may be the other side of value (Novy-Marx 2013), its implications are quite different in terms of price-level impact.

In contrast, the cross-section of prices of investment-sorted portfolios is substantially different. We find that the stocks of firms with relatively high investment are significantly overpriced. This result echoes the finding of Polk and Sapienza (2006) that the investment characteristic may be the strongest signal of price-level mispricing, at least with respect to the CAPM.

A natural follow-up question is to what extent simple long-run return measures such as the cumulative abnormal return (CAR) can proxy for price-level mispricing. We find that the generic CAR is a poor proxy for mispricing, but a simple modification of CAR that discounts distant
abnormal returns more does an adequate job capturing the cross-sectional dispersion in the mis-pricing of B/M sorted portfolios. However, by ignoring the covariance between realized duration and abnormal returns, these measures fail to capture the level of mispricing of the portfolios.

1.1 Literature review

The vast majority of prior research uses the abnormal return with respect to a factor model such as the CAPM as the metric with which to evaluate trading strategies. Despite the popularity of the abnormal return metric, a simple example shows that abnormal return could be a poor proxy for a price-level measure of model mispricing. Consider the hedge fund managers that, according to Brunnermeier and Nagel (2004), purchased technology stocks in the early stages of the dot-com boom and sold them before the crash. Though these positions and their associated characteristics may forecast positive abnormal returns relative to existing models such as the CAPM, the purchased stocks are likely to have been already overpriced, not underpriced relative to that model. Thus, a stock that is overpriced relative to the CAPM could generate positive abnormal returns in the short run, and vice versa.

Nevertheless, this illustration suggests that the initial model-specific mispricing of an asset may be recovered from the behavior of subsequent abnormal returns over the long run. Our measure highlights the correct way to aggregate the subsequent abnormal returns to arrive at the initial mispricing.

Of course, there is a long history of accumulating realized abnormal returns in order to proxy for price-level deviations from a benchmark model in the corporate finance literature, namely the well-known cumulative abnormal return (CAR) and buy-and-hold abnormal return (BHAR) methodologies. Work in this area includes Fama (1998), Barber and Lyon (1997), Lyon, Barber, and Tsai (1999), Brav (2000), and Bessembinder, Cooper, and Zhang (2019) among others. Though our method also aggregates future abnormal returns, by correctly discounting the stochastic payoffs associated with mispricing, our approach substantially improves on the long-run return measures.

Papers have introduced alternative price-level measures of mispricing. Lee, Myers, and Swaminathan (1999) and others infer the intrinsic value of the firm from its fundamentals. In stark
contrast, our formula is an identity linking mispricing to abnormal returns. Black (1986) comments that markets are efficient if prices are within factor of two of intrinsic value but provides no way in which to carefully measure such deviations. Nevertheless, his prior provides a useful benchmark for the economic significance of the price-level misspecifications we measure.

The closest mispricing metric to ours is the price-level alpha construct of Cohen, Polk, and Vuolteenaho (2009) (CPV). Unlike that paper, our mispricing identity begins with a clear definition of mispricing that is linked to a specification of the SDF, does not require unknown quantities such as the risk exposures and volatility in the absence of mispricing, and does not rely on the Campbell-Shiller (1988) approximation. The first quality of our new identity gives our price-level error $\delta$ a straightforward interpretation as a price-level measure of model misspecification and the latter two qualities allow us to improve on the statistical power of CPV, as we discuss in more detail in the paper.

Our exact framework can be used to revisit prior observations about market efficiency. Fama (1970) defines a market as efficient if “prices ‘fully reflect’ all available information” but goes on to test the efficient market hypothesis using returns, finding that the market is semi-strong form efficient. Shiller (1984, pp. 458-459) writes, “because real returns are nearly unforecastable, the real price of stocks is close to the intrinsic value . . . is one of the most remarkable errors in the history of economic thought.” Summers (1986) provides a numerical example that supports this argument, and Campbell (2017) shows how an expected return that follows a persistent AR(1) process may have low volatility but a large effect on the log dividend-price ratio.

Our identity provides a more sophisticated framework in which to understand Shiller’s point. It shows that mispricing can be large even if each post-formation alpha can be small if alphas are persistent. Furthermore, mispricing can be large even if alphas are small on average if alphas tend to comove strongly with $\phi$. The latter channel has been overlooked in the literature but can be quantitatively important: Cho (2019) shows empirically that as arbitrageurs trade away the alphas of equity anomalies such as value and momentum, they can expose the anomalies to systematic risks that the arbitrageurs are exposed to. That is, in the presence of arbitrage with limited capital, the initial mispricing that takes the form of alphas can persist in the form of risk premia associated with distorted factor betas.
Several recent papers study related topics. Cochrane (2014) shows how mean-variance characterizations can be applied to the stream of long-run payoffs or return opportunities even in a dynamic framework. Chernov, Lochstoer, and Lundeby (2019) propose a new asset-pricing test that requires the linear SDF specification of a factor model to price both one-period and multi-period factor returns that they implement via a managed-portfolio test. Their restriction puts further discipline on a factor model specification and is clearly distinct from our question of price-level specification errors. van Binsbergen and Opp (2019) study a structural model of a production economy in which the cost of equity faced by firms may be distorted due to abnormal returns and study its implications for the real economy. In contrast, we derive an identity that holds under minimal assumptions and does not require structural assumptions about the production technology and the nature of financial constraints when estimating mispricing.

2 Framework: Asset Pricing with Price Levels

This section proposes a measure of asset price level, price-level risk, and price-level mispricing. The form of the price level and price-level risk arise naturally from our definition of mispricing in the price level, which we describe after specifying the asset pricing environment.

2.1 The environment

Consider an asset with dividends (or coupons for bonds) \( \{D_{t+j}\}_{j=1}^{\infty} \). Our goal is to relate the mispricing (pricing error) of the asset at time \( t \) to its subsequent returns. We measure mispricing with respect to the (candidate) stochastic discount factor (SDF) \( \{M_{t+j}\}_{j=1}^{\infty} \), where we use \( M_{t,t+j} = \prod_{s=1}^{j} M_{t+s-1,t+s} \) to denote the cumulative SDF. The SDF we analyze could either be a candidate SDF that we are interested in comparing to the SDF implied by market prices in the sense of Hanson and Jagannathan (1991, 1997), or it could be the SDF of a particular investor in a market in which the law of one price fails for some assets due to frictions (e.g., Gărleanu and Pedersen 2011 and Geanakoplos and Zame 2014).
2.2 Fundamental value and mispricing

The benchmark value of the asset that we compare the price $P_t$ to is the present value of future dividends to a buy-and-hold investor with the SDF, and we call this (intrinsic) value $V_t$:

$$V_t = \sum_{j=1}^{\infty} E_t \left[ M_{t+j} D_{t+j} \right]. \quad (3)$$

We define mispricing in the price level (pricing error) $\delta_t$ as the percentage of market price attributable to the deviation of price from value:

$$\delta_t = \frac{P_t - V_t}{P_t}. \quad (4)$$

Hence, $\delta_t > 0$ if the asset is overpriced and $\delta_t < 0$ if it is underpriced. $\delta_t$ can range from $-\infty$ (if $V_t > 0$ and $P_t = 0$) to 1 (if $V_t = 0$ and $P_t > 0$), the opposite of the range for net returns, $[-1, \infty)$.

2.3 Assumptions

To link $\delta_t$ to subsequent returns, we make two relatively mild assumptions. The first is the existence of a risk-free asset that satisfies the fundamental theorem of asset pricing.

**Assumption 1.** At each $t+j-1$, there is a risk-free asset with return $R_{f,t+j}$ at time $t+j$ that satisfies the fundamental theorem of asset pricing:

$$E_{t+j-1} \left[ M_{t+j} \left(1 + R_{f,t+j} \right) \right] = 1. \quad (5)$$

This assumption does not state that a particular proxy for the risk-free rate (e.g., one-month Treasury bill rate) has to satisfy the return pricing equation, although we will follow the common practice of using the Treasury bill rate proxy. It merely states that there exists a correct risk-free rate measure. The second assumption is a weak form of a no-bubble condition.

**Assumption 2.** The present value of the deviation of price and value at the limit $j \rightarrow \infty$ is zero:

$$\lim_{j \rightarrow \infty} E_t \left[ M_{t,t+j} \left( P_{t+j} - V_{t+j} \right) \right] = 0.$$
This assumption is weaker than having two separate no-bubble conditions on price and value, 
\[ \lim_{j \to \infty} E_t [M_{t+j} P_{t+j}] = 0 \] and 
\[ \lim_{j \to \infty} E_t [M_{t+j} V_{t+j}] = 0, \] which imply Assumption (2).

