On the Long-term Interaction between Patent Screening and its Enforcement*

Gerard Llobet
CEMFI and CEPR

Álvaro Parra
UBC

Javier Suarez
CEMFI and CEPR

May 20, 2019

Abstract

In a dynamic model of innovation in which today's entrants are tomorrow's incumbents, we study the long-term outcomes emerging from the interaction between the ex-ante screening conducted by a patent office and the ex-post enforcement carried out by courts. The granting of a patent and its future enforcement provide incentives for firms to innovate. However, the enforcement of previous patents on which the firm might build on hinders innovation. To characterize the global effect of patent protection we solve for the long-run steady state equilibrium of the model and study the screening incentives faced by the patent office and the enforcement incentives faced by judges. Although a patent office and judges are assumed to maximize welfare, their incentives and instruments at their disposal differ. Whereas the patent office deals with every patent application, a judge deals with individual cases and only after an infringement claim has been made. We show that, when judges are expected to make mistakes, the patent office chooses to allow the patenting of low-quality easy-to-challenge innovations in order to induce entry, foster R&D activity, and improve long-term welfare. An individual judge perceives other judges as strategic substitutes. We show that depending on the patent office's screening level, judges may either exert more or less effort than the socially desirable level. In equilibrium, depending on the model parameters, the patent office and judges' effort may be strategic complements or strategic substitutes.

JEL Codes: L26, O31, O34.
Keywords: Intellectual Property, Innovation, Imitation, Patent Screening, Patent Enforcement, Industry Dynamics.

*This work is based on a previous paper entitled "Entrepreneurship Innovation, Patent Protection, and Industry Dynamics." Financial support from the European Commission (grant CIT5-CT-2006-028942) and the Spanish Ministry of Science and Innovation through the ECO2008-00801, the Consolider-Ingenio 2010 Project "Consolidating Economics" and the Regional Government of Madrid through grant S2015/HUM-3491 is gratefully acknowledged. Address for correspondence: CEMFI, Casado del Alisal 5, 28014 Madrid, Spain. Tel: +34-914290551. Fax: +34-914291056. Email: llobet@cemfi.es, suarez@cemfi.es.

1 Introduction

The proliferation of patents in highly technological markets makes entry of new firms difficult, among other reasons, because of the risk of infringing some patents. One example is the market for smartphones, in which producers are entangled in endless legal battles.[1] Some practitioners doubt about the effectiveness of the patent system in generating the right incentives to innovate and refer to this problem as the "tragedy of the anti-commons," describing strategic patenting and patent stacking as obstacles to innovation (Heller and Eisenberg (1998)).

Patent proliferation has been spurred by the strong protection of innovators' intellectual property rights (IP), especially in the United States. It has been argued that the creation of a unique Court of Appeals of the Federal Circuit in 1982, as well as the 1984's Semiconductors Act and the extension of patent duration to 20 years, strengthened the protection of IP. However, whether these reforms have really promoted innovation is theoretically and empirically controversial.[2] In fact, some quantitative assessments indicate that these reforms may have been detrimental to innovation (Levin *et al.* (1985), Hall and Ziedonis (2001)).

In this paper we study the effects of the protection of intellectual property rights in a tractable industry dynamics model. We focus on the effects on the speed of innovation and on social welfare in markets in which entrants face uncertainty on whether their product might infringe some of the existing IP rights. We consider an industry made up of a continuum of business niches where each niche can be thought of as a distinct product. The successful developers of improved versions of each product contribute to welfare and appropriate temporary monopoly profits like in a standard quality ladder model with limit pricing (Grossman and Helpman, 1991). These temporary monopolies are based on the protection granted by IP and are threatened by the entry of the developers of even better versions of the product (innovators) as well as imitators.

[1] See "The Great Patent Battle," *The Economist*, 10/21/2010.

[2] See Gallini (2002) for a review of the reforms and their effect on patenting activity.

The success of the genuine innovators is compromised by the competition coming from the opposition of incumbent monopolists, who use their IP to fight off entrants.[3] Patent protection against future innovation in this case constitutes a double-edged sword. Prospective entrants anticipate that in markets in which more incumbents hold patents conflicts with them will be more inevitable and the resulting profits will tend to be lower; this discourages their entry. However, if the firm can surmount this barrier, stronger protection extends incumbency and provides additional incentives to innovate.

Patent protection against imitation also has two effects on innovation. As in the previous case, it extends incumbency of genuine innovators who benefit from monopoly profits. Successful imitation, however, by eliminating some incumbent patent holders reduces the barriers to entry that future innovators face as they are more likely to access unprotected market niches.

In the paper we show that in a frictionless environment the First Best provides maximum protection against imitation and no protection against further innovation. Interestingly, when the incumbent is granted some protection against further innovation, it is optimal to reduce the protection against imitation. The reason is that successful imitation helps eliminate entrenched incumbents that might fend off future innovators.

The second part of the paper provides a model that endogenously determines the protection that patent holders obtain. In this model, a Patent Office decides the screening it carries out on the patent applications received. Insufficient screening translates into some imitators obtaining a patent that they can use to fight off the incumbent in the niche where they aim to operate. The conflict between these two firms as well as the conflict that may arise between an incumbent fighting a genuine innovator are resolved by a judge.

Each court case is decided by a (bayesian) judge aiming to find out whether the potential entrant is a genuine innovation or an innovator that was not screened out by the

[3] We assume that the strength of IP protection affects the incumbents' probability of expelling the innovators and imitators who challenge their business niches. This modeling allows us to abstract from the traditional distinction between patent length and patent breadth (Scotchmer (2004)).

Patent Office. Judges entertain a prior probability that an entrant is a genuine innovator based on the screening decision of the Patent Office. However, figuring out more precisely the nature of the entrant requires incurring costly effort that the judge weights in relation to the potential benefits. These benefits arise from avoiding Type-I and Type-II errors. Type-I errors arise when the judge's prior that the entrant is an imitator is high and should not be allowed but, instead, it is a genuine innovator that is deterred. Type-II errors arise when a judge's prior indicates that the entrant is likely to be an innovator who should replace the incumbent but, instead, it is an imitator that occupies the niche. Of course, these costs are a function of the decisions carried out by future judges in the same niche. This creates a dynamic problem for which we characterize the equilibrium level of effort and we provide conditions under which these efforts are strategic complements or substitutes.

Finally, we embed the decisions of the judges into the choice of the level of screening of the Patent Office. That is, the Patent Office can choose whether to engage in costly screening itself or, instead, rely on the work of future judges who will be later required to decide on specific cases. In those situations in which judges endogeneously exert high effort and, as a result, are likely to take the right decision the level of screening of bad patents will be lower.

The rest of the paper proceeds as follows. Section 2 introduces our baseline industry dynamics setup, analyzing its equilibrium and steady-state properties. It also explores the welfare implications of IP protection when the behavior of courts and the Patent Office is taken as exogenous. Section 3 studies the endogenous screening decision of the Patent Office while Section 4 analyzes the trade-offs that individual judges face and their equilibrium behavior. Section 5 generalizes the model and studies the robustness of our results. Section 6 concludes. All proofs are relegated to the appendix.

2 The Baseline Model

2.1 Set Up

We characterize the evolution of an industry in a discrete-time model with discount factor $\beta < 1$. This industry is comprised of a continuum of business niches of measure one. Each niche can be interpreted as the market of a different product.[4] Every niche is protected by a patent. Niches can be divided in two categories depending on the degree of competition. A monopolized niche is one in which the latest innovation was novel; i.e., the innovation substantially improved upon the existing product. In a monopolized niche the incumbent earns a per period profit flow of $\pi > 0$. A competitive niche is one in which the latest innovation turned out to be obvious; i.e., the innovation did not provide a meaningful improvement over the existing product (e.g., an imitation). Due to competition, firms participating in competitive niches earn zero profits.[5] At each date t we denote the proportion of monopolized niches by $x_t \in [0, 1]$.

Firms in monopolized niches might lose their incumbency status because they have been imitated by an obvious innovation or successfully replaced by an innovator that holds a patent for a superior substitute product. In every period t these firms arise from a measure e_t of potential entrants which develop a new product (innovation) and apply for a patent. Entry has a cost normalized to 1. With probability α the innovation of a potential entrant is novel. With probability $1 - \alpha$, the innovation is obvious. A *patent office* screens all applications and decides whether to grant a patent. We assume that every novel innovation is granted a patent, whereas a proportion λ of obvious products succeed in the application. Only innovations that receive a patent can enter a niche. Entrants only learn about the quality of their patent when they reach the market. The entry of these firms is untargeted; consequently, the probability that an entrant at t lands

[4] This simplification allows us to abstract from cross-product competition and to focus on competition related with concomitant and future entry into each niche.

[5] In Section 3 models competition in each niche as a quality ladder under price competition for a single unit of good (Grossman and Helpman, 1991). Under zero marginal cost of production, profits (and prices) are equal to the quality improvement brought to the market by the innovator. Novel innovations increase the quality by π and obvious innovators by zero.

in a monopolized niche is x_t.

Entry on a niche can be fought by filing an infringement claim. We assume that litigation is costless but, whenever indifferent, the incumbent does not file a lawsuit. This means that only incumbents that might obtain a future stream of profits—i.e., incumbents in monopolized niches—will engage in litigation. *Judges* review the infringement claims and make a probabilistic decision. If the judge rules in favor of the incumbent, the firm preserves its monopoly status and the entrant's innovation goes to waste. If the judge rules in favor of the entrant, it replaces the existing incumbent and receives a profit flow according to the quality of its innovation. When the entrant's innovation is novel, a judge rules in its favor with probability μ_1. Obvious entrants succeed with probability μ_0. We assume that $\mu_1 \geq \mu_0$. To ease the exposition, we initially model the variables governing the decisions of the patent office and judges as exogenous parameters. In later sections we endogenize the behavior of both types of agents.

Under these assumptions, the value of being the incumbent in a monopolized niche at date t—if it has not been infringed or infringement has been fend off in court—that we denote as v_t can be recursively written as

$$v_t = \pi + \beta[1 - e_{t+1}(\alpha\mu_1 + (1-\alpha)\lambda\mu_0)]v_{t+1}. \tag{1}$$

That is, the present value of the monopoly profits that a patent yields corresponds to a current flow of π and a discounted future value, βv_{t+1}, that is weighted by the probability that at date $t+1$ the incumbent surmounts the entry of both types of innovators. The law of motion for the proportion of monopolized niches, x_t, can be written as

$$x_{t+1} = [1 - e_{t+1}(\alpha\mu_1 + (1-\alpha)\lambda\mu_0)]x_t + \alpha e_{t+1}[1 - x_t(1-\mu_1)]. \tag{2}$$

The first term of this expression accounts for the proportion of monopolized niches at t, x_t, that either are not challenged by entry or survives the challenge of an entrant. The second term accounts for the competitive niches that become monopolized by novel innovators at t and the previously monopolized niches that are simply replaced by another novel patent.