### 2.4 The law of motion for mispricing

Under our definitions and Assumption (1), mispricing follows a simple law of motion. Eq. (3) and the law of iterated expectations implies the fundamental theorem of asset pricing holds for value:

\[ 1 = E_t \left[ M_{t+1} \frac{V_{t+1} + D_{t+1}}{V_t} \right]. \] (6)

Next, use Eq. (4) to substitute the empirically unobserved quantities \( V_t \) and \( V_{t+1} \) with \( V_t = (1 - \delta_t) P_t \) and \( V_{t+1} = (1 - \delta_{t+1}) P_{t+1} \) to obtain,

\[ \delta_t = 1 - E_t \left[ M_{t+1} (1 + R_{t+1}) \right] + E_t \left[ M_{t+1} \frac{P_{t+1}}{P_t} \delta_{t+1} \right]. \] (7)

Finally, use \( 1 = E_t \left[ M_{t+1} (1 + R_{f, t+1}) \right] \) (Assumption (1)) to express mispricing \( \delta_t \) at time \( t \) in terms of excess return \( R_{t+1}^e \) and mispricing \( \delta_{t+1} \) at time \( t + 1 \):

\[ \delta_t = -E_t \left[ M_{t+1} R_{t+1}^e \right] + E_t \left[ M_{t+1} \frac{P_{t+1}}{P_t} \delta_{t+1} \right]. \] (8)

The law of motion in Eq. (8) is intuitive. Since \( E_t \left[ M_{t+1} R_{t+1}^e \right] \) is the conditional abnormal return at time \( t + 1 \) adjusted for the gross risk-free rate \( E_t \left[ M_{t+1} R_{t+1}^e \right] = (1 + R_{f, t+1})^{-1} \alpha_{t+1} \), where \( \alpha_{t+1} \) is the conditional abnormal return), Eq. (8) says that underpricing (overpricing) at time \( t \) is either “paid out” as a positive (negative) abnormal return or contributes to the remaining mispricing at time \( t + 1 \). The discount factor on \( \delta_{t+1} \) is the SDF times the capital gain, which is intuitive given that \( \delta_{t+1} \) is normalized by \( P_{t+1} \). Hence, \( \delta_{t+1} \) matters more at time \( t \) if it arises in a state in which \( P_{t+1} \) is high (hence the capital gain term) or has a higher present value (hence the SDF term).
2.5 Relating mispricing to subsequent returns

Iterating the law of motion for mispricing (Eq. (8)) forward and using Assumption (2) to set
\[ \lim_{j \to \infty} E_t \left[ M_{t,t+j} \frac{P_{t+j}}{P_t} \delta_{t+j} \right] = 0 \]
expresses mispricing as a discounted sum of future excess returns:
\[
\delta_t = - \sum_{j=1}^{\infty} E_t \left[ M_{t,t+j-1} \frac{P_{t+j-1}}{P_t} \frac{\alpha_{t+j}}{1 + R_{f,t+j}} \right].
\]
(9)

This formula motivates our measures of price level and price-level risk.

For intuition, take a time \( t + j - 1 \) conditional expectation within the expectation and use
\[ E_{t+j-1} \left[ M_{t+j} R_{t+j}^e \right] = \left( 1 + R_{f,t+j} \right)^{-1} \alpha_{t+j} \]
to write,
\[
\delta_t = - \sum_{j=1}^{\infty} E_t \left[ M_{t,t+j-1} \frac{P_{t+j-1}}{P_t} \frac{\alpha_{t+j}}{1 + R_{f,t+j}} \right].
\]
(10)

Hence, mispricing at time \( t \) is the present value of subsequent abnormal returns. Hence, our identity highlights that abnormal returns occurring sooner, in more valuable states, or after relatively large capital gains are associated with larger price-level deviations. Specifically, the gross risk-free rate expresses the conditional abnormal return earned at time \( t + j \) as a time \( t + j - 1 \) value, the time \( t + j - 1 \) price \( P_{t+j-1} \) translates that abnormal return into time \( t + j - 1 \) abnormal cash flow, and the cumulative SDF expresses that abnormal cash flow in today’s value. Finally, the formula normalizes the present value of abnormal cash flows with today’s price \( P_t \). We call the discount factor \( M_{t,t+j-1} \frac{P_{t+j-1}}{P_t} \) “realized duration,” since in expectation (and in the absence of abnormal returns) it measures the contribution of dividends at time \( t + j \) and onward to the price of the asset at time \( t \).

Eq. (10) also suggests that truncating the infinite sum with a finite sum up to some large \( J \) provides a close approximation,
\[
\delta_t = - \sum_{j=1}^{J} E_t \left[ M_{t,t+j-1} \frac{P_{t+j-1}}{P_t} \frac{\alpha_{t+j}}{1 + R_{f,t+j}} \right]
\]
(11)
since realized duration is likely to be small at that \( J \) and the abnormal return is also likely to be small at that \( J \) even if \( \delta_t \) is large. This truncation makes it easy to use Eq. (9) and its decomposi-
tion (discussed next) in empirical applications.

### 2.6 Price level and price-level risk

To obtain the price level and price-level risk measures, decompose the right-hand side of Eq. (9) into three components and rearrange to obtain the following:

\[
\begin{align*}
&\sum_{j=1}^{\infty} E_t [\phi_{t,t+j-1} M_{t+j}] E_t [1 + R_{t+j}] = \delta_t \\
&\quad \quad \{\text{Price level}\} \\
\end{align*}
\]

\[
\begin{align*}
&\sum_{j=1}^{\infty} E_t [\phi_{t,t+j-1} M_{t+j}] E_t [1 + R_{f,t+j}] \\
&\quad \quad \{\text{Price level due to risk-free rates}\} \\
\end{align*}
\]

\[
\begin{align*}
&- \sum_{j=1}^{\infty} \text{Cov}_t (\phi_{t,t+j-1} M_{t+j}, R_{e,t+j}) \\
&\quad \quad \{\text{Price-level risk}\} \\
\end{align*}
\]

Or

\[
\begin{align*}
&\sum_{j=1}^{\infty} E_t [\phi_{t,t+j-1} M_{t+j}] E_t [R_{e,t+j}] = \delta_t - \sum_{j=1}^{\infty} \text{Cov}_t (\phi_{t,t+j-1} M_{t+j}, R_{e,t+j}) \\
&\quad \quad \{\text{Excess price level}\} \\
&\quad \quad \{\text{Price-level risk}\} \\
\end{align*}
\]

where

\[
\phi_{t,t+j} = M_{t,t+j} \frac{P_{t+j}}{P_t} 
\]
The first component is a simple discounted sum of future contemporaneous risk premia, and
the second component corrects the first for the fact that future risk premia can covary with past
realized duration $\phi$. The third component is a residual component of price-level risk that depends
more importantly on the degree to which realized duration covaries with excess returns; it reduces
to the covariance of realized duration with excess return adjusted by the expected SDF (i.e., the
inverse of the gross risk-free rate),

$$\sum_{j=1}^{\infty} E_t \left( \phi_{t,t+j-1} M_{t+j}, R^e_{t+j} \right) = \delta_t + \sum_{j=1}^{\infty} Cov_t \left( \phi_{t,t+j-1}, R^e_{t+j} \right).$$  (17)

When $J = 1$, the price-level relation collapses to the familiar expected return relation:

$$E_t \left[ R^e_{t+1} \right] = \alpha_{t+1} + \lambda_{M,t+1} \beta_{M,t+1},$$  (18)
where $\lambda_{M,t+1} = \frac{\text{Var}_t(M_{t+1})}{E_t[M_{t+1}]}$ is the price of risk and $\beta_{M,t+1} = -\frac{\text{Cov}_t(M_{t+1}, R_{t+1})}{\text{Var}_t(M_{t+1})}$ is the quantity of risk measured with respect to the SDF.

### 2.7 Comparison to other ways of inferring mispricing in the price level

Some readers may be interested in how our price-level measure of mispricing, $\delta_t = \frac{P_t - V_t}{P_t}$, compares to existing measures of mispricing or long-term return. We spend the next few pages to make the comparison, but those readers who would rather see the empirical applications of our novel price level approach may skip the following discussion and go directly to Section 3.

**Market-to-book ratio**

The market-to-book ratio, closely related to long-run reversal, is one popular measure of mispricing (Rosenberg, Reid, and Lanstein 1984, De Bondt and Thaler 1985, and Lakonishok, Shleifer, and Vishny 1994). However, market-to-book ratio is a highly imperfect measure of mispricing, since many other factors than mispricing can also influence the ratio. The decomposition of Vuolteenaho (2002) and Cohen, Polk, and Vuolteenaho (2003) shows that the log market-to-book ratio $mb_t$ is approximately,

$$mb_t = \sum_{j=1}^{\infty} \rho^{j-1}E_t[\text{roe}_{t+j}] - \sum_{j=1}^{\infty} \rho^{j-1}E_t[\alpha_{t+j} + \lambda_{M,t+j} \beta_{M,t+j}] + \frac{1}{2} \sum_{j=1}^{\infty} \rho^{j-1}\text{Var}_t(r_{t+j}),$$

(19)

where $\text{roe}_{t+j}$ is the log return on equity, $\alpha_{t+j}$ is abnormal return, $\lambda_{M,t+j} \beta_{M,t+j}$ is the risk premium implied by the SDF $M$, and $r_{t+j}$ is log return. Hence, besides the distortion in the discount rate due to $\alpha_{t+j}$, other factors such as earnings growth, risk, and volatility can affect the cross-sectional and time-series variation in the market-to-book ratio.