The measure of innovating firms e_t is determined as follows. At each date there is an infinite number of potential innovative entrepreneurs who may attempt to engender and develop an innovation. Entry, and its associated development cost, occurs at the beginning of the period before production (and profits) take place.[6] To profitably enter a monopolized niche, a novel innovator has to overcome the opposition (in court) of the incumbent monopolist. A novel innovator that challenges a competitive niche faces no opposition of existing firms, becoming a monopolist with probability one. Thus, an innovator's (ex-ante) probability of success in becoming a monopolist at date t can be written as

$$p_t = \alpha[1 - x_t(1 - \mu_1)], \tag{3}$$

where α is the probability of obtaining a novel innovation and $1 - x_t(1 - \mu_1)$ corresponds to the probability of landing in a competitive niche, $1 - x_t$, plus the probability of landing on a monopolized niche x_t times the probability of wining in court μ_1.

Finally, the mass of innovations subject to development at any date t, e_t, is governed by a free-entry condition. That is, every period in equilibrium the net gain from entering and developing an innovation must be non-positive; i.e., $-1 + p_t v_t \leq 0$. If this inequality is strict, no innovations are developed in the corresponding date and the innovation flow e_t must be zero. We summarize this requirement in the following equation,

$$e_t[-1 + p_t v_t] = 0, \tag{4}$$

to which we will refer as the *complementary slackness* condition.

2.2 Steady State Equilibrium Analysis

In this section we define the dynamic equilibrium of the industry and analyze its steady-state properties. Equilibrium conditions determine four key endogenous variables at each date t: the flow rate of entry e_t, the proportion of monopolized niches x_t, the probability

[6] Opposite to classical models in the patent-race literature (e.g., Loury (1979) and Lee and Wilde (1980)), we abstract from the timing of innovation.

that an innovator becomes a monopolist p_t, and the value of being a monopolist v_t. We denote the steady-state value of these variables with the subscript ss.

The next assumption restricts the parameter values for π so that the solution is interior and the discussion of the model is meaningful. The following lemma characterizes the steady-state equilibrium of the model.

Assumption 1. $\pi \in \left(\frac{1-\beta}{\alpha\mu_1}, \frac{1}{\alpha}\left(\beta + (1-\beta)\frac{\alpha+(1-\alpha)\lambda\mu_0}{\alpha\mu_1+(1-\alpha)\lambda\mu_0}\right)\right)$.

Lemma 1. *There exists a unique steady-state equilibrium with* $e_{ss} \in (0,1)$ *if and only if Assumption 1 holds. This equilibrium is given by* $v_{ss} = p_{ss}^{-1}$,

$$x_{ss} = \frac{\alpha}{\alpha + (1-\alpha)\lambda\mu_0}, \qquad (5) \qquad\qquad p_{ss} = \alpha\frac{\alpha\mu_1 + (1-\alpha)\lambda\mu_0}{\alpha + (1-\alpha)\lambda\mu_0}, \qquad (6)$$

$$e_{ss} = \frac{\pi p_{ss} - (1-\beta)}{\beta\left(\alpha\mu_1 + (1-\alpha)\lambda\mu_0\right)}. \qquad (7)$$

In equilibrium, entry occurs until the expected value of developing an innovation, $p_{ss}v_{ss}$, equals the entry costs. It is interesting to observe that despite the fact that entry affects the proportion of monopolized niches and the value of participating in the market, in steady state both of these values are not affected by the payoff parameter π. That is, an increase in the expected-discounted payoffs is completely absorbed by increased entry; the values of x_{ss} and v_{ss} remain unchanged. In the expressions above, λ and μ_0 always appear together; $\lambda\mu_0$ represents the rate at which obvious innovation successfully enters a niche. The next proposition summarizes the main comparative statics of the model.

Proposition 1. *In a steady-state equilibrium with interior* e_{ss}*, the effects of marginal changes in the parameters on the steady-state variables* x_{ss}*,* v_{ss}*, and* e_{ss} *have the signs shown in the following table:*

	π	β	α	$\lambda\mu_0$	μ_1
x_{ss}	0	0	+	−	0
p_{ss}	0	0	+	+	+
v_{ss}	0	0	−	−	−
e_{ss}	+	+	+	?	+

The proportion of niches served by firms holding valid patents, x_{ss}, is increasing in α and decreasing in $\lambda\mu_0$. In the steady state, the proportion of monopolized niches reflects the composition of the pool of entrants that receive a patent. As a result, the higher the proportion of entrants with genuine innovations the bigger the proportion of niches that they will occupy. In the other direction, the higher the probability that a firm with an invalid patent arises and it is allowed to produce, $(1-\alpha)\lambda\mu_0$, the more often valid patent holders will be challenged and defeated in court.

The effect of the previous parameters on the value v_{ss} is inversely related to p_{ss} as, by free entry, $p_{ss}v_{ss} = 1$. The probability of entering the market profitably p_{ss} is increasing in both the value of obtaining a novel innovation α and the probability that such innovation is upheld by courts, μ_1. Perhaps surprising, p_{ss} is also increasing in the rate that obvious innovation enter a niche $\lambda\mu_0$. This occurs as a higher success rate $\lambda\mu_0$ decreases the proportion of monopolized niches x_{ss}, increasing the probability that an novel innovator lands in a competitive market and is not contested in court.

As expected, entry increases in the flow of profits and the discount factor. An increase in judges' probability of ruling in favor of a novel innovator, μ_1, or in the probability of obtaining a novel innovation, α, foster entry, as they makes entry more likely to succeed. However, the effect of $\lambda\mu_0$ is in general ambiguous and it is characterized in the next proposition.

Proposition 2. *In a steady-state equilibrium, the effect of an increase in the entry rate of obvious innovations $\lambda\mu_0$ on entry e_{ss} can be increasing, decreasing or inverted-U shaped. In particular, it is decreasing when $\mu_1 = 1$.*

The previous result points to an interesting non-monotonic relationship between entry and the protection that incumbents receive against obvious innovations. The main driver behind this result is that a change in $\lambda\mu_0$ conveys two effects of opposite signs. On the one hand, an increase in $\lambda\mu_0$ fosters entry—through the decrease in x_{ss}—as it reduces the number of innovations challenged in court. On the other hand, an increase in $\lambda\mu_0$

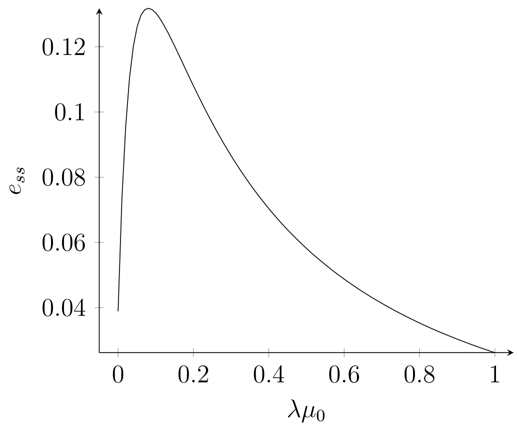


Figure 1: Entry is maximized at an interior value of $\lambda\mu_0$.

Notes: Parameter values are $\alpha = 0.1$, $\pi = 2.4$, $\beta = 0.8$, and $\mu_1 = 0.85$.

decreases the value of v_{ss} because the expected duration of incumbency is reduced, as incumbents become more likely to be replaced by obvious innovators.

It can be shown that when the probability of success in court of an entrant with a novel innovation (μ_1) is close to one, the second effect dominates and entry monotonically decreases with $\lambda\mu_0$. When $\mu_1 = 1$, novel innovators are indifferent between landing in monopolized or competitive niches, as they are granted access regardless. This makes the benefit of increasing the number of competitive niches irrelevant and, consequently, only the lower value-of-incumbency effect persist. As illustrated by Figure 1, however, for lower values of μ_1 the first effect may dominate when $\lambda\mu_0$ is low. In those cases, the innovation flow is maximized at some interior value $\lambda\mu_0$. These results imply non-trivial trade-offs for our discussion below on the socially optimal level of protection against imitation, $1 - \lambda\mu_0$, and its link to the socially optimal level of protection against innovation, $1 - \mu_1$.

3 The Patent Office

In this section we study the patent office screening problem. We first assume that the office takes the judges' decisions parameter μ_0 and μ_1 as given. In the next section we endogenize judges' decision and study the interaction between both institutions.

We assume that the Patent Office goal is to maximize society's welfare. In order to perform a meaningful welfare analysis, we need to formalize the demand side of the industry. We do this along the lines of a standard quality ladder model with limit pricing.[7] In particular, we assume that there is a unit mass of infinitely-lived homogeneous consumers willing to buy at most one unit of the product from each niche $j \in [0, 1]$ at each date t. Utility is additive across goods and dates, the intertemporal discount factor is $\beta < 1$, and the net utility flow from buying good j at price P_{jt} is $U_{jt} = Q_{jt} - P_{jt}$, where Q_{jt} is the quality of the good. For simplicity, production costs are assumed to be zero.

The successful entry of a novel innovation in a given niche improves the quality of the best good available in that niche by π units. The novel innovator, however, is able to charge a price $P_{jt} = \pi$ that captures the full quality advantage of their product vis-a-vis the best competing alternative. In contrast, the successful entry of an obvious innovation (imitator) introduces competition for latest technology, decreasing the market price P_{jt} to zero, transferring the benefits to consumers.

In this environment, the total per-period welfare in steady state is equal to

$$W = e_{ss} \left(p_{ss} \frac{\pi}{1 - \beta} - 1 - \kappa(\lambda) \right), \tag{8}$$

where $\kappa(\lambda)$ represents the cost of screening a patent application, which is continuously differentiable, convex, satisfying $\kappa(1) = \kappa'(1) = 0$ and $\kappa'(0) = -\infty$. Every period e_{ss} innovations take place at a cost of 1. Because only novel innovations increase the quality of existing products and innovations only reach the market if the innovating firm manages to win in court, each innovation gives society benefits with probability p_{ss}. Finally, the rents associated with successful novel innovation are the present discounted value of a perpetual increase in quality π. These rents are split between firms and consumers. When the innovation arrives the rent is perceived by the innovating firm. When the innovating firm is replaced, either by a novel or an obvious patent, the rents of the incumbent are competed away and the benefit of the increase in quality is accrued to consumers. In

[7] In section 5.2 we discuss an alternative market environment where firms invest in cost-reducing innovations. Unlike in the quality ladder model, that case exhibits a dead-weight loss derived from market power and we analyze its implications.

the absence of screening costs (i.e., $\kappa(\lambda) = 0$), the free-entry condition (4) implies that the parenthesis in (8) is always positive.[8] Thus, the net effect of innovation in society is positive as long screening cost are not prohibitively high. We will assume so for the remainder of the paper.