**Price-level alpha**

Cohen, Polk, and Vuolteenaho (2009) (CPV) were the first to propose an identity for measuring mispricing in the price level. They define the fundamental value of a stock as the present value of future dividends discounted with the discount factors that would have prevailed in the absence
of mispricing:

$$V_t = \sum_{j=1}^{\infty} E_t \left[ \frac{D_{t+j}}{\prod_{s=1}^{j} (1 + R_{v,t+s})} \right],$$  \hspace{1cm} (20)$$

where $R_{v,t+s}$ is the return on $V_t$. They then show that price-level alpha $\alpha_{t}^{\text{price}}$, defined as the log deviation of price from value, approximately equals

$$\alpha_{t}^{\text{price}} = -\sum_{j=1}^{\infty} \rho^{j-1} E_t \left[ r_{t+j} - r_{v,t+j} \right]$$

$$= -\sum_{j=1}^{\infty} \rho^{j-1} E_t \left[ R_{t+j} - \lambda_{M,t+j} \beta_{M,v,t+j} \right] + \frac{1}{2} \sum_{j=1}^{\infty} \rho^{j-1} \left( \text{Var}_t \left( r_{t+j} \right) - \text{Var}_t \left( r_{v,t+j} \right) \right),$$  \hspace{1cm} (21)$$

where $r$ denotes log return, $R$ denotes simple return, and $\beta_{M,v,t+j}$ is quantity of risk in the absence of mispricing. Hence, if the distortion in the volatility of log return due to mispricing is small,

$$-\sum_{j=1}^{\infty} \rho^{j-1} E_t \left[ R_{t+j} \right] \approx \alpha_{t}^{\text{price}} = -\sum_{j=1}^{\infty} \rho^{j-1} E_t \left[ \lambda_{M,t+j} \beta_{M,v,t+j} \right].$$  \hspace{1cm} (22)$$

CPV use this relation to run a cross-sectional regression that explains the price level based on price-level risk with respect to the CAPM, using cash-flow betas measured by the exposure of a portfolio’s return on equity to the market’s return on equity to estimate market betas in the absence of mispricing. Their regression is useful in rejecting the null of no mispricing, since under that null the volatility distortion should be zero, but using their identity to draw a conclusion beyond the rejection of the null is difficult due to the potentially large volatility distortion along with measurement error introduced from estimating $\beta_{M,v,t+j}$ with their cash-flow beta proxy. Our identity addresses this problem by expressing mispricing in terms of observable quantities (other than the SDF loadings).

**Estimating the fundamental value directly**

Another approach to estimating fundamental mispricing is to estimate the fundamental value directly from cash-flow data:

$$V_t = \sum_{j=1}^{\infty} E_t \left[ M_{t,j} D_{t+j} \right].$$  \hspace{1cm} (23)$$

Estimating this quantity unconditionally in the data is problematic for two reasons. First, the predictability of dividends means that conditional covariance is smaller than the unconditional
covariance. Second, truncation of the infinite sum at some finite $J$ leaves out a large fraction of the value.\(^3\) To estimate the conditional expectation, one can estimate the dynamics of the factors in the SDF and dividends in a VAR and use it to obtain the conditional fundamental values at each given point in time. However, depending on the cash flow, a simple VAR with a few number of lags may not capture the persistent nature of dividend cash flows, and the constant coefficients in the estimated VAR model could understate the extent to which the SDF covaries with cash flows in a crisis scenario.

**Cumulative abnormal return (CAR)**

One popular measure of long-term return is the cumulative abnormal return defined as the simple sum of abnormal returns over a period of time:

\[
CAR_t = - \sum_{j=1}^{\infty} E_t \left[ \alpha_{t+j} \right] \quad (24)
\]

(written with a sign flip so that, like our $\delta_t$, positive abnormal returns means a negative $CAR_t$).

How well can CAR proxy for fundamental mispricing of the portfolio?

To see how CAR relates to fundamental mispricing, rewrite Eq. (10) as

\[
\delta_t = - \sum_{j=1}^{\infty} E_t \left[ w_{t,t+j} \right] E_t \left[ \alpha_{t+j} \right] - \sum_{j=1}^{\infty} Cov_t \left( w_{t,t+j}, \alpha_{t+j} \right), \quad (25)
\]

where

\[
w_{t,t+j} = \frac{P_{t+j-1}}{P_t} \left( 1 + R_{f,t+j} \right). \quad (26)
\]

Hence, CAR is an exact measure of mispricing when $E_t \left[ w_{t,t+j} \right] = 1$ and $Cov_t \left( w_{t,t+j}, \alpha_{t+j} \right) = 0$. These conditions would approximately hold, for example, if abnormal return tends to be stable (i.e., conditional abnormal return equals unconditional abnormal return), the gross monthly risk-free rate is close to 1, and the portfolio has a very high duration such that cumulative capital gain approximately equals cumulative return (in which case $E_t \left[ M_{t+s} \frac{P_{t+s}}{P_{t+s-1}} \right] \approx E_t \left[ M_{t+s} \left( 1 + R_{t+s} \right) \right]$).

\(^3\)For example, suppose $V_t$ follows a Gordon growth model with expected dividend growth rate $g$ and constant discount rate $R$. $\sum_{j=J}^{\infty} E_t \left[ D_t (1+g)^j / (1+R)^j \right] = \left( \frac{1+g}{1+R} \right)^J E_t \left[ D_t \frac{1+g}{1+R} \right] = \left( \frac{1+g}{1+R} \right)^J V_t$. This means, if $g = 5\%$ and $R = 10\%$, about 40% (10\%) of $V_t$ comes from dividends occurring 20 (50) years after $t$. 

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1 \Rightarrow E_t \left[ \prod_{s=1}^{j-1} \left( M_{t+s} \frac{P_{t+s}}{P_{t+s-1}} \right) \right] \approx 1 \text{ when abnormal returns are small and both the SDF and returns exhibit little serial covariance). Nevertheless, Cho (2019) shows that returns on anomaly trading strategies depend importantly on shocks to the capital of arbitrageurs proxied by aggregate funding liquidity and the aggregate arbitrageur’s portfolio, suggesting that these conditions are violated.}

“Discounted” CAR

Under slightly less strong assumptions than the ones specified for CAR, mispricing $\delta_t$ is approximately a discounted sum of subsequent abnormal returns with a simple geometric discount factor. To see this, start with Eq. (25) and continue to assume $\text{Cov}_t \left( w_{t,t+j}, \alpha_{t+j} \right) = 0$. Next, note that if monthly risk-free rates are small and both the SDF and returns exhibit little serial covariance,

$$E_t \left[ w_{t,t+j} \right] \approx \prod_{s=1}^{j-1} E_t \left[ M_{t+s} \frac{P_{t+s}}{P_{t+s-1}} \right] = \prod_{s=1}^{j-1} E_t \left[ M_{t+s} (1 + R_{t+s}) \right] \frac{1}{1 + D_{t+s}/P_{t+s}}. \quad (27)$$

Then, replace $\frac{1}{1 + D_{t+s}/P_{t+s}}$ with the Campbell-Shiller (1988) discount factor $\rho = \frac{1}{1 + D/P}$ (where $D/P$ is the long-run average of the dividend-price ratio) and assume that abnormal returns are small ($E_t \left[ M_{t+s} (1 + R_{t+s}) \right] \approx 1$) to obtain

$$E_t \left[ w_{t,t+j} \right] \approx \rho^{j-1} \quad (28)$$

Hence, under these strong assumptions, we can write mispricing as a sum of subsequent abnormal returns discounted at a constant rate:

$$\delta_t \approx -\sum_{j=1}^{\infty} \rho^{j-1} E_t \left[ \alpha_{t+j} \right]. \quad (29)$$

We call this discounted CAR and analyze this potential metric in Section 7 together with simple CAR.

An ex-post identity for mispricing

Our identity uses the SDF to discount future cash flows, so some readers would wonder whether defining ex-post realized returns as the discount factor yields a similar identity. Such an approach
yields an identity that holds both ex-ante and ex-post, but it involves the unobserved return in the absence of mispricing:

\[ \delta_t = -\sum_{j=1}^{\infty} \frac{1}{\Pi_{s=1}^{j} (1 + R_{v,t+s})} (R_{t+j} - R_{v,t+j}), \]  

(30)

where \( R_v \) is the return on fundamental value \( V \).

3 Data

3.1 Basic data structure

We use monthly stock price data from the Center for Research in Security Prices (CRSP) and annual accounting data from CRSP/Compustat Merged (CCM) to create a basic merged dataset. We use one-month Treasury bill rates from Kenneth French’s data library (originally from Ibbotson Associates) as the annual risk-free rate and the market excess return from the same data library as the market factor.

We use the basic data to compute returns on characteristic-sorted portfolios. Unlike conventional return-based studies, however, we keep track of post-formation returns over 180 months (15 years), not just for one month. Specifically, we form value-weighted decile portfolios each month at \( t \) based on the NYSE decile cutoffs and compute the post-formation returns on these portfolios over \( t+1, \ldots, t+180 \).

Post-formation returns at \( t+j \) for the portfolio formed at \( t \) are returns on portfolios formed at \( t+j-1 \) by rebalancing the time-\( t \) stocks based on their cumulative capital gain from time \( t \) to time \( t+j-1 \). The rebalancing based on cumulative capital gain is the correct approach for our purpose of inferring the initial price level of the portfolio based on subsequent returns, since it mirrors how the returns earned by investing the dividend payments for an individual asset do not enter into our formula.

In summary, our data is three dimensional, as we illustrate in Table 1: we have post-formation returns for 10 different portfolios, for \( T \) different portfolio formation periods, and for \( J \) different post-formation periods. Our baseline data use 1953m6–2003m12 as the portfolio formation peri-
ods ($T = 606$ months) and $J = 180$ post-formation months, which imply post-formation returns over 1953m7–2018m12. Our portfolio sort begins in 1953m6, since this is when we can first compute all accounting-based characteristics based on the annual Compustat dataset.