The Patent Office chooses the value of λ that maximizes W taking the courts' behavior as given. The derivative of the welfare function with respect to the screening rate λ is (recall that $\lambda = 0$ represents maximal screening):

$$\frac{\partial W}{\partial \lambda} = \frac{\partial e_{ss}}{\partial \lambda}\left(p_{ss}\frac{\pi}{1-\beta} - 1 - \kappa(\lambda)\right) + e_{ss}\left(\frac{\pi}{1-\beta}\frac{\partial p_{ss}}{\partial \lambda} - \kappa'(\lambda)\right). \tag{9}$$

The level of screening of a patent office directly affects welfare through two channels: it determines entry and the probability of success. From Proposition 1 we know that p_{ss} is increasing in λ. That is, for a given entry flow e_{ss}, a lower level of screening increases the proportion of competitive markets, reducing the number of novel innovations challenged in court, decreasing the number of innovations that go to waste. From Proposition 2 we know that the net effect on entry can be ambiguous depending on the courts decisions.

Proposition 3. *In the absence of screening costs (i.e., $\kappa(\lambda) = 0$ for all λ), the patent office maximizes welfare by setting: i) $\lambda = 0$, if $\mu_1 = 1$; ii) if $\mu_1 < 1$, when interior,*

$$\lambda^* = \alpha\frac{2(1-\beta) + (1-3\mu_1)\alpha\pi + (1-\mu_1)\sqrt{\alpha\pi(\alpha\pi + 8(1-\beta))}}{2\mu_0(1-\alpha)(\alpha\pi - (1-\beta))} \tag{10}$$

If perfect screening of obvious patents were infinitely costly, the optimally screening is naturally interior. Proposition 3 goes further by saying that, even in the absence of screening costs, the patent office may want to allow obvious patents (imitation). The intuition behind this result is that obvious patents may foster entry by increasing the number of competitive niches, thus increasing the probability of success of future novel innovators. When $\mu_1 = 1$, however, no novel innovation goes to waste and the benefit of increasing p_{ss} is nil. Consequently, only the effect of screening on steady state entry

[8] This is so, as $\pi/(1-\beta) > v_{ss}$ and $p_{ss}v_{ss} = 1$.

matters. From Proposition 2, we know that entry is decreasing in λ so that the optimal solution is full screening.

To conclude this section, as a benchmark, we solve the problem of a planner that can control both the patent office's screening rate and the judges' decisions.

Corollary 1. *In the absence of screening cost, a social planner that can decide both the patent office screening rate and the judges' ruling rates (i.e.,* $\max_{(\lambda,\mu_0,\mu_1)\in[0,1]^3} W$*) chooses to fully screen for obvious innovators, either at the Patent office or in court, and to always rule in favor of the new novel innovator.*

This result arises from a combination of the previous results. As shown in Proposition 1, higher values of μ_1 yield an increase in the probability that an entrant is successful, p_{ss}, and, consequently, an increase in total entry, e_{ss}. Both effects contribute to increase social welfare, as indicated in equation (9). Hence, a Patent Office that could regulate the behavior of courts at no cost should choose $\mu_1 = 1$. Using Proposition 3 we know that obvious entrants should not receive any protection in that case; i.e., $\lambda\mu_0 = 0$.

4 Endogenous Courts

In the previous sections we assumed that the result of a litigation started by an incumbent facing an entrant was governed by the two exogenous probabilities μ_0 and μ_1. In this section we endogenize these probabilities as the result of the decision to gather evidence by the judge that oversees the case. Because judges only make a decision if an infringement claim is made, we model the interaction between the Patent Office and courts as a Stackelberg game in which judges make their decisions after observing the screening rate of the Patent Office.[9] We show that when judges best respond to the Patent Office screening effort, a new incentive to allow obvious innovations appears: it induces judges to make better rulings.

[9]Even if the screening rate λ is not publicly announced by the Patent Office, Judges can infer the rate by simply knowing α and the number of infringement claims made.

4.1 Signals and Beliefs

Suppose that each infringement claim is overseen by a different and independent judge; that is, each judge takes the effort of the other judges as given. When a case reaches the court, the judge in charge does not directly observe the quality of the entrant's patent. The judge, however, takes as given the screening rate of the Patent Office, λ. As a result, the prior belief about the probability that the patent of an entrant is invalid (obvious) equals to

$$\gamma = \frac{(1-\alpha)\lambda}{\alpha + (1-\alpha)\lambda} \in [0, 1-\alpha]\,. \tag{11}$$

The judge can exert effort to gather evidence and receive a costly signal σ. The outcome of the signal is binary, $\sigma \in \{0,1\}$. It takes a value zero when the judge finds no evidence of infringement (indicating that the innovation is likely to be novel) and one when the judge finds otherwise. The relation between the signal's outcomes and the judge's effort, s, is as follows

$$\Pr[\sigma = 1|\text{novel}] = \frac{1-s}{2} \qquad \text{and} \qquad \Pr[\sigma = 1|\text{obvious}] = \frac{1+s}{2}.$$

That is, as effort increases, the signal becomes more precise regarding the true quality of the innovation of the entrant. If no effort is exerted, $s = 0$, the signal becomes completely uninformative.

We assume that judges are pro-entrant. That is, a judge always rules in favor of the entrant if no evidence of infringement is found, $\sigma = 0$. In that case, the entrant always replaces the incumbent. When the signal produces some evidence of potential infringement, the judge makes a probabilistic decision equal to its posterior belief

$$\theta \equiv \Pr[\text{novel}|\sigma = 1] = \frac{(1-\gamma)(1-s)}{(1-\gamma)(1-s) + \gamma(1+s)}.$$

Under this decision rule, the implied success probabilities in court for a novel and an obvious innovator are

$$\begin{aligned} \mu_1(s) &= \Pr[\sigma = 0|\text{novel}] + \theta \Pr[\sigma = 1|\text{novel}] = \frac{1+s}{2} + \frac{1-s}{2}\theta, \\ \mu_0(s) &= \Pr[\sigma = 0|\text{obvious}] + \theta \Pr[\sigma = 1|\text{obvious}] = \frac{1-s}{2} + \frac{1+s}{2}\theta. \end{aligned} \tag{12}$$

As a result, $\mu_1(s) \geq \mu_0(s)$ and this difference grows with the judge's effort s.

4.2 A Single Judge's Problem

We assume that each judge decides how much effort to exert in order to maximize social welfare net of the cost of this effort. When taking the decision, a judge takes the Patent Office screening rate λ and the effort of other judges $\hat{s}$ as given. This means that the judge only takes into account the impact that the *particular* case under consideration has in total welfare. To illustrate the difference, although (8) considers the entry cost that an entrant incurs, a judge ignore these costs. A judge only reviews a case after entry has occurred. Entry cost of the current entrant are, thus, sunk at the moment in which the judge makes its ruling.

The impact that a judge has in total welfare can be summarized as the combination of statistical errors I and II. In particular, we define the Type-I error, E_I, as the result of forbidding the production of a firm with a novel innovation. This error is easy to characterize. If a judge forbids such a firm to enter, the innovation is never implemented, reducing social welfare by π on a permanent basis. That is, $E_I = \pi/(1-\beta)$.

The Type-II error, E_{II}, corresponds to allowing an obvious innovation to replace a monopolist and turning the niche into a competitive one. Changing the state of a niche from monopolized to competitive affects the probability of future entry and, consequently, the stream of future innovations. To derive the Type-II error, therefore, we need to compute, the present value of social welfare in each state. More precisely, $E_{II} = \beta(w_M - w_C)$, where w_M and w_C are the present value of the social welfare that a niche generates in the monopoly and competition state, respectively. The difference in values is discounted one period, as the stream of entry is affected starting next period. The values w_M and w_C are given by the solution to

$$\begin{aligned} w_C &= \alpha e_{ss}(\hat{s}) \left(\frac{\pi}{1-\beta} + \beta w_M \right) + (1 - \alpha e_{ss}(\hat{s}))\beta w_C, \\ w_M &= \alpha \mu_1(\hat{s}) e_{ss}(\hat{s}) \left(\frac{\pi}{1-\beta} + \beta w_M \right) + (1-\alpha)\lambda \mu_0(\hat{s}) e_{ss}(\hat{s}) \beta w_C + \phi \beta w_M. \end{aligned} \tag{13}$$

where $\phi \equiv 1 - e_{ss}(\hat{s})(\alpha\mu_1(\hat{s}) + (1-\alpha)\lambda\mu_0(\hat{s}))$. Because a single judge is atomistic, it takes as given the future decisions of the judges, $\hat{s}$. As a result, the probability that an obvious and novel entrant prevail in court in the future and the future entry rate are also exogenous for a single judge and denoted in the previous expression as $e_{ss}(\hat{s})$, $\mu_1(\hat{s})$, and $\mu_0(\hat{s})$, respectively.

The social value of a competitive niche before entry takes place depends on whether the niche faces entry of a novel innovator. With probability $\alpha e_{ss}(\hat{s})$, a novel innovation arrives, providing society an increase in discounted surplus of $\pi/(1-\beta)$, and turns the niche into a monopolized one starting next period. With probability $1-\alpha e_{ss}(\hat{s})$ either no innovation or an obvious innovation arrives, providing no value for society and keeping the niche as competitive in the next period.

For a monopolized niche, the social value depends on whether it faces entry or not and the identity of the entrant. With probability $\alpha\mu_1 e_{ss}(\hat{s})$ a novel innovator enters the niche and succeeds in court. In this case society receives the social value of a novel innovation at the beginning of a period $\pi/(1-\beta)$ and the continuation value of a monopolized niche. With probability $(1-\alpha)\lambda\mu_0 e_{ss}(\hat{s})$, the niche faces successful entry by an obvious innovator, turning the niche competitive. Finally, in the last term, ϕ represents the probability that (successful) entry does not occur, maintaining the niche monopolized.

Solving the previous values allow us to characterize the Type-II error, which, as it turns out, has a negative sign. Social welfare increases when a niche turns competitive, as eliminating a monopoly reduces the barriers to entry, fostering future innovation. The next lemma formalizes this result.

Lemma 2. *The steady-state value of the Type-II error is negative and equal to*

$$E_{II}(\hat{s},\gamma) = -\frac{\pi}{1-\beta}\frac{(1-\gamma)\alpha\beta(1-\mu_1(\hat{s}))e_{ss}(\hat{s})}{(1-\gamma)(1-\beta)+\alpha\beta\left(1-\gamma+\gamma\mu_0\left(\hat{s}\right)\right)e_{ss}\left(\hat{s}\right)}. \tag{14}$$

A judge decides how much effort s to exert in order to minimize the social cost of both types of error plus the cost of its effort $c(s)$. That is, a judge solves

$$\min_{s\in[0,1]} J(s,\hat{s},\gamma) = (1-\gamma)(1-\mu_1(s))E_I + \gamma\mu_0(s)E_{II}(\hat{s},\gamma) + c(s). \tag{15}$$

For tractability, and in order to obtain analytical results, we assume in the remaining of this section that the effort is binary, $s \in \{0,1\}$. In Section 6, we show that the main findings are robust to allowing a continuous effort by judges. Without loss of generality, due to the binary effort assumption, we also assume for the rest of this section that a judge's cost of effort is linear in its effort level; i.e., $c(s) = c \cdot s$. The binary effort assumption delivers simple expressions for $\mu_0(s)$ and $\mu_1(s)$. In particular, we have $\mu_0(0) = \mu_1(0) = 1-\gamma/2$ and $\mu_1(1) = 1 > 0 = \mu_0(1)$. We now turn to the optimal decision of the judges on whether to exert effort or not.

Start by noticing that when $s = 1$, using (15), we have $J(1,\hat{s},\gamma) = c$ for any value of γ and $\hat{s}$. That is, when the signal is fully informative, a judge commits no mistakes and the only social cost of the its decision is the cost of the judge's effort. When a judge decides not to exert effort we have

$$J(0,\hat{s},\gamma) = \frac{\pi}{1-\beta}\frac{\gamma(1-\gamma)}{2}\Phi(\hat{s},\gamma) \tag{16}$$

where $\phi(\hat{s},\gamma) > 0$ for all $(\hat{s},\gamma) \in [0,1]^2$ and is given by equation (22) in the Appendix.[10] If the patent office fully screens innovators (i.e., $\gamma = 0$; all the entrants assessed by a judge are novel innovators) or if all innovations are obvious ($\gamma = 1$), the bayesian judge can trivially back out the quality of the entrant without exerting any effort and, therefore, the right decision is always made. That is, $J(0,\hat{s},0) = J(0,\hat{s},1) = 0$. For any other value of γ, $J(0,\hat{s},\gamma)$ is positive, indicating that the judge's social cost function is non-monotonic in γ. The next assumption guarantees that the judge's effort cost is not too high. That is, for any set of parameters there exists a prior γ that guarantees that the judge prefers to exert effort.

Assumption 2. $c < \pi/8(1-\beta)$.

[10] The proof of Lemma 3 characterizes the $\phi(\hat{s},\gamma)$ function.

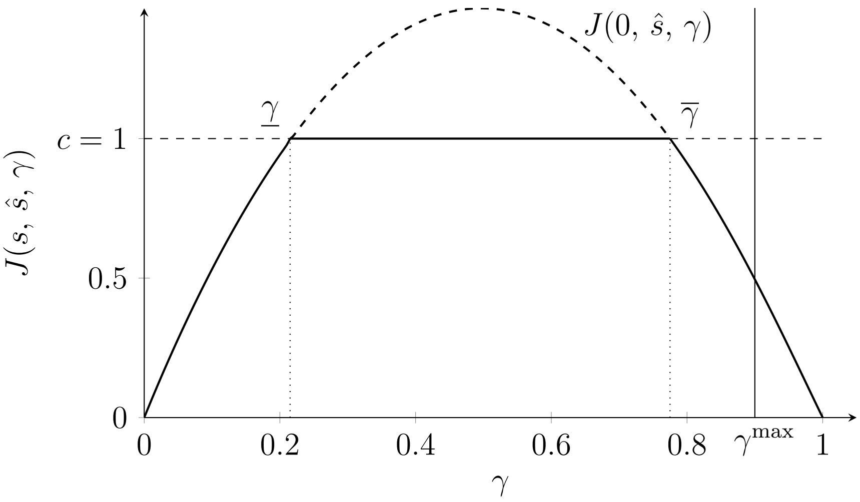


Figure 2: The cost of an individual judge, as function of its prior γ, when making the optimal choice s given aggregate effort $\hat{s}$.

Note: Parameter values are $\alpha = 0.1$, $\beta = 0.8$, $\pi = 2.4$, $c = 1$, and $\hat{s} = 0.6$.

Lemma 3. *Under Assumption 2, for any value aggregated effort by other judges* $\hat{s} \in [0, 1]$*, there exists thresholds* $\underline{\gamma}, \overline{\gamma} \in (0, 1)$ *with* $\overline{\gamma} > \underline{\gamma}$ *such that the judge exerts effort if* $\gamma \in [\underline{\gamma}, \overline{\gamma}]$ *and perform no effort otherwise.*[11]

The informativeness of the judge's effort depends on the screening level of the patent office and, indirectly, on the initial composition of the pool of entrants, determined by α. When the majority of new patent holders are either novel or obvious innovators, the prior belief changes little with the judge's effort. In this case, the judge prefers not to exert effort and simply rule according to that prior. In contrast, when the proportion of novel and obvious innovators is balanced, a judge's effort becomes valuable and more effort is exerted to better screen innovators.

Figure 2 depicts the judge's problem. The parabola depicts the judge's cost of not exerting effort as a function of its prior γ —which is induced by the Patent Office screening rate λ. The cost of full effort is represented by $c = 1$. The cutoffs $\underline{\gamma}$ and $\overline{\gamma}$ are represented by the points where c intersects $J(0, \hat{s}, \gamma)$. The solid line shows the minimal cost for the judge for each level of screening rate γ. It is interesting to observe that depending on the

[11]When a judge's prior γ takes either of the values $\{\underline{\gamma}, \overline{\gamma}\}$, an individual judge is indifferent between exerting effort or not. We break this indifference by assuming that judges exert effort.

initial screening rate, the Patent Office's effort and the judge's effort could be strategic complements or substitutes from the judges' perspective. Since from (11), γ increases in λ, an increase of the screening rate at $\underline{\gamma}$ (i.e., a lower λ) decreases the judges' effort indicating strategic substitutability. In contrast, for values of γ above $\overline{\gamma}$, an increase in patent screening can raise the effort performed by judges; an instance of strategic complementarity.

Lemma 4. *For a given prior $\gamma \in (0, 1)$ and aggregated effort by other judges $\hat{s} \in [0, 1)$ the cost of exerting no effort $J(0, \hat{s}, \gamma)$ increases with: a reduction in the probability of novel patent α and a raise in the value of a novel innovation $\pi/(1 - \beta)$.*

To conclude this section, we explore how the main parameters of the model affect's a judge's effort decision. The cost of making mistakes increases when novel innovations become scarce or the value of novel innovation—i.e., the scale of the cost of making mistakes (see eq. (16))—increases. In both situations the range of screening rate under which the judge performs full effort $[\underline{\gamma}, \overline{\gamma}]$ widens.

4.3 Aggregated-Judge Equilibrium

The analysis in Lemma 3 refers to a single judge, taking the behavior of other judges, $\hat{s}$, as given. We now proceed to characterize the steady-state symmetric equilibrium by finding the effort level of an individual judge s^* that is consistent with the aggregate behavior of all the judges. Before characterizing the equilibrium observe that the pair of thresholds $(\underline{\gamma}, \overline{\gamma})$ are a function of the conjectured level of effort by the other judges $\hat{s}$ and the cost of effort c. To construct the equilibrium at the aggregate level we need to develop an understanding of how aggregate behavior, $\hat{s}$, affects effort by an individual judge.

Lemma 5. *For a given prior γ, aggregated judge effort is a strategic complement to individual judge effort if $\hat{s} > (1 - \sqrt{\gamma})/(1 + \sqrt{\gamma}) \in [0, 1]$ and it is a strategic substitute otherwise.*

Similar to the Patent Office's screening rate, aggregated effort by other judges can encourage or deter effort by a single judge. Strategic complementary occurs when $J(s, \hat{s}, \gamma)$ is increasing in $\hat{s}$, as an increase $\hat{s}$ would induce full effort by an individual judge for a wider range of γ (in Figure 2 this would be represented by an upward shift of the parabola). In contrast, strategic substitutability calls for coordination at lower levels of effort. From Lemma 5, we can see that aggregated effort is a strategic complement to individual effort when $\hat{s} = 1$ and strategic substitute when $\hat{s} = 0$. Which implies that judges may face a coordination problem as stated in the next lemma.

Lemma 6. *When aggregated effort is binary,* $\underline{\gamma}(1) < \underline{\gamma}(0)$ *and* $\overline{\gamma}(1) > \overline{\gamma}(0)$.

Compared to no aggregated effort, when aggregated judges coordinate in high effort, an individual judge will also perform high effort for a large range of γ. This creates a coordination problem for some priors, engendering a multiplicity of equilibrium as stated by the next proposition.

Proposition 4. *The set of equilibria in the aggregated-judge game, as a function of the judges' prior, is given by:*

$$s^* = \begin{cases} 0 & \text{if } \gamma < \underline{\gamma}(1) \text{ or } \gamma > \overline{\gamma}(1), \\ 1 & \text{if } \gamma \in (\underline{\gamma}(0), \overline{\gamma}(0)), \\ \{0,1\} & \text{if } \gamma \in [\underline{\gamma}(1), \underline{\gamma}(0)] \cup [\overline{\gamma}(0)), \overline{\gamma}(1)]. \end{cases}.$$

Figure 3 illustrates Proposition 4 by depicting the aggregate behavior of judges $\hat{s}^*$ as a function of the patent office screening rate γ. When the proportion of obvious patents is very low, judges find that the incremental benefit of their effort is small. Judges exert no effort and rule according to their prior, which is favorable to entrants. As the pool of patents becomes more mixed, individual judges face increasing incentives to exert effort. In the range $(\underline{\gamma}(0), \overline{\gamma}(0)]$ judges perform maximum effort and make no mistake in their rulings. As γ increases further and the pool of patents consists mostly of obvious patents, the benefit of the judges' effort starts to decrease. For very high values of γ, judges coordinate in no effort and rulings tend to favor incumbents.

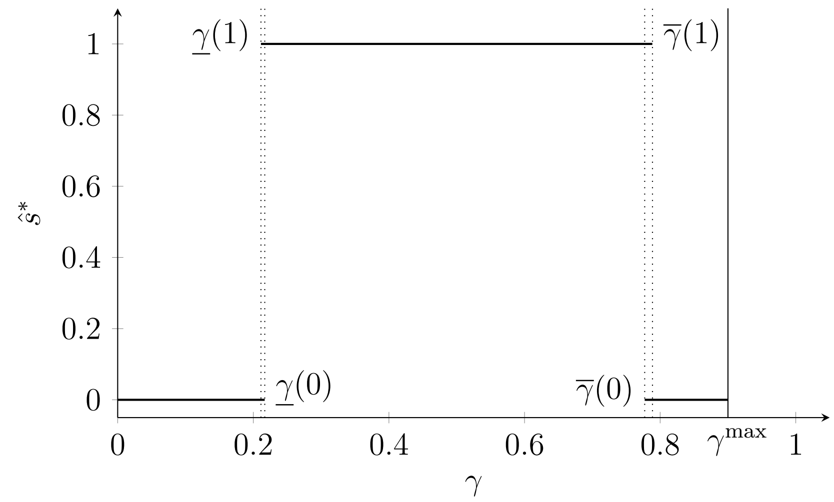


Figure 3: Equilibrium judge effort as a function of γ.

Note: Parameter values are $\alpha = 0.9$, $\beta = 0.8$, and $\pi = 2.4$.