### 3.2 Basic data adjustments

We use domestic common stocks (CRSP share code 10 or 11) listed on the three major exchanges (CRSP exchange code 1, 2, or 3). We replace missing prices with the average bid-ask price when available and drop observations with missing share or price information in the previous month. We code missing returns as zero returns and add delisting returns to returns. If delisting returns are missing but the CRSP delisting code is 500 or between 520 and 584, we use $-35\%$ ($-55\%$) as the delisting returns for NYSE and AMEX stocks (for Nasdaq stocks) (Shumway (1997) and Shumway and Warther (2002)). To compute capital gains, we use the CRSP split-adjustment factor (CFACSHR) to ensure that capital gains are not affected by split events.

### 3.3 Characteristics and portfolios

The main characteristic with which we sort stocks is the book to market (B/M) ratio computed each year in June. B/M ratio is the stock’s book value of equity in the previous fiscal year divided by market value of equity in December of the previous calendar year. Book value of equity is defined as the stockholders’ equity $SEQ$ (Compustat item 144) plus balance sheet deferred taxes and investment tax credit $TXDITC$ (item 35) minus book value of preferred stock $(BE = SEQ + TXDITC - BPSTK)$. Book value of preferred stock $BPSTK$ equals the preferred stock redemption value $PSTKRV$ (item 56), preferred stock liquidating value $PSTKL$ (item 10), preferred stock carrying value $UPSTK$ (item 130), or zero depending on data availability. If $SEQ$ is unavailable, we set it equal to total assets $AT$ (item 6) minus total liability $LT$ (item 181). If $TXDITC$ is unavailable, it is assumed to be zero. We treat zero or negative book values as missing.

We look at four other characteristics than B/M ratio. Size is market equity calculated every month. The momentum characteristic is calculated every month and is the cumulative gross return over the previous 12 months excluding the month before the portfolio formation. We define profitability and investment as in Fama and French (2015). Profitability is computed each
year in June. Profitability in calendar year $y$ is operating profits in fiscal year $y-1$ over book value of equity in fiscal year $y-2$, where operating profits sales $SALE$ (Compustat item 12) minus cost of goods sold $COGS$ (item 41), interest and related expenses $XINT$ (item 134) (if available), and selling, general, and administrative expenses $XSGA$ (item 132) (if available). Investment is also computed each year in June, and investment in calendar year $y$ is total assets in fiscal year $y-1$ divided by total assets in fiscal year $y-2$.

For each characteristic, we form 10 value-weighted portfolios based on the distribution of the characteristic among NYSE stocks. When doing so, to mitigate the effect of stale prices, we exclude microcaps defined as market equity below the bottom 10% cutoff among NYSE stocks. (Our results are very similar if we instead exclude stocks with liquidity level based on Amihud (2002) below the bottom 10% NYSE cutoff.) Hence, the 10 portfolios sorted by size are only based on the upper 90% size groups. To mitigate the effect of data errors in accounting variables, we also exclude stocks with B/M values below 0.01 or above 100. We do this for all portfolio sorts, not just for those based on the B/M ratio. However, to avoid a look-ahead bias, if after portfolio formation a stock’s B/M ratio falls outside the 0.01–100 range or size falls below the 10 percentile NYSE cutoff, we still keep it in the portfolio.

### 3.4 Descriptive statistics

Table 2 reports descriptive statistics for the aggregate variables and B/M portfolios. Panel A shows that the monthly realized market risk premium and volatility over 1953m6–2003m12 are 0.58% and 4.37%, implying a Sharpe ratio of 0.133. This is lower than the typical Sharpe ratio of the market based on annual returns, since monthly volatility is higher. The aggregate descriptive statistics for 1953m6–2018m12 are similar.

Panel B describes the post-formation returns and capital gains on B/M-sorted portfolios. Value portfolios have higher monthly returns than growth portfolios in the first few years after portfolio formation, but the difference becomes small beyond 5 years. Return volatility is similar across the B/M deciles. Finally, do growth stocks have higher capital gains than value stocks? The table shows that this is not the case. Value stocks have higher average gross capital gains in the first 5 years, implying that their high dividend yields do not offset high returns.
4 Do Prices Look Different from Expected Returns?

Does asset pricing with price levels show patterns distinct from that based on returns? We study this question for characteristic-sorted portfolios, starting with the book-to-market (B/M) ratio. We measure risk from the perspective of a mean-variance investor who holds the market (CAPM) and assume that the market factor has a constant loading on the SDF. This is not necessarily because we believe that the unconditional CAPM is the right model of risk but because it is the model that the return-based asset pricing literature tends to use as the baseline specification of risk.

4.1 The cross-section of returns

For comparison, we start with the cross-section of returns on B/M portfolios formed in 1953m6–2003m12. To do this, we estimate the CAPM betas using portfolio-specific time series regression,

\[ R_{i,t+1}^e = \alpha_i + \beta_i R_{m,t+1}^e + \epsilon_{i,t+1}, \]

over 1953m7–2004m1 and plot the cross-sectional relation between \( R_{i,t+1}^e \) and \( \hat{\beta}_i R_{m,t+1}^e \).

Figure 1a shows that value firms have high returns and low betas, whereas growth firms have low returns and high betas, a pattern documented previously (Campbell (2017)). Because of this, the cross-sectional relation between realized and predicted excess returns is negative.

The portfolios also have a common pattern. All but the bottom B/M decile portfolio appear to be “underpriced” in the sense that they deliver returns higher than what their risk exposure to the market factor requires, and all portfolios are very far from the 45 degree line that denotes the case where predicted mean excess return equals the realized excess return on average. Among different portfolios, the realized mean excess returns of the highest B/M portfolios (decile 9 and 10) are the furthest away from the predicted levels.

4.2 The cross-section of prices

Do prices look different from returns? We study the cross-section of prices of B/M portfolios formed in 1953m6–2003m12 based on post-formation returns over 1953m7–2018m12. Eq. (13)
shows that in the absence of mispricing,

\[-1 \times - \sum_{j=1}^{\infty} E_t \left[ \phi_{t,t+j-1} M_{t+j} \right] E_t \left[ R_{t+j}^{e} \right] = - \sum_{j=1}^{\infty} \text{Cov}_t \left( \phi_{t,t+j-1} M_{t+j}, R_{t+j}^{e} \right), \]

where \( \phi_{t,t+j-1} = M_{t,t+j-1} \frac{P_{t+j-1}}{P_t} \) is realized duration. We estimate this relation by replacing the population mean (covariance) with a sample mean (covariance) and truncating the infinite sum at \( J = 180 \) months. The expression based on an unconditional mean is likely to be very similar to that based on the conditional mean, since excess portfolio returns, portfolio capital gains, and market excess returns are all difficult to predict ex-ante.

To be consistent with the implicit assumptions of the earlier return regression, we specify the SDF \( M_{t+j} \) to be linear in the market factor and have constant loadings:

\[ M_{t+j} = b_0 - b_1 R_{m,t+j}^{e}. \]

Under the assumption that the SDF explains excess market returns as well as the risk-free rate, the two parameter values are approximately

\[ b_0 = 1, \ b_1 = \frac{\lambda_m}{\sigma_m^2} = \frac{E \left[ R_{m,t+j}^{e} \right]}{\text{Var} \left( R_{m,t+j}^{e} \right)} \]

in the monthly data with a small risk-free rate and a small squared monthly Sharpe ratio of the market (Cochrane 2005). Hence, we use the in-sample value of \( \hat{b}_1 = \frac{\hat{\lambda}_m}{\hat{\sigma}_m^2} = 3.054 \) (Table 2) to estimate the excess price level and price-level risk we use in the plots:

\[
\begin{align*}
\text{(Estimated) excess price level} &= - \sum_{j=1}^{J} \hat{E}_T \left[ \hat{\phi}_{t,t+j-1} \hat{M}_{t+j} \right] \hat{E}_T \left[ R_{(t),t+j}^{e} \right] \\
\text{(Estimated) price-level risk} &= - \sum_{j=1}^{J} \text{Cov}_T \left( \hat{\phi}_{t,t+j-1} \hat{M}_{t+j}, R_{(t),t+j}^{e} \right)
\end{align*}
\]

where \( R_{(t),t+j}^{e} \) is the time \( t+j \) post-formation excess return on portfolio formed at time \( t \). Of course, our identity is well-suited for going beyond the assumption of constant SDF parameters and the approach of testing the relation unconditionality. For instance, one can model the dy-
namics of returns and the SDF using a VAR to estimate the excess price levels and price-level risk conditionally. Here, we implement the unconditional version for a better comparison with the prototypical unconditional return regression.

Figure 1b plots the excess price level and the price-level risk for B/M-sorted portfolios formed in 1953m7–2003m12. As expected, the excess price levels of value stocks tend to be lower than those of growth stocks, as future cash flows of value stocks are more heavily discounted than those of growth stocks. Nevertheless, unlike the risk in one-month returns, value stocks do not necessarily have lower risk in the price level. In fact, the highest B/M portfolio has the second largest price-level risk, although the lowest B/M portfolio still has the highest price-level risk among all portfolios. Because of this, the relation between $-1$ times the excess price level and price-level risk is slightly positive, in contrast to the negative relation between beta and returns in the cross-sectional return plot. The portfolios are also closer to the 45 degree line that denotes the case when price-level risks perfectly explain the excess price levels.