Lemma 7. *When multiple equilibria exists, the high effort equilibrium is welfare dominated—from the perspective of problem* (15)*—by the no effort equilibrium.*

Consider a screening rate with multiple equilibria. When judges coordinate in high effort, its social cost is equal to the the cost of its effort, c. Because a single judge can always unilaterally deviate to high effort and obtain a social cost of c, it has to be the case that the no-effort equilibrium is payoff dominant (in the sense that induces a lower social cost) with respect to the high effort equilibria. For the next section we apply the refinement that judges coordinate in their payoff dominant equilibria, which in this context means: $s^* = 1$ if $\gamma \in [\underline{\gamma}(0), \overline{\gamma}(0)]$ and $s^* = 0$ otherwise.

4.4 Screening and Enforcement Equilibrium

In this section we characterize the screening decision of the Patent Office once the best response of the court is internalized. That is, the Patent Office maximizes (8) given the aggregate behavior $s^*(\lambda)$ of the judges. Observe that (8) ignores the judges' cost of effort. We do so for two reasons. First, it is the natural benchmark to the original exercise in Section 3. Also, this eases exposition by isolating a new incentive. The next section studies the relation between the Patent Office and courts when the effort of judges is continuous and the patent office fully incorporates the costs of the court.

Conditional on no effort by the judges ($s^* = 0$)—that is, ignoring the best response of the judges—problem (8) delivers an screening level λ° which is interior due to the assumptions in $\kappa(\lambda)$. Conditional on choosing a level of λ that induces high effort by judges, both the probability of entry $p_{ss} = \alpha$ and the entry level $e_{ss} = (\alpha\pi - 1 + \beta)/\alpha\beta$ are maximal and do not depend on the level of screening λ. Thus, the problem of the Patent Office becomes

$$\max_{\lambda \in \{\lambda : s^*(\lambda)=1\}} W = \frac{\alpha\pi - (1-\beta)}{\alpha\beta} \left(\alpha \frac{\pi}{1-\beta} - 1 - \kappa(\lambda) \right).$$

It is clear that from this problem, conditional on high effort by judges, the patent office chooses the lowest feasible screening rate, which in terms of judges' priors, corresponds to $\overline{\gamma}(0)$. Call this screening rate $\overline{\lambda}$. Next proposition characterizes the screening and enforcement equilibrium.

Proposition 5. *If $\lambda^\circ \leq \overline{\lambda}$, the screening rate set by the patent office is $\overline{\lambda}$ and judges exert high effort $s^* = 1$. If $\lambda^\circ > \overline{\lambda}$, the screening rate set by the patent office depends on the degree of convexity of $\kappa(\lambda)$. When $\kappa(\lambda)$ is not too convex, $\overline{\lambda}$ is optimal and judges exert high effort $s^* = 1$.*

The convexity requirement for $\kappa(\lambda)$ to induce effort by the judges is made explicit in the appendix. The previous proposition extends Proposition 3 to the case in which the decision rules by the court μ_0 and μ_1 are endogenously determined. In equilibrium, the patent office chooses to allow low quality innovators not only to to induce entry—as it does in the exogenous court scenario—but also allows low quality innovations to induce effort by the court. It is interesting to observe that, when judges are induced to exert effort, the patent office screening rate and the judges effort are strategic complements from the judges' perspective. As we shall see in the next sections, this finding is quite robust.

5 Robustness and Extensions

5.1 Continuous Judge Effort

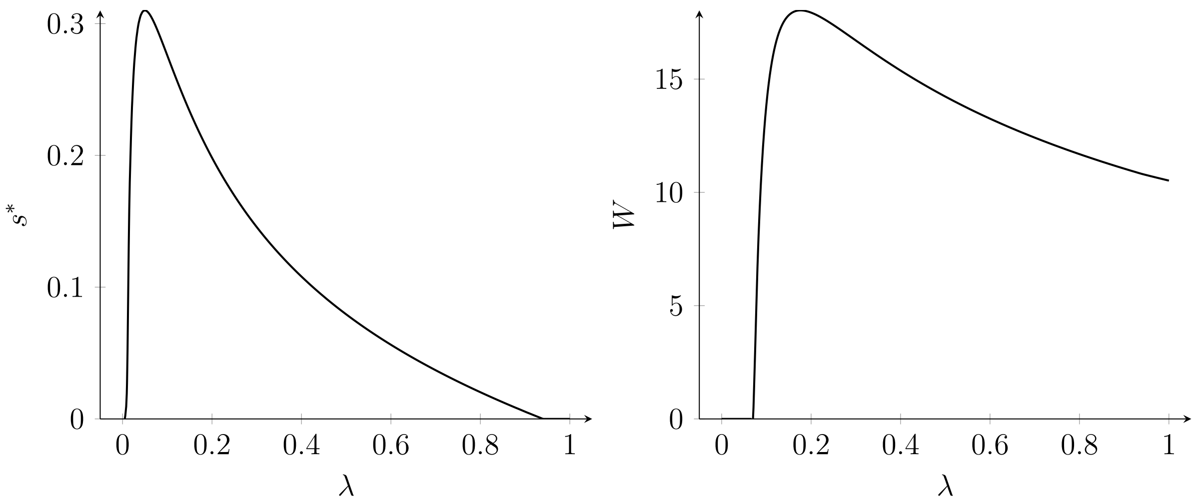
In this section we illustrate the Patent Office's incentives to allow for obvious innovations persists when the judges' effort can take a continuous value. Taking the first order condition of the judge's problem (15) and then imposing symmetry among judges, we obtain the aggregated effort as a function of the screening rate λ. Figure 4a shows that the aggregate effort of the judges takes an inverted-U shape. This result highlights the fact that the screening rate of the Patent Office can be either strategic complement or substitute to the judges effort.

We now turn to the Patent Office problem which, in the context where judges' decisions are endogenized, becomes

$$W = e_{ss}(\lambda, s^*)\left(p_{ss}(\lambda, s^*)\frac{\pi}{1-\beta} - 1 - \kappa(\lambda) - \tau(\lambda)x_{ss}(\lambda, s^*)c(s^*)\right). \tag{17}$$

where $\tau(\lambda) = \alpha + (1-\alpha)\lambda$ represents the probability that an entrant receives a patent. The problem above is analogous to (8) with the main difference that now the cost of the patent system consists not only of the screening costs of the patent office $\kappa(\lambda)$ but also the cost of effort by the judges, $c(s^*)$. Because judges review a patent conditional on an infringement claim being made, out of the total number of entrants, e_{ss}, they review and incur in the corresponding cost for those that get a patent, $\tau(\lambda)$, and land in a monopolized niche x_{ss}. It is interesting to observe that the Patent Office faces a new incentive when internalizes the costs of the patent system. Although lower screening reduces the Patent Office's costs $\kappa(\lambda)$ it also increases the probability that an innovation reaches court $\tau(\lambda)x_{ss}$. This new effect puts an extra pressure for the Patent Office to screen. Despite this new effect, Figure 4b shows that it is still optimal for the Patent Office to choose an interior screening level.

Figure 5a and 5b and show how the optimal screening rate of the patent office λ and the effort of judges varies with changes in the screening cost (parameterized by k) and the



(a) Aggregated Judge-Effort

(b) Welfare

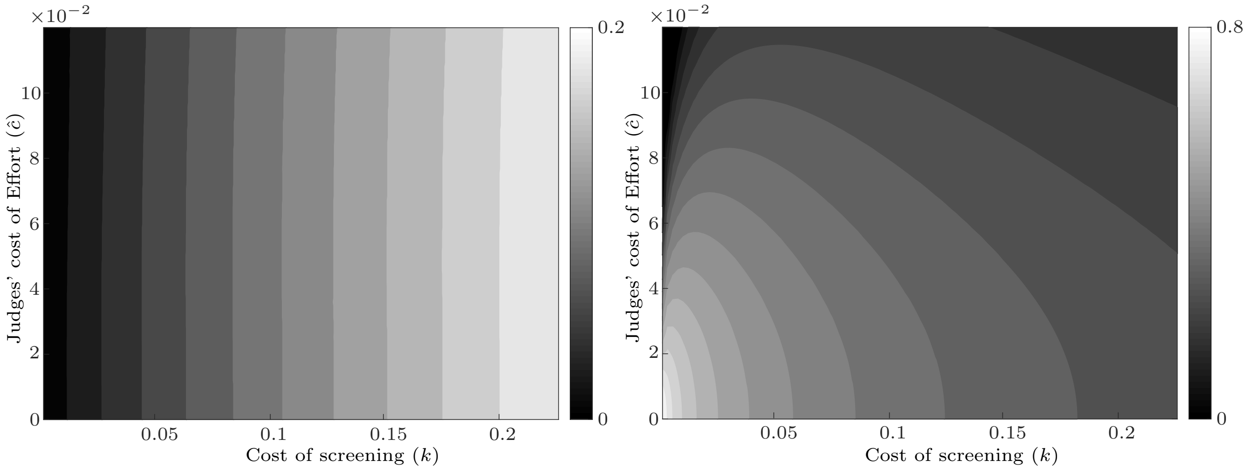
Figure 4: Optimal judge effort and welfare for different values of λ.

Notes: (a) The cost of screening is given by $\kappa(\lambda) = \exp(k(1-\lambda)/\lambda) - 1$. (b) A judge's cost of effort is given by $c(s) = (\pi/(1-\beta))c\,(\exp(c \cdot s/(1-s)) - 1)$.

cost of effort (parameterized by $\hat{c}$). As expected, an increase in the screening cost reduces the level of screening and an increase in the cost of effort, reduces the judges effort. An increase in the cost of the judges' effort encourages the Patent Office to screen more, highlighting the fact that the Patent Office regards the effort of judges as a substitute of its own. It is interesting to observe, however, that from the judge's perspective the patent office screening rate can be strategic complement or substitutes. Consistent with Figure 4a, an increase in the cost of screening — which, recall, decreases the level of screening — initially encourage judges to exert more effort (strategic substitutes) but, when the cost of screening is sufficiently high, the decrease in screening by the patent office discourages Judges to exert effort (strategic complement). As shown in Figure 5b the strategic complementary region is the largest.

5.2 Cost-saving Innovations

In the previous sections we have studied innovation in a quality ladder. In that context, novel innovators were able to extract all the surplus from consumers, avoiding dead-weight losses and simplifying the welfare analysis. In this section we turn to cost-saving



(a) Optimal screening rate λ^* **(b)** Optimal Judges effort s^*

Figure 5: Optimal screening and effort as a function of screening and effort cost.

Notes: (a) The cost of screening is given by $\kappa(\lambda) = \exp(k(1-\lambda)/\lambda) - 1$. (b) A judge's cost of effort is given by $c(s) = \hat{c}(\pi/(1-\beta))\,(\exp(\hat{c}\cdot s/(1-s)) - 1)$.

innovations, which provide a useful setup to study the welfare losses associated to market power.

As before, we now assume that there is a continuum of niches of size 1. In each niche the good produced is homogeneous. Firms compete in prices and face an isoelastic equal to $q = a/p$. Each novel innovation decreases the existing marginal cost by a factor of $1-\delta$ where $\delta \in (0,1)$; that is, if mc represents the baseline marginal cost, after m novel innovations the marginal cost becomes $mc_m = \delta^m mc$.

Lemma 8. *The profit flow π and dead-weight loss ℓ generated by a novel innovation are independent of the baseline marginal cost mc and the number of innovations m. In particular, they are equal to $\pi = a(1-\delta)$ and $\ell = a\left(\ln(\delta^{-1}) - (1-\delta)\right) > 0$.*

Because profits are invariant to the number of innovations, the firm behavior described in Section 2 goes through without alterations. The objective functions of the Patent Office and courts, however, need to be modified. Figure 6 depicts the product market payoff associated with each innovation. We will use this figure to construct these objective

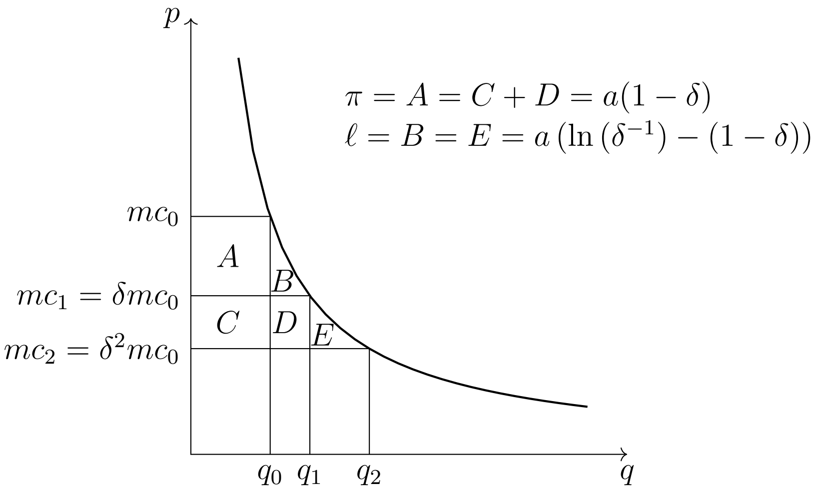


Figure 6: Cost-saving innovation: product market payoffs.

functions. The social welfare function of the Patent Office is given by

$$W = e_{ss}\left((1-\alpha)\lambda x_{ss}\mu_0\frac{\ell}{1-\beta} + \alpha\left(x_{ss}\mu_1\frac{\pi+\ell}{1-\beta} + (1-x_{ss})\frac{\pi}{1-\beta}\right) - 1 - K\right) \quad (18)$$

where $K = k(\lambda) + \tau(\lambda)x_{ss}c(s)$ is the total cost of the patent system. To explain the welfare function consider Figure 6 when mc_1 is the latest technology available in the market. The first term in the main parenthesis captures the arrival of an obvious entrant, which occurs with probability $1-\alpha$ and who obtains a patent with probability λ. With probability $1-x_{ss}$ the obvious entrant lands in a competitive niche, not affecting welfare creation. With probability x_{ss} the obvious entrant lands in a monopolized niche. In that case, the market price goes down from mc_0 to mc_1. The area A depicts the profits of the replaced incumbent which are transferred to consumers as surplus. Because this surplus is not created, we ignore it from the welfare equation. The area B was the existing dead-weight loss, which with the arrival of the obvious innovation is transferred to consumers permanently, increasing welfare by $\ell/(1-\beta)$.

The second expression represents the payoffs when the entrant is novel, which occurs with probability α. Novel innovators always get a patent. With probability x_{ss} the entrant lands in a monopolized niche and gets challenged in court. The entrant wins in court with probability μ_1. In that case, the price goes down from mc_0 to mc_1. As before, the area A is transferred from the incumbent to consumers and the original dead-weight loss B

is now captured by consumers permanently. In contrast, the novel entrant captures the area $C + D$ as profits. When a new innovation arrives in the future, these profits will be eventually transferred to consumers. That is, the welfare value created are the areas $B + C + D$ at perpetuity, or $(\pi + \ell)/(1 - \beta)$. Finally, when the novel innovator lands in a competitive niche—i.e.; the existing price equals the latest technology mc_1—- it does not get challenged in court and captures the area $C + D$ as profits. As before, these profits will eventually be transferred to consumers when a new innovation arrives. The entrant, thus, creates a welfare value $\pi/(1 - \beta)$.

In order to better understand the Patent Office's incentives we can re-arrange Eq. (18) using the steady state properties of the game, to obtain the expression

$$W = e_{ss} \left(p_{ss} \frac{\pi + \ell}{1 - \beta} - 1 - K \right).$$

From here we can see that the judge problem is analogous to that in (8) but now the welfare expression also captures the dead-weight loss recouped with the arrival of an innovation. It is immediate that the main message of Proposition 3 applies: When courts' behavior is fixed and when $\mu_1 < 1$ the patent the optimal screening rate might be interior even in the absence of screening cost.

We can now analyze how the judge's endogenous decision changes when innovations are cost reducing and a dead-weight loss might arise. Recall that entrants are only sued by an incumbent in a monopolized niche. Therefore, letting the entrant stay in the market will always increase welfare by (at least) $\ell/(1 - \beta)$ regardless of the entrant's type. As in the benchmark case, a Type-I error arises when a novel innovation is prevented from production. Since this case only occurs in already monopolized niches, this error leads to a loss of $E_I^{CS} = (\pi + \ell)/(1 - \beta)$, where CS stands for the cost-saving setup.

The Type-II error represents the "loss" in social welfare when an obvious innovation is allowed to replace a current monopolist. In this case, this cost has now two components. First, there is a short run gain, derived from eliminating the dead-weight loss that the incumbent generated. Second, there is the same dynamic effect explained in the benchmark case that increases the future value of the niche. That is,

$$E_{II}^{CS} = -\frac{\ell}{1-\beta} + \beta(w_M - w_C),$$

where the value of a monopolistic and competitive niche are respectively defined as

$$\begin{aligned} w_C &= \alpha e_{ss}(\hat{s}) \left(\frac{\pi}{1-\beta} + \beta w_M \right) + (1 - \alpha e_{ss}(\hat{s}))\beta w_C, \\ w_M &= \alpha \mu_1(\hat{s}) e_{ss}(\hat{s}) \left(\frac{\pi + \ell}{1-\beta} + \beta w_M \right) + (1-\alpha)\lambda\mu_0(\hat{s}) e_{ss}(\hat{s}) \left[\frac{\ell}{1-\beta} + \beta w_C \right] + \phi\beta w_M, \end{aligned} \tag{19}$$

and $\phi \equiv 1 - e_{ss}(\hat{s})(\alpha\mu_1(\hat{s}) + (1-\alpha)\lambda\mu_0(\hat{s}))$. The difference with the benchmark case is that, here, the value of a monopolistic niche now depends on the dead-weight loss ℓ. Each time a monopolist is replaced by another firm the deadweight loss associated to its innovation is obtained and transferred to consumers by means of a lower price. This effect increases w_M and narrows the difference between the value of a monopolistic and a competitive niche.

As a result, the dead-weight loss generates two effects on E_{II}^{CS} going in opposite directions. The static gain enhances the return from eliminating an existing monopolist. The dynamic effect reduces the incremental value of a competitive niche compared to a monopolized one. After some algebra we can show that the static effect dominates. That is, if we compare the expression for the Type-II error in the benchmark case, $E_{II}(\hat{s}, \gamma)$, and the one that arises in the cost-saving innovation case we have that $E_{II}^{CS}(\hat{s}, \gamma) < E_{II}(\hat{s}, \gamma)$.

Using the previous expressions we can now turn to the decision of a judge in the binary effort scenario. As before, we can compute the costs associated to the effort decision based on the effort carried out by all other judges, $\hat{s}$, which we denote as $J^{CS}(s, \hat{s}, \gamma)$. As in the standard case, it is easy to see that when a judge chooses $s = 1$ no error is made, meaning that the only cost that he/she bears is the one related to effort. That is, $J^{CS}(1, \hat{s}, \gamma) = c$. When a judge chooses effort $s = 0$, however, the expression for the cost becomes

$$J^{CS}(0, \hat{s}, \gamma) = J(0, \hat{s}, \gamma) + \frac{\gamma(1-\gamma)}{2} \frac{\ell}{1-\beta} \Delta(\hat{s}, \gamma), \tag{20}$$

where $\Delta(\hat{s}, \gamma)$ is a function of the two relevant choice variables, the effort of other judges and the level of screening carried out by the Patent Office together with the rest of parameters of the model.

The previous expression allows us to study whether the existence of a dead-weight loss fosters or hinders the high-effort choice decision of a judge. In particular, suppose that without dead-weight loss, $\ell = 0$, the equilibrium implies a choice of effort of 1 by all judges. This means that $J(1, 1, \gamma) < J(0, 1, \gamma)$. Whether this equilibrium exists or not when ℓ is positive depends on the sign of $\Delta(\hat{s}, \gamma)$. That is, if $\Delta(1, \gamma) < 0$ there exists a sufficiently large dead-weight loss for which $J^{CS}(1, 1, \gamma) \geq J^{CS}(0, 1, \gamma)$ and no effort will be exerted in equilibrium. It turns out that $\Delta(1, \gamma) < 0$ if

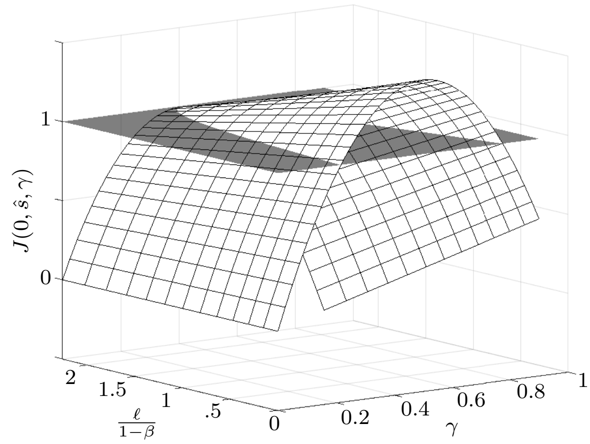
$$\frac{\alpha\pi}{1-\beta} < \frac{2-\gamma}{1-\gamma}. \tag{21}$$

The next proposition summarizes this discussion.