This plot is a graphical way to see that the CAPM does a relatively good job describing the cross-section of average price levels of book-to-market-sorted portfolios, consistent with the finding of Cohen, Polk, and Vuolteenaho (2009) based on an approximate identity relating price level to price-level risk. However, the pricing errors defined as the vertical distance between the portfolio and the 45 degree line (or equivalently, horizontal distance between the portfolio and the 45 degree line) are negative for value portfolios and positive for growth portfolios. The cross-sectional “price” regression in the next section shows that value portfolios are significantly underpriced than growth portfolios when we restrict $b_1$ to be the in-sample realized value and not significant when we allow $b_1$ to be estimated in the cross-section.
4.3 Components of price-level risk

Why do value stocks have a high price-level risk despite having a low market exposure immediately after portfolio formation? Given the price-level risk decomposition

\[- \sum_{j=1}^{\infty} Cov_t \left( \phi_{t,t+j-1} M_{t+j}, R^e_{t+j} \right) = - \sum_{j=1}^{\infty} E_t \left[ \phi_{t,t+j-1} - E_t \left[ \phi_{t,t+j-1} \right] \right] Cov_t \left( M_{t+j}, R^e_{t+j} \right)\]

Discounted sum of contemporaneous risk

\[- \sum_{j=1}^{\infty} E_t \left[ \left( \phi_{t,t+j-1} - E_t \left[ \phi_{t,t+j-1} \right] \right) \left( M_{t+j} - E_t \left[ M_{t+j} \right] \right) \left( R^e_{t+j} - E_t \left[ R^e_{t+j} \right] \right) \right] \]

Intertemporal correction

\[- \sum_{j=1}^{\infty} E_t \left[ \left( \phi_{t,t+j-1} \right) - E_t \left[ \phi_{t,t+j-1} \right] \right] \left( R^e_{t+j} - E_t \left[ R^e_{t+j} \right] \right) \]

Residual price-level risk

what is the relative contribution of the discounted premia for contemporaneous risk and residual price-level risk for the total price-level risk?

Figure 2a shows that the discounted sum of premia for contemporaneous risk is the main component of price-level risk. This component is the highest for the highest B/M portfolio because the market betas of value portfolios cross those of growth portfolio around 6 years following portfolio formation and stay higher (Figure 2b), as was also noted in CPV. That is, the misalignment of returns and risk tends to be temporal and is corrected afterward, making the price level and price-level risk line up reasonably well.

The component that corrects the discounted sum of contemporaneous risks for the possible intertemporal covariance between contemporaneous risk and realized duration contributes positively to total price-level risk and tends to be similar across different portfolios. As explained earlier, the last residual component of price-level risk captures the intertemporal covariance between realized duration and excess returns. This component is large and positive for the growth portfolio and negative for higher B/M portfolios, contributing importantly to the cross-sectional variation in price-level risk. Understanding the economic mechanism for this variation, which does not arise in cross-sectional return regressions, would to be an important task for price level analysis going forward.
4.4 Other characteristics

How does the cross-section of returns compare to the cross-section of prices for portfolios sorted by other characteristics? Figure 3 shows that over our sample period, all characteristic sortings except the size sort lead to a negative cross-sectional relation between excess returns and market betas, similar to the B/M sort. And even the size sort leads to a much smaller variation in market betas that is difficult to reconcile with the large variation in returns.

Figure 4 shows that the cross-sectional fit tends to improve when looking at prices. In all but the investment sort, higher price-level risk tends to be associated with a lower excess price level such that the price levels line up with price-level risk exposure to the market factor in the right direction. A closer look at the portfolios reveals additional interesting patterns. Among the momentum-sorted portfolios, the low momentum (decile 1) portfolio has the lowest excess price level, despite it having very low returns in the month immediately after portfolio formation. Similarly, the low profitability portfolio (decile 1) has the lowest excess price level among all profitability-sorted portfolios. This suggests that low momentum or low profitability stocks are relatively undervalued at the time of portfolio formation due to reasons such as overreaction to negative news but generate negative abnormal returns in the short run as the overreaction exacerbates.

On the other hand, the price-level analysis also reveals where the CAPM does poorly in explaining asset price levels. First, portfolio price levels tend to be below the 45 degree line, with the exception of the smaller portfolios in the size sort. This fact suggests either that the risk adjustment based on the CAPM is misspecified or that estimating price-level risk based on unconditional covariance rather than conditional leads to an upper bias in the estimated risk. However, cross-sectional price regressions in the next section show that this intercept component of mispricing tends to be statistically insignificant. Second, risk does not explain prices well in the cross-section of investment-sorted portfolios. This suggests that firm investment is a signal for fundamental mispricing with respect to the CAPM, as argued in Polk and Sapienza (2006).
5 Cross-sectional “Price” Regression

Having seen the graphical evidence, we now formally test if characteristic-sorted portfolios have statistically significant mispricings (pricing errors) with respect to the CAPM. We do this with a cross-sectional price regression that mirrors the conventional cross-sectional return regression based on unconditional moments,

\[
c + \tilde{\delta}_i = -\sum_{j=1}^{J} E \left[ \phi_{i,t,t+j-1} M_{t+j} \right] E \left[ R^e_{i,t+j} \right] + \left\{ -\sum_{j=1}^{J} \text{Cov} \left( \phi_{i,t,t+j-1} M_{t+j}, R^e_{i,t+j} \right) \right\}
\]

\[= -\sum_{j=1}^{J} E \left[ M_{t,t+j-1} \frac{P_{t+j-1}}{P_t} M_{t+j} R^e_{i(t),t+j} \right].
\]

The expression allows for an intercept \(c\) to absorb shifts in \(\delta\) that is common across all portfolios.

Eq. (37) reduces to the familiar cross-sectional return regression if we choose \(J = 1\). In this case, \(M_{t,t+j-1} \frac{P_{t+j-1}}{P_t} = 1\) so that \(c + \tilde{\delta}_i = -E \left[ M_{t+j} R^e_{i,t+1} \right]\). Rearranging and letting \(\lambda_M = \text{Var}(M_{t+1}) / E[M_{t+1}], \beta_{i,M} = -\frac{\text{Cov}(M_{t+1}, R^e_{i,t+1})}{\text{Var}(M_{t+1})}, \lambda_0 = -\frac{c}{E[M_{t+1}]}, \text{and } \alpha_i = -\frac{\tilde{\delta}_i}{E[M_{t+1}]},\)

\[E \left[ R^e_{i,t+1} \right] = \alpha_i + \lambda_0 + \lambda_M \beta_{i,M}.
\]

Our baseline specification uses \(J = 180\) months, and we show how the result changes with smaller values of \(J\).

5.1 The GMM

We estimate Eq. (37) using the general method of moments (GMM). We again specify

\[M_{t+j} = 1 - b_1 R^e_{m,t+j};\]

and use zero pricing error as the moment condition for each portfolio:

\[\tilde{\delta}_i(c, b_1) = -c - \sum_{j=1}^{J} E \left[ M_{t,t+j-1} \frac{P_{t+j-1}}{P_t} M_{t+j} R^e_{i(t),t+j} \right] = 0.
\]
We find values that solve
\[
\min_{c,b_1} \sum_{i=1}^{N} \delta_i^2 (c, b)
\]  \hspace{1cm} (41)
and use the Bartlett kernel used by Newey and West (1987) with a bandwidth of \( J \) months to estimate the spectral density matrix, allowing for serial correlation due to overlapping samples as well as contemporaneous correlations among the portfolios. The variance-covariance matrix for parameter estimates is given by
\[
V_b = \frac{1}{T} (D' D)^{-1} D' S D (D' D)^{-1},
\]  \hspace{1cm} (42)
where \( S \) is the estimated spectral density matrix and \( D \) is the matrix representing the estimated derivative of the moments with respect to changes in the parameters. The variance-covariance matrix of pricing errors is given by
\[
V_{\delta} = \frac{1}{T} \left( I - D (D' D)^{-1} D' \right) S \left( I - D (D' D)^{-1} D' \right).
\]  \hspace{1cm} (43)

5.2 Cross-section of prices of B/M portfolios

Table 3 shows the result for 10 value-weighted B/M portfolios across alternative choices for the cutoff \( J \) (up to \( J = 180 \) months) (Panel A). When \( J = 1 \), the result simplifies to the cross-sectional return regression, which shows that the coefficient \( b_1 \) that best explains the cross-sectional variation in returns is negative. The last three columns show either the market factor volatility or the mean that generates \( b_1 = \) factor mean/\( \text{(factor volatility)}^2 \) if we were to use in-sample values for the other. Since the estimated \( b_1 \) is negative, there is no factor volatility that generates \( b_1 \). The market risk premium that generates \( b_1 \) given the in-sample market volatility of 0.0437 is \(-1.1\%\).

From around \( J = 36 \) months (3 years), however, the cross-sectional price regression implies a positive and significant \( b_1 \). The annualized market risk premium that explains the cross-section of prices at \( J = 15 \) years is 13.8\%, which is high but in our opinion not implausible. The results for \( J = 10 \) and 15 years are similar, which supports our assumption that the discounted sum of
abnormal returns beyond 10 or 15 years,

\[- \sum_{j=J+1}^{\infty} E_t \left[ \frac{P_{t+j-1}}{P_t} - E_{t+j-1} \left[ M_{t+j} R^e_{t+j} \right] \right], \quad (44)\]

is small. The point estimates of $\tilde{\delta}$ shows that growth portfolios are overpriced by 12% and value portfolios are underpriced by 5.2%, but these values, as well as their difference of 17.2%, are statistically insignificant. The intercept of 28% is significantly positive, suggesting that the CAPM model does an adequate though not a perfect job in explaining the cross-section of prices.