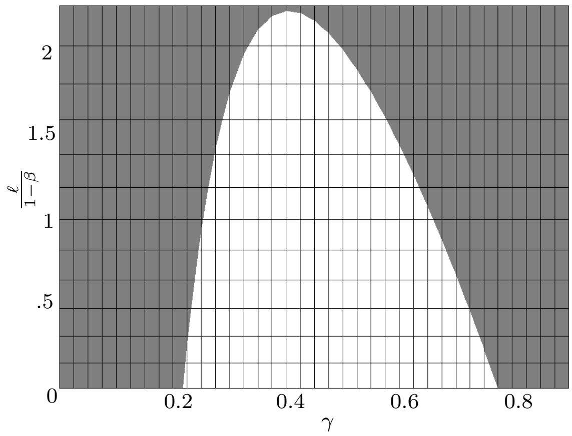
Proposition 6. *If* (21) *holds there exists a sufficiently high value of the dead-weight loss, ℓ, for which the high-effort equilibrium will fail to exist.*

The intuition for this result is as follows. Suppose that a judge decides not to exert any effort when all the judges make the right ruling and allow in the future only novel innovators to replace existing incumbents. In that case, the main gain from not exerting effort stems from allowing obvious innovations to compete with the incumbent and eliminate the dead-weight loss. The incidence of this effect is increasing in γ, which makes the right-hand side of (21) more likely to hold. The cost of not exerting effort, however, stems from increasing the probability that a genuine innovator fails to replace a current monopolist, preventing society from reaping the gains from the innovation. When this effect is small a judge will be more inclined towards lower effort.

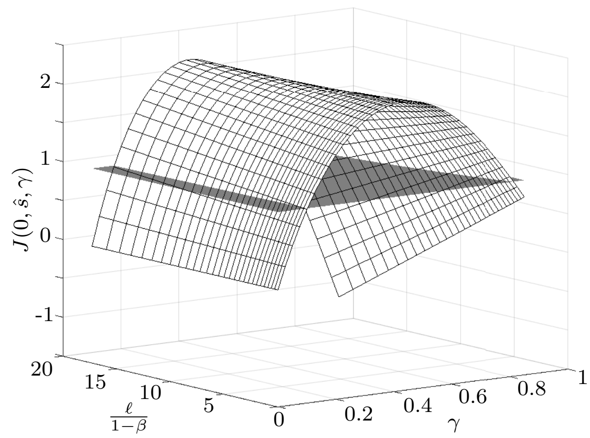
These two effects can be illustrated in Figure 7. The upper left panel of the figure shows how the cost of exerting $s = 0$ changes with different values of ℓ and γ when $\Delta(1, \gamma) < 0$. As expected, this cost is hump shaped with respect to the value of γ, as already illustrated in Figure 2. Effort will be exerted when the horizontal line, which



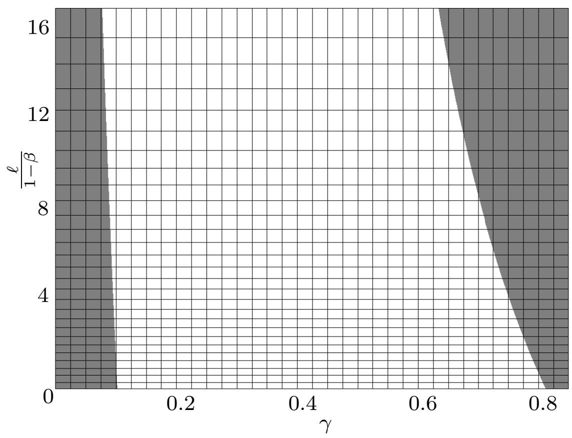
(a) $\Delta(1, \gamma) < 0$



(b) Fig. 7a: $\underline{\gamma}(1)$ and $\overline{\gamma}(1)$ as a function of ℓ



(c) Sign of $\Delta(1, \gamma)$ changing with γ



(d) Fig. 7c: $\underline{\gamma}(1)$ and $\overline{\gamma}(1)$ as a function of ℓ

Figure 7: Optimal judge effort and welfare for different values of λ.

Notes: Parameters in figures (a) and (b) are $\pi = 2.4$, $\beta = .8$, and $a = .1$; in figures (c) and (d) are $\pi = 1.2$, $\beta = .95$, and $a = .15$

indicates the cost of exerting effort, c, is lower. In the other dimension, and consistent with the previous proposition, higher values of ℓ lead to a lower cost. Consequently, as it can be seen from the upper right panel, the region where the equilibrium exhibits a positive effort level by all judges, indicated by the white area, shrinks as the dead-weight loss increases.

The lower panels show that, often, the situation might involve positive and negative values of $\Delta(1, \gamma)$ as γ changes. That is, (21) might hold only if γ is sufficiently high. The lower right panel of the figure shows one such case where the region where an equilibrium with positive effort exists, corresponds to $\Delta(1, \gamma) > 0$ for low values of γ, whereas the opposite occurs when γ is large. Consistent with the effects highlighted in the previous discussion, we can see that the higher bound shrinks as ℓ increases. The lower bound, however, exhibits the opposite behavior and, due to $\Delta(1, \gamma) > 0$, it expands as ℓ increases, since this leads to a higher cost by the judge of not exerting effort.

6 Concluding Remarks

Innovation is considered key to industry dynamics. Entry, exit, and innovation are complex interrelated phenomena in every industry, and especially so in the youngest and more technology-intensive industries. Many of these industries rely on IP as the source of temporary monopoly power that allows the successful innovators to obtain a return for their previous R&D investments. IP protection, however, is a double cutting edge knife for the dynamics of innovative industries, as the protection of incumbent innovators may be an obstacle to the success of novel innovators.

This paper contributes to the growing literature that analyzes the role of IP protection by embedding it in an industry dynamics setting in which innovation and imitation are different, interrelated processes modeled along similar lines. We find that welfare and innovation are maximized with zero protection against further innovation and, conditional on this, with full protection against imitation. However, if some protection against innovation is unavoidable, allowing for some imitation may be socially beneficial. We show how these results are the outcome of the combined decisions of a Patent Office that engages in ex-ante screening of incoming innovators and courts that are asked ex-post to analyze the merits of the cases brought to them when entrants are in conflict with market incumbents.

References

COCKBURN, I. M. and MACGARVIE, M. J. (2011). Entry and patenting in the software industry. *Management Science*, **57** (5), 915–933.

GALLINI, N. T. (2002). The economics of patents: Lessons from recent u.s. patent reform. *Journal of Economic Perspectives*, **16** (2), 131–154.

GROSSMAN, G. M. and HELPMAN, E. (1991). Quality ladders in the theory of growth. *Review of Economic Studies*, **58** (1), 43–61.

HALL, B. H. and ZIEDONIS, R. H. (2001). The patent paradox revisited: An empirical study of patenting in the u.s. semiconductor industry, 1979-1995. *Rand Journal of Economics*, **32** (1), 101–28.

HELLER, M. A. and EISENBERG, R. S. (1998). Can patents deter innovation? the anticommons in biomedical research. *Science*, **280**, 698–701.

HOPENHAYN, H., LLOBET, G. and MITCHELL, M. (2006). Rewarding sequential innovations: Prizes, patents and buyouts. *Journal of Political Economy*, **104** (6), 1041–1068.

LEE, T. and WILDE, L. L. (1980). Market structure and innovation: A reformulation. *Quarterly Journal of Economics*, **94** (2), 429–36.

LEVIN, R. C., COHEN, W. M. and MOWERY, D. C. (1985). R&d appropriability, opportunity, and market structure: New evidence on some schumpeterian hypotheses. *American Economic Review*, **75** (2), 20–24.

LOURY, G. C. (1979). Market structure and innovation. *Quarterly Journal of Economics*, **93** (3), 395–410.

SCOTCHMER, S. (2004). *Innovation and Incentives*. Cambridge, MA: MIT Press.

Appendix

Proof of Lemma 1. To show existence we start by imposing the steady condition $x_t = x_{ss}$ for all t in eq. (2), obtaining eq. (5). Replacing x_{ss} into (3) we obtain eq. (6). Using the complementary slackness condition (4), we obtain $v_{ss} = p_{ss}^{-1}$. Finally, using condition (4) again and imposing steady state to (1), we find the expression for the entry flow (7). Because all these equations are linear, we have a unique solution.

To determine when the entry flow is positive, observe that (7) is positive if and only if $\pi > (1-\beta)/p_{ss}$. From Proposition 1 we know that p_{ss} is increasing in $\lambda\mu_0$. Thus a sufficient condition for entry follows from taking $\lambda\mu_0 = 0$ so that $p_{ss} = \alpha\mu_1$, which delivers the result. ■

Proof of Propositions 1 and 2. Most comparative statics are direct, as they follow from direct differentiation. The derivative of the entry flow with respect to $\lambda\mu_0$ is

$$\frac{\partial e_{ss}}{\partial\left(\lambda\mu_0\right)} = \frac{(1-\alpha)}{\beta\left(\alpha\mu_1 + (1-\alpha)\lambda\mu_0\right)^2}\left[(1-\beta) - p_{ss}^2\frac{\pi}{\alpha}\right]$$

For the sign we just care about the parenthesis, which is a monotonic function of π. To show that the derivative can be positive for any value of $\lambda\mu_0$, take the maximum value of π for which there is no entry. That is, take $\pi = (1-\beta)/p_{ss}$. In this case the parenthesis becomes $(1-\beta)(1-p_{ss}/\alpha)$. But as $p_{ss} < \alpha$ whenever $\mu_1 < 1$ and the derivative is strictly positive. By continuity, the results holds for π values under which there is entry.

To show that the derivative is negative when $\mu_1 = 1$, observe that in this case $p_{ss} = \alpha$ and the parenthesis becomes $(1-\beta) - \alpha\pi$, which is negative for any values of $\lambda\mu_0$ given Assumption 1. Finally, an example of an inverted-U shape relation is given in the main text. ■

Proof of Proposition 3. The proof that $\lambda^* = 0$ whenever $\mu_1 = 1$ is given in the text. For an interior solution we compute the derivative (9) and solve for the values of λ such that (9) is equal to zero. Because the first order condition corresponds to a third degree, we obtain three candidate solution. Call them $\{0, -, +\}$. The first solution,

$$\lambda_0 = \frac{-\alpha\left(\pi\alpha\mu_1 - (1-\beta)\right)}{\mu_0\left(1-\alpha\right)\left(\pi\alpha - (1-\beta)\right)} < 0$$

corresponds to values for which (8) is equal to zero, thus never a maximum. Evaluated at λ_0, $W(\lambda_0) = 0$ as both $e_{ss}(\lambda_0) = 0$ and $p_{ss}(\lambda_0) = (1-\beta)/\pi$ hold. Using these facts, it is readily verifiable that the second order condition at $\lambda = 0$ is positive. Therefore, the critical point corresponding to a (local) maximum must lie to the right of λ_0

The second and third solutions are given by

$$\lambda_{-,+} = \lambda_0 + \alpha\frac{\left(1-\mu_1\right)\left(\pi\alpha \pm \sqrt{\alpha\pi\left(\pi\alpha + 8\left(1-\beta\right)\right)}\right)}{2\mu_0\left(1-\alpha\right)\left(\pi\alpha - (1-\beta)\right)}$$

Given Assumption 1, the denominator above is positive and the sign of the fraction depends on solution. For the negative solution the numerator is negative, and positive for the positive solution. Therefore, we have that $\lambda_- < \lambda_0 < \lambda_+$. Thus, given the analysis above, λ_+ is the solution for a local maximum. ■