Panel B adds the restriction that the slope coefficient on the SDF is the in-sample value 3.054, which corresponds to the case we saw with figures in the previous section. The restriction makes the intercept insignificant but implies that high B/M portfolios are significantly underpriced, with the highest B/M value portfolio being underpriced by 10%. The difference in the pricing error between the growth and the value portfolios is 25.4% and statistically significant.

Panel C uses the alternative restriction that the intercept is zero. This restriction brings the estimated $b_1$ of 2.86 closer to the in-sample value, suggesting that adding this theoretical restriction may increase the efficiency of the estimator. The restriction again implies that the value portfolio is significantly underpriced (by 9.7%) and that the difference in the pricing error between the growth and the value portfolios is 26.5% and statistically significant.

5.3 Comparison to Cohen, Polk, and Vuolteenaho (2009)

Cohen, Polk, and Vuolteenaho (2009) (CPV) similarly study the cross-section of price levels of B/M sorted portfolios. They find that with a high annual market risk premium of 21.4%, their price-level measure of mispricing (“price-level alpha”) based on 180 months of post-formation returns has a difference of 12.1% between the growth and the value portfolios and that this difference is statistically insignificant (standard error of 38.1%) (Table III on p.2755). The estimated price-level alphas tend to be non-monotonic and are the highest for decile 5 (19.2% compared to $-0.5\%$ in decile 1) and lowest in decile 8 ($-16.9\%$ compared to $-12.6\%$ in decile 10).

Our results clearly highlight that price-level regressions based on our new identity are better suited for a careful analysis of asset price levels. Equipped with the new identity, we show that the
cross-sectional variation in price levels of B/M portfolios can be explained with a lower implied market risk premium and substantially smaller standard errors. The cross-sectional pricing errors tend to be monotonic, and the difference in the pricing error between the growth and the value portfolio is 17.2% with a standard error of 10.3%. The significant improvement in statistical power, which later allows us to reject the CAPM model for investment-sorted portfolios, and the greater observed efficiency can be attributed to two important differences between our identity and that of CPV, also discussed earlier in Section 2.7. First, our identity relates mispricing \( \delta \) to observed subsequent returns and does not require estimating unobserved quantities. In contrast, the price regression in CPV relates their measure of price level to price-level risk measured by the market exposures that post-formation returns would have in the absence of mispricing, which they estimate using the exposure of a portfolio’s return on equity to the market’s return on equity. Second, our identity is exact, but the identity in CPV assumes that return volatility in the presence of mispricing is similar to that in the absence of mispricing and also relies on the assumptions on which the Campbell-Shiller (1988) identity is derived.4

5.4 Other characteristics

Table 4 reports the cross-sectional price regression result for four other characteristics: size, momentum, profitability, and investment. As observed graphically, the one-period return regression implies a negative price of risk for all characteristic sorts other than size. In contrast, by \( J = 10 \) years, all characteristic sorts imply a positive and statistically significant price of risk. The intercept term is significant for the size and investment sorts but insignificant for the momentum and profitability sorts.

Among other characteristics, momentum shows the most interesting patterns in mispricing relative to the CAPM. Although the high momentum (top decile) portfolio generates a large positive abnormal return with respect to CAPM in the month following portfolio formation, the portfolio is significantly overpriced, not underpriced. This fact suggests that momentum stocks are already overpriced at the time of portfolio formation but generate a positive abnormal return as they become even more overpriced. On the other hand, the low momentum (bottom decile) portfolio is estimated to be slightly overpriced, consistent with its large negative abnormal return.

in the month after portfolio formation. This finding suggests that the momentum phenomena is more nuanced than current explanations put forth in the literature. The high momentum phenomenon appears to be an outcome of overreaction to positive news, whereas the low momentum phenomenon appears to be more consistent with underreaction to negative news.

The cross-section of prices of investment-sorted portfolios is also revealing. Those results show that we need a very high annualized market risk premium of 16.5% to explain the cross-sectional dispersion in the price levels of investment and that even with this risk premium, high investment stocks are significantly overpriced. This result echoes the finding of Polk and Sapienza (2006) that the investment characteristic may be the strongest signal for price-level mispricing in the stocks with respect to the CAPM model.

5.5 Longer sample period

How well do price levels line up with price-level risk in a longer sample? For size and momentum characteristics, the CRSP monthly data allow us to go as far back as 1926 to form portfolios. For the B/M ratio, we use the book equity data provided by Davis, Fama, and French (2000) to extend the sample. The resulting data are overlapping samples of post-formation returns for portfolios formed in 1926m6–2003m12 (post-formation returns over 1926m7–2018m12).

Consistent with our results for the baseline sample period, the price of risk that explains the cross-sectional variation in price levels is positive, and the estimated SDF loading $b_1$ is very stable across the characteristic sorts. The pricing errors tend to be statistically insignificant, and although the pricing error of high momentum stocks are slightly positive, it is small and not statistically significant. Hence, the overreaction interpretation of the positive momentum phenomenon applies only to the more recent period. This result is consistent with the findings of Lou and Polk (2019) who argue that arbitrageurs trading momentum can cause the anomaly to transition from an underreaction to an overreaction phenomenon.
Comparing $\delta$ to Long-Run Return Measures

Measures such as the cumulative abnormal returns (CAR),

$$\text{CAR} = - \sum_{j=1}^{J} E_t [\alpha_{t+j}] ,$$

proxy for the degree to which the assets generate abnormal returns over a long period of time following a corporate or market event of interest but do not have a clear theoretical interpretation. (The convention is to not have a negative sign before the sum, but we put it there for an easier comparison to $\delta$.) Given that our mispricing measure $\delta_t = (P_t - V_t) / P_t$ also takes the form of a discounted sum of future abnormal returns, how well does CAR proxy for the initial mispricing of the assets?

Figure 5a plots the estimated $\delta$s based on the in-sample value of the SDF loading $b_1$ and CARs of B/M sorted portfolios with $J = 180$ months. Whereas $\delta$ tends to fall smoothly and monotonically with the B/M ratio, the variation in CAR is both large and less monotonic in the cross-section of B/M deciles. Most strikingly, there is a large level difference between $\delta$s and CARs, with the latter being 20 to 40% lower than $\delta$.

It is interesting to ask whether we can do better by introducing a constant discount factor to the CAR formula to obtain a discounted CAR (“DCAR”) with $\rho = 0.9^{1/12}$,

$$\text{DCAR} = - \sum_{j=1}^{J} \rho^{j-1} E_t [\alpha_{t+j}] ,$$

which we showed to be a crude approximation of $\delta$ in Section 2.7. We find that DCAR traces the cross-sectional variation in $\delta$ better, but there is still a substantial level difference between $\delta$s and DCAR of about 20% (Figure 5b). Hence, DCAR can be useful in inferring the cross-sectional difference in $\delta$ based on a simple formula but a portfolio’s DCAR alone does not have a straightforward interpretation as mispricing.

The large level difference between $\delta$ and the two CAR measures arises due to the condition-
ality of abnormal returns. To see this, recall that $\delta$ can be written as

$$\delta_t = -\sum_{j=1}^{J} E_t \left[ M_{t,t+j-1} \frac{P_{t+j-1}}{P_t} \frac{\alpha_{t+j}}{1+R_{f,t+j}} \right],$$

where $M_{t,t+j-1} \frac{P_{t+j-1}}{P_t}$ is realized duration. Hence, the positive upward shift in $\delta$ compared to CAR measures implies that realized duration tends to predict conditional abnormal returns with a negative sign, which works as a positive correction. We suspect that this in part arises from the long-run reversal (De Bondt and Thaler 1985); past capital gains tend to predict returns with a negative sign, and if part of this predictability arises due to conditional abnormal return rather than market exposure, it contributes positively to $\delta$.

### 7 Conclusion

Our novel model misspecification measure, delta, precisely links future alpha to current price-level deviations. In stark contrast to existing flawed measures, our approach correctly recognizes that abnormal returns occurring sooner, in more valuable states, or after relatively large capital gains should be associated with larger price-level distortions. We find that the CAPM deltas of size-, book-to-market-, momentum-, and profitability-sorted portfolios are statistically insignificant while investment-sorted portfolios generate economically large price-level distortions that are precisely measured. By providing an exact metric of the extent to which a candidate asset-pricing model is approximately right, our new approach along with its associated empirical results could ultimately help economists distinguish among risk-based, behavioral-based, and institutional-friction-based explanations for well-known empirical patterns in markets. More broadly, our identity for price-level mispricing could have fruitful applications to detecting and studying the bubbles and crashes in the aggregate financial market.
References


Table 1: Post-formation Returns: An Illustration

The table describes our three-dimensional data structure. Our data consist of overlapping samples of returns over 180 post-formation months on portfolios formed in 607 different months (1953m7–2003m12) for 10 different characteristic deciles.