Proof of Lemma 2. Using the value functions for w_M and w_C compute $\Gamma = w_M - w_C$

$$\Gamma = -e_{ss}\alpha\frac{\pi}{1-\beta}\left(1-\mu_1\right) + \left[1 - e_{ss}\left(\alpha + (1-\alpha)\lambda\mu_0\right)\right]\beta\Gamma,$$

Solving for the value of Γ delivers (14). ■

Proof of Lemma 3. We start by showing that (16) is positive when $\gamma \in (0,1)$. Assumption 1 guarantees $\hat{e}_{ss}(\hat{s}) > 0$ for $\gamma \in [0,1)$. Thus, for every $\gamma \in [0,1)$

$$\Phi = \frac{(1-\beta)(1-\gamma) + \alpha\beta\left((2-\gamma)\mu_1(\hat{s}) + \gamma\mu_0(\hat{s}) - 1\right)\hat{e}_{ss}(\hat{s})}{(1-\beta)(1-\gamma) + \alpha\beta\left(1-\gamma+\gamma\mu_0(\hat{s})\right)\hat{e}_{ss}(\hat{s})} > 0. \tag{22}$$

which implies $J(0,\hat{s},0) = 0$ and $J(0,\hat{s},\gamma) > 0$ for $\gamma \in (0,1)$. When $\gamma = 1$ we use L'Hospital's rule to show that (22) converges to a non-negative constant so that $J(0,\hat{s},1) = 0$. To complete the proof we need to show that there exists γ such that $J(0,\hat{s},\gamma) > c$. Take $\gamma = 1/2$,

$$J(0,\hat{s},1/2) = \frac{\pi}{1-\beta}\frac{(1-\beta) + \alpha\beta e_{ss}(\hat{s})}{8(1-\beta) + 14\alpha\beta e_{ss}(\hat{s})}$$

which is increasing in $e_{ss}(\hat{s})$. The result follows from taking $e_{ss}(\hat{s}) = 0$ and Assumption 2. ■

Proof of Lemma 4 and 5. Both lemmas follow from differentiating (16) when $\mu_0(\hat{s})$ and $\mu_1(\hat{s})$ are given by (12) and

$$e_{ss}(\hat{s}) = \frac{(1-\gamma)(1-\beta)(\tilde{\pi}p_{ss}(\hat{s}) - 1)}{\alpha\beta k_1} \quad \text{where} \quad p_{ss}(\hat{s}) = \alpha\frac{k_1}{k_2}, \quad \tilde{\pi} = \frac{\pi}{1-\beta},$$

$k_1 = (1-\gamma)\mu_1(\hat{s}) + \gamma\mu_0(\hat{s})$, and $k_2 = 1-\gamma+\gamma\mu_0(\hat{s})$. By Proposition 1 e_{ss} is increasing in α. Therefore, it is sufficient to analyze the effect of e_{ss} on Φ. Differentiating Φ, we obtain

$$\frac{\partial\Phi}{\partial e_{ss}} = -\frac{\alpha\beta(1-\beta)(1-\gamma)(2-\gamma)(1-\mu_1(\hat{s}))}{\left((1-\beta)(1-\gamma) + \beta\alpha k_2 e_{ss}(\hat{s})\right)^2} < 0$$

Which implies that (16) is decreasing in α.

For the second comparative static, differentiating (16) width respect to $\tilde{\pi}$ we obtain:

$$\frac{\partial J(0,\hat{s},\gamma)}{\partial\hat{\pi}} = \frac{\gamma(1-\gamma)}{2}\left(\Phi(\hat{s},\gamma) + \hat{\pi}\frac{\partial\Phi}{\partial e_{ss}}\frac{\partial e_{ss}}{\partial\hat{\pi}}\right) \quad \text{where} \quad \frac{\partial e_{ss}}{\partial\hat{\pi}} = \frac{(1-\gamma)(1-\beta)p_{ss}(\hat{s})}{\alpha\beta k_1}.$$

The parenthesis above is positive for all $\gamma < 1$ and equal to

$$k_1^2\left(\frac{k_2 - (2-\gamma)(1-\mu_1(\hat{s}))\alpha}{k_2}\right) + k_1(k_2+k_3)(\hat{\pi}p_{ss}(\hat{s}) - 1) + k_2k_3(\hat{\pi}p_{ss}(\hat{s}) - 1)^2 > 0$$

where $k_3 = ((2-\gamma)\mu_1(\hat{s}) + \gamma\mu_0(\hat{s}) - 1) > 0$.

Finally, differentiating with respect to $\hat{s}$ we obtain

$$\frac{\partial J(0,\hat{s},\gamma)}{\partial\hat{s}} = K\left(\gamma(\hat{s}+1)^2 - (\hat{s}-1)^2\right)$$

where K is a positive constant. The parenthesis is increasing in $\hat{s}$ and is negative when $\hat{s} = 0$ and positive $\hat{s} = 1$. The condition in the lemma follows from solving for the value of $\hat{s}$ that makes the parenthesis equal to zero. ■

Proof of Lemma 6. Replacing the values $\hat{s} \in \{0,1\}$ we obtain (omitting γ from the argument of $J(s,\hat{s},\gamma)$ for ease of notation):

$$J(0,1) = \frac{\pi}{1-\beta}\frac{\gamma(1-\gamma)}{2} \quad \text{and} \quad J(0,0) = J(0,1)\frac{\gamma\left(2-\gamma^2\right)(1-\beta) + 2(1-\gamma)(2-\gamma)\pi\alpha}{(2-\gamma^2)\left((2-\gamma)\pi\alpha - \gamma(1-\gamma)(1-\beta)\right)}$$

Using our positive entry assumption it is possible to show that the term accompanying $J(0,1)$ in the expression for $J(0,0)$ is positive and less than one. Thus, $J(0,0,\gamma) < J(0,1,\gamma)$ for every γ. Because $\underline{\gamma}(\hat{s})$ and $\overline{\gamma}(\hat{s})$ are the solutions to $J(0,\hat{s},\gamma) = c$, the result follows. ■

Proof of Lemma 7. See main text. ■

Proof of Proposition 5. Foe ease in notation we do the proof in the γ space instead of λ. The two relate according to equation (11). We start by showing that, $e_{ss}(\hat{s},\gamma)$ is decreasing in γ (i.e., λ) when when $\hat{s}=0$. Let $\tilde{\pi}=\pi/(1-\beta)$, in this scenario $e_{ss}(0,\gamma)$ is given by:

$$e_{ss}(0,\gamma)=\frac{2(1-\gamma)(1-\beta)(\tilde{\pi}p_{ss}-1)}{\alpha\beta(2-\gamma)}$$

where $p_{ss}=\alpha(2-\gamma)/(2-\gamma^2)>0$ is decreasing in γ. It is straight forward to verify that $\mu_1^{-1}>p_{ss}$ so that, by assumption 1, entry is positive. Differentiating with respect γ we obtain

$$\frac{\partial e_{ss}(0,\gamma)}{\partial\gamma}=-\frac{2(1-\beta)\left(\gamma^2-2\right)^2\left(\tilde{\pi}\left(\gamma^2+2(1-\gamma)\right)p_{ss}^2/\alpha-1\right)}{\alpha\beta\left(4-2(1+\gamma)\gamma+\gamma^3\right)^2}$$

which is negative by the positive entry assumption. We now show that, in the absence of screening costs (i.e., $\kappa(\lambda)=0$ for all λ) the welfare function is decreasing in λ. in this scenario we have

$$W_{\kappa=0}=\frac{2(1-\gamma)(1-\beta)(\tilde{\pi}p_{ss}-1)^2}{\alpha\beta(2-\gamma)}\quad\text{thus}\quad\frac{\partial W_{\kappa=0}}{\partial\gamma}=-\frac{e_{ss}(0)(\tilde{\pi}p_{ss}\phi_1-1)}{(2-\gamma)(1-\gamma)}$$

where $\phi_1=\left(6-12\gamma+9\gamma^2-2\gamma^3\right)/\left(2-\gamma^2\right)$. As before, it can be verified that $\mu_1^{-1}>p_{ss}\phi_1$, so that this derivative is negative.

We can now prove the statement. Start by observing that $e_{ss}(0,0)=e_{ss}(1,\lambda)$ for all λ. Similarly, $p_{ss}(\hat{s}=0,\lambda=0)=\alpha=p_{ss}(\hat{s}=1,\lambda)\geq p_{ss}(\hat{s},\lambda)$ for any $\hat{s}$ and λ. Therefore, when judges perform full effort or when every obvious innovation is screen out of the market both entry probability p_{ss} and entry e_{ss} are maximized. Let γ° be the corresponding λ° in the γ space. For any $\gamma<\overline{\gamma}(0)$ choosing $\overline{\gamma}(0)$ weakly increases e_{ss} and p_{ss} and strictly decreases the screening cost. Therefore, if $\gamma^\circ\leq\overline{\gamma}(0)$ then choosing $\overline{\gamma}(0)$ dominates any screening rate in which $\hat{s}=0$. If $\gamma^\circ>\overline{\gamma}(0)$, although $\overline{\gamma}(0)$ offers larger entry and probability of success than γ° the screening-cost savings of choosing γ° may overcome the benefits. Therefore, if we bound the increase in cost, the full effort screening rate dominates. [Add condition from notes] ■

Proof of Lemma 8. Let $m>1$ be an arbitrary number of innovations in the quality ladder and $mc_m=\delta^m mc$ for any $mc>0$. In a monopolized niche, due to the unit elasticity of demand, the incumbent wants to charge the highest price feasible. In this case $p=mc_{m-1}$. Then, the incumbent profits are given by $\pi=(p-mc_m)q=a(1-\delta)$ which is independent of the number of innovations and the baseline cost mc. The dead-weight loss in the market is given by

$$\int_{mc_m}^{mc_{m-1}}q(p)dp-\pi=a\left(\ln\left(\delta^{-1}\right)-(1-\delta)\right)$$

also independent of m and mc. ■

Proof of Proposition 6. The function $\Delta(\hat{s},\gamma)$ in (20) is given by

$$\Delta(\hat{s},\gamma)=\frac{\beta\alpha\left((2-\gamma)\mu_1(\hat{s})+\gamma\mu_0(\hat{s})-1\right)e_{ss}(\hat{s})-(1-\beta)}{\beta\alpha\left((1-\gamma)+\gamma\mu_0(\hat{s})\right)e_{ss}(\hat{s})+(1-\beta)(1-\gamma)}.$$

To have an equilibrium where every judge perform effort a necessary condition is $J^{CS}(0,1,\gamma)>c$. In this case we have

$$\Delta(1,\gamma)=\frac{(1-\gamma)\pi\alpha-(2-\gamma)(1-\beta)}{(1-\gamma)\pi\alpha}$$

which is negative whenever condition (21) holds. Since $\lim_{\delta\to0}\ell=\infty$, $J^{CS}(0,1,\gamma)$ can be made arbitrarily small (or even negative) with a sufficiently large ℓ, inducing judges not to exert effort. ■