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Formation month $t$</th>
<th>Calendar month $t + j$</th>
<th>Number of post-formation months $j$</th>
<th>Return</th>
<th>Capital gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>BM1</td>
<td>1953m6</td>
<td>1953m7</td>
<td>1</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>BM1</td>
<td>1953m6</td>
<td>1953m8</td>
<td>2</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>BM1</td>
<td>1953m6</td>
<td>1968m6</td>
<td>180</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>BM1</td>
<td>1954m7</td>
<td>1964m8</td>
<td>1</td>
<td>...</td>
<td>...</td>
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<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>BM1</td>
<td>2003m12</td>
<td>2018m12</td>
<td>180</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>BM2</td>
<td>1953m6</td>
<td>1953m7</td>
<td>1</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>BM10</td>
<td>2003m12</td>
<td>2018m12</td>
<td>180</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>
Table 2: Descriptive Statistics

Panel A reports descriptive statistics for the monthly market factor and monthly risk-free rates over 1953m6–2003m12 and over 1953m7–2018m12. Both the market factor and the risk-free rate measured by 1 month Treasury bill rate are from Ken French’s data library. \( \hat{b}_1 = \frac{R^e_m}{\sigma^2(R^e_m)} \) is the approximate slope coefficient that the stochastic discount factor has on the market factor. Panel B reports the average excess return, standard deviation of excess return, and average gross capital gain for different portfolio formation periods (1 month, 2 to 60 months, 61 to 180 months) and B/M deciles.

<table>
<thead>
<tr>
<th>Sample period</th>
<th>( \overline{R}_m )</th>
<th>( \sigma(R^e_m) )</th>
<th>( \overline{R}_f )</th>
<th>Sharpe ratio</th>
<th>( \hat{b}_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1953m6-2003m12</td>
<td>0.0058</td>
<td>0.0437</td>
<td>0.0043</td>
<td>0.133</td>
<td>3.054</td>
</tr>
<tr>
<td>1953m7-2018m12</td>
<td>0.0060</td>
<td>0.0429</td>
<td>0.0035</td>
<td>0.139</td>
<td>3.245</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>B/M decile</th>
<th>Low BM</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>High BM</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \overline{R}_{t+1} )</td>
<td>0.0050</td>
<td>0.0062</td>
<td>0.0062</td>
<td>0.0059</td>
<td>0.0059</td>
<td>0.0076</td>
<td>0.0076</td>
<td>0.0067</td>
<td>0.0094</td>
<td>0.0096</td>
</tr>
<tr>
<td>( \overline{R}_{t+j}: j = 2, ..., 60 )</td>
<td>0.0049</td>
<td>0.0053</td>
<td>0.0057</td>
<td>0.0051</td>
<td>0.0061</td>
<td>0.0062</td>
<td>0.0067</td>
<td>0.0073</td>
<td>0.0078</td>
<td>0.0088</td>
</tr>
<tr>
<td>( \overline{R}_{t+j}: j = 61, ..., 180 )</td>
<td>0.0048</td>
<td>0.0054</td>
<td>0.0052</td>
<td>0.0053</td>
<td>0.0054</td>
<td>0.0056</td>
<td>0.0055</td>
<td>0.0060</td>
<td>0.0064</td>
<td>0.0055</td>
</tr>
<tr>
<td>( \sigma(R^e_{t+1}) )</td>
<td>0.0520</td>
<td>0.0477</td>
<td>0.0469</td>
<td>0.0467</td>
<td>0.0441</td>
<td>0.0442</td>
<td>0.0421</td>
<td>0.0443</td>
<td>0.0457</td>
<td>0.0496</td>
</tr>
<tr>
<td>( \sigma(R^e_{t+j}: j = 2, ..., 60) )</td>
<td>0.0492</td>
<td>0.0471</td>
<td>0.0461</td>
<td>0.0455</td>
<td>0.0438</td>
<td>0.0448</td>
<td>0.0437</td>
<td>0.0437</td>
<td>0.0446</td>
<td>0.0488</td>
</tr>
<tr>
<td>( \sigma(R^e_{t+j}: j = 61, ..., 180) )</td>
<td>0.0454</td>
<td>0.0469</td>
<td>0.0473</td>
<td>0.0466</td>
<td>0.0461</td>
<td>0.0473</td>
<td>0.0465</td>
<td>0.0471</td>
<td>0.0479</td>
<td>0.0511</td>
</tr>
<tr>
<td>( \overline{P}<em>{t+j}/\overline{P}</em>{t+j-1}: j = 1, ..., 60 )</td>
<td>1.0074</td>
<td>1.0072</td>
<td>1.0072</td>
<td>1.0064</td>
<td>1.0071</td>
<td>1.0070</td>
<td>1.0073</td>
<td>1.0077</td>
<td>1.0079</td>
<td>1.0095</td>
</tr>
<tr>
<td>( \overline{P}<em>{t+j}/\overline{P}</em>{t+j-1}: j = 61, ..., 180 )</td>
<td>1.0068</td>
<td>1.0068</td>
<td>1.0065</td>
<td>1.0063</td>
<td>1.0064</td>
<td>1.0064</td>
<td>1.0062</td>
<td>1.0066</td>
<td>1.0070</td>
<td>1.0064</td>
</tr>
</tbody>
</table>

35
The table reports the outcome of the cross-section price regression with respect to the CAPM model for B/M-sorted portfolios formed in 1953m6–2003m12. We track post-formation returns up to 15 years, covering 1953m7–2018m12. The regression minimizes the squared sum of cross-sectional deviations in portfolio excess price levels from price-level risk:

$$\min_{c,b_1} \sum_{i=1}^{N} \tilde{\delta}_i^2 (c, b_1)$$

where

$$\tilde{\delta}_i (c, b_1) = -\sum_{j=1}^{J} E \left( \phi_{t,T+j-1} M_{t+j} \right) E \left( R_{t,T+j}^e \right) + \left\{ -\sum_{j=1}^{J} \text{Cov} \left( \phi_{t,T+j-1} M_{t+j}, R_{t,T+j}^e \right) \right\} - c = -\sum_{j=1}^{J} E \left( \phi_{t,T+j-1} M_{t+j} R_{t,T+j}^e \right) - c,$$

$$\phi_{t,T+j-1} = \left( \prod_{s=1}^{J-1} M_{t+s} \right) \frac{R_{t,T+j}}{R_{t,T}}$$
is realized duration of the portfolio, and $$M_{t+j} = 1 - b_1 R_{m,t+j}^e$$ is one-period stochastic discount factor that depends linearly on the market factor. When $$J = 1$$, the regression reduces to a conventional cross-sectional return regression. As $$J$$ gets larger, the regression becomes a price-level regression. The table reports the estimated cross-sectional pricing errors for different deciles of the B/M-sorted portfolios, estimated parameters, and monthly market return volatility $$\sigma_m^2$$ and premium $$\lambda_m$$ implied by $$b_1 = \lambda_m / \sigma_m^2$$ and the in-sample value of the premium (to infer volatility) or volatility (to infer premium). In the parentheses are standard errors estimated based on the Bartlett kernel used by Newey and West (1987) with a bandwidth of $$J$$ months.

**Panel A: No restriction**

|  | Lo BM | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Hi BM | 1 – 10 | Parameters | Implied risk and premium |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| J |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 1mo | 0.000 | -0.000 | 0.000 | 0.001 | 0.001 | -0.000 | 0.000 | 0.001 | -0.002 | -0.003 | 0.003 | -0.02 | -5.84 | NaN | 0.011 | 0.134 |
|  | (0.001) | (0.001) | (0.001) | (0.001) | (0.001) | (0.001) | (0.001) | (0.001) | (0.001) | (0.001) | (0.001) | (0.01) | (3.37) | | |
| 1yr | 0.028 | 0.014 | 0.013 | 0.014 | 0.011 | -0.009 | -0.003 | -0.006 | -0.029 | -0.034 | 0.063 | -0.05 | 1.33 | 0.067 | 0.003 | 0.030 |
|  | (0.011) | (0.006) | (0.006) | (0.005) | (0.006) | (0.004) | (0.005) | (0.008) | (0.011) | (0.020) | | | (0.05) | (1.86) | | |
| 3yrs | 0.077 | 0.055 | 0.033 | 0.028 | -0.001 | -0.013 | -0.024 | -0.046 | -0.062 | 0.139 | 0.08 | 4.13 | 0.038 | 0.008 | 0.094 |
|  | (0.031) | (0.019) | (0.015) | (0.010) | (0.012) | (0.010) | (0.017) | (0.016) | (0.024) | (0.055) | | | (0.06) | (1.10) | | |
| 5yrs | 0.105 | 0.077 | 0.041 | 0.021 | -0.009 | -0.022 | -0.037 | -0.060 | -0.056 | 0.161 | 0.21 | 5.51 | 0.033 | 0.011 | 0.126 |
|  | (0.053) | (0.033) | (0.022) | (0.010) | (0.014) | (0.012) | (0.017) | (0.029) | (0.026) | (0.029) | (0.081) | | | (0.06) | (1.15) | | |
| 10yrs | 0.119 | 0.083 | 0.042 | 0.010 | -0.010 | -0.026 | -0.041 | -0.060 | -0.065 | -0.053 | 0.172 | 0.28 | 5.92 | 0.032 | 0.011 | 0.135 |
|  | (0.074) | (0.041) | (0.027) | (0.006) | (0.015) | (0.016) | (0.025) | (0.024) | (0.033) | (0.029) | (0.103) | | | (0.09) | (1.29) | | |
| 15yrs | 0.120 | 0.084 | 0.042 | 0.009 | -0.010 | -0.027 | -0.041 | -0.060 | -0.065 | -0.052 | 0.172 | 0.28 | 6.01 | 0.031 | 0.011 | 0.138 |
|  | (0.076) | (0.041) | (0.024) | (0.004) | (0.013) | (0.018) | (0.026) | (0.034) | (0.035) | (0.027) | (0.103) | | | (0.09) | (1.23) | | |
Cross-section of Prices of B/M Portfolios (Continued)

**Panel B: Restrict \( b_1 = 3.054 \)**

<table>
<thead>
<tr>
<th>( J )</th>
<th>Lo BM</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>Hi BM</th>
<th>1 − 10</th>
<th>Intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td>1mo</td>
<td>0.003</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>-0.001</td>
<td>-0.001</td>
<td>-0.000</td>
<td>-0.003</td>
<td>0.005</td>
<td>-0.003</td>
<td>-0.000</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.002)</td>
<td>(0.000)</td>
<td></td>
</tr>
<tr>
<td>1yr</td>
<td>0.032</td>
<td>0.018</td>
<td>0.013</td>
<td>0.015</td>
<td>0.007</td>
<td>-0.010</td>
<td>-0.007</td>
<td>-0.010</td>
<td>-0.028</td>
<td>-0.031</td>
<td>0.063</td>
<td>-0.001</td>
</tr>
<tr>
<td></td>
<td>(0.014)</td>
<td>(0.007)</td>
<td>(0.006)</td>
<td>(0.005)</td>
<td>(0.006)</td>
<td>(0.007)</td>
<td>(0.006)</td>
<td>(0.007)</td>
<td>(0.010)</td>
<td>(0.021)</td>
<td>(0.002)</td>
<td></td>
</tr>
<tr>
<td>3yrs</td>
<td>0.073</td>
<td>0.051</td>
<td>0.035</td>
<td>0.034</td>
<td>0.002</td>
<td>-0.010</td>
<td>-0.020</td>
<td>-0.042</td>
<td>-0.047</td>
<td>-0.076</td>
<td>0.148</td>
<td>-0.01</td>
</tr>
<tr>
<td></td>
<td>(0.034)</td>
<td>(0.020)</td>
<td>(0.013)</td>
<td>(0.012)</td>
<td>(0.014)</td>
<td>(0.011)</td>
<td>(0.012)</td>
<td>(0.019)</td>
<td>(0.015)</td>
<td>(0.020)</td>
<td>(0.049)</td>
<td></td>
</tr>
<tr>
<td>5yrs</td>
<td>0.103</td>
<td>0.075</td>
<td>0.050</td>
<td>0.049</td>
<td>0.001</td>
<td>-0.011</td>
<td>-0.034</td>
<td>-0.059</td>
<td>-0.075</td>
<td>-0.101</td>
<td>0.204</td>
<td>-0.01</td>
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<tr>
<td></td>
<td>(0.051)</td>
<td>(0.031)</td>
<td>(0.013)</td>
<td>(0.013)</td>
<td>(0.018)</td>
<td>(0.016)</td>
<td>(0.025)</td>
<td>(0.012)</td>
<td>(0.021)</td>
<td>(0.023)</td>
<td>(0.071)</td>
<td></td>
</tr>
<tr>
<td>10yrs</td>
<td>0.134</td>
<td>0.083</td>
<td>0.054</td>
<td>0.044</td>
<td>0.003</td>
<td>-0.011</td>
<td>-0.044</td>
<td>-0.061</td>
<td>-0.098</td>
<td>-0.104</td>
<td>0.238</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>(0.073)</td>
<td>(0.040)</td>
<td>(0.021)</td>
<td>(0.009)</td>
<td>(0.015)</td>
<td>(0.016)</td>
<td>(0.022)</td>
<td>(0.035)</td>
<td>(0.033)</td>
<td>(0.025)</td>
<td>(0.096)</td>
<td></td>
</tr>
<tr>
<td>15yrs</td>
<td>0.154</td>
<td>0.084</td>
<td>0.054</td>
<td>0.038</td>
<td>0.001</td>
<td>-0.014</td>
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**Panel C: Restrict \( c = 0 \)**

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Table 4: Cross-section of Prices of Portfolios Sorted by Other Characteristics

The table reports the outcome of the cross-section price regression with respect to the CAPM model for characteristic sorted portfolios formed in 1953m6–2003m12. We track post-formation returns up to 15 years, covering 1953m7–2018m12. Size is market capitalization, momentum is cumulative return over month -12 to -1, and profitability and investment are operating profitability and asset growth. Table 3 describes further details on the price regression and the reported values. In the parentheses are standard errors based on the Bartlett kernel used by Newey and West (1987) with a bandwidth of \(J\) months.

### Panel A: Size

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Cross-section of Prices of Portfolios Sorted by Other Characteristics (Continued)

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Table 5: Cross-section of Prices: Longer Sample Period

The table reports the outcome of the cross-section price regression with respect to the CAPM model for B/M, size, and momentum sorted portfolios formed in 1926m6–2003m12. We track post-formation returns up to 15 years, covering 1926m7–2018m12. Table 3 describes further details on the price regression and the reported values. In the parentheses are standard errors based on the Bartlett kernel used by Newey and West (1987) with a bandwidth of \( J \) months.

### Panel A: Book to market

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Cross-section of Prices: Longer Sample Period (Continued)

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Figure 1: Cross-section of Returns and Price Levels of B/M Portfolios

The first figure plots the cross-sectional relation between realized mean excess return and mean excess return predicted by the CAPM model for 10 B/M-sorted portfolios for portfolios formed in 1953m6–2003m12 (returns 1953m7–2004m1). The second figure plots the cross-sectional relation between realized excess price level and estimated price-level risk for the same portfolios based on 180 post-formation returns for each portfolio formation month (overlapping samples of post-formation returns cover 1953m7–2018m12). The vertical distance from the 45 degree line represents an abnormal return $\alpha$ in the first figure and pricing error $\delta$ in the second figure, where

$$
\delta = - \sum_{j=1}^{J} E[\phi_{i,t+j-1}M_{t+j}] E[R_{i,t+j}] + \left\{ - \sum_{j=1}^{J} \text{Cov}(\phi_{i,t+j-1}M_{t+j}, R_{i,t+j}) \right\},
$$

$$
\phi_{i,t+j-1} = \left( \prod_{s=1}^{j-1} M_{t+s} \right) \frac{R_{i,t+j}}{\bar{R}_{ij}}, M_{t+j} = 1 - b_{1}R_{m,t+j}, \text{and } b_{1} = \frac{\lambda_{m}}{\sigma_{m}^{2}} \text{ uses the in-sample value reported in Table 2.}
$$

The vertical axis in the second figure is $-1$ times the excess price level, which is negative for all portfolios.
These figures describe the price-level risk exposure to the market factor for 10 portfolios sorted by B/M ratio in 1953m6–2003m12 based on overlapping samples of post-formation returns over 1953m7–2018m12. The first figure shows that a large fraction of price-level market exposure is due to the discounted sum of contemporaneous risk exposures:

\[-\sum_{j=1}^{\infty} \text{Cov}_t (\phi_{t,j-1} \text{ } M_{t+j}, R_{t+j}^e) = -\sum_{j=1}^{\infty} E_t [\phi_{t,j-1}] \text{Cov}_t (M_{t+j}, R_{t+j}^e)\]

Price-level risk

\[-\sum_{j=1}^{\infty} E_t \left[ (\phi_{t,j-1} - E_t [\phi_{t,j-1}]) (M_{t+j} - E_t [M_{t+j}]) (R_{t+j}^e - E_t [R_{t+j}^e]) \right]\]

Discounted sum of contemporaneous risk

\[-\sum_{j=1}^{\infty} E_t \left[ (\phi_{t,j-1} - E_t [\phi_{t,j-1}M_{t+j}]) (R_{t+j}^e - E_t [R_{t+j}^e]) \right]\]

Intertemporal correction

\[-\sum_{j=1}^{\infty} E_t \left[ (\phi_{t,j-1}E_t [M_{t+j}]) (R_{t+j}^e - E_t [R_{t+j}^e]) \right]\]

Residual price-level risk

The second figure describes why the high B/M (value) portfolio has a higher discounted sum of contemporaneous risk than the low B/M (growth) portfolio, despite the former having a smaller market exposure in the month after portfolio formation. It is because the value portfolio’s market beta grows over post-formation periods, whereas the growth portfolio’s market beta declines quickly after portfolio formation.
Figure 3: Cross-section of Returns of Other Portfolios

The figures show that the cross-section of mean excess returns on size, momentum, operating profitability (“profitability”), and asset growth (“investment”) sorted portfolios over 1953m7–2004m1 (portfolio formation 1953m6–2003m12) tend not to have a positive relation to the level predicted by their exposure to the market factor.
The figures show that the cross-section of excess price levels of size, momentum, operating profitability (“profitability”), and asset growth (“investment”) sorted portfolios over 1953m7–2004m1 (overlapping samples of post-formation returns over 1953m7–2018m12) tend to fall with price-level risk exposure to the market factor.
The two figures compare estimated mispricing \( \delta \) to the cumulative abnormal return (CAR),

\[
CAR = - \sum_{j=1}^{J} E_t [\alpha_{t+j}],
\]

and against the discounted CAR (DCAR),

\[
DCAR = - \sum_{j=1}^{J} \rho^j E_t [\alpha_{t+j}]
\]

with \( \rho = 0.9^{1/12} \) and \( J = 180 \) months for ten portfolios sorted by the B/M ratio. These portfolios are formed in 1953m6–2003m12 and the abnormal returns are estimated separately for each post-formation month up to \( J = 180 \) months. We estimate the expected conditional abnormal returns simply using unconditional abnormal returns, which implicitly assumes that the conditional component of the abnormal return is not contemporaneously correlated with the market factor. The mispricing \( \delta \) is estimated based on the in-sample value of the SDF loading \( b_1 = E[R_m^e] / Var(R_m^e) \).