Inventory Dynamics and Supply Chain Coordination

A shorter version of this paper is forthcoming in *Management Science*.

**Harish Krishnan**  
Operations and Logistic Division  
Sauder School of Business  
University of British Columbia

and

**Ralph Winter**  
Strategy and Business Economics Division  
Sauder School of Business  
University of British Columbia

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Harish Krishnan and Ralph A. Winter*
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Abstract

This paper extends the theory of supply chain incentive contracts from the static newsvendor framework of the existing literature to the simplest of dynamic settings. We analyze the coordination of the inventory and pricing incentives within a supply chain. A manufacturer distributes a product through retailers who compete on both price and fill-rates. Unsold inventory is carried over from one period to the next. We show that inventory durability is the key factor in determining the underlying nature of incentive distortions and their contractual resolutions. When the product is highly perishable, retailers are biased towards excessive price competition and inadequate inventories. Vertical price floors or inventory buy-backs (subsidies for unsold inventory) can coordinate incentives in both pricing and inventory decisions. When the product is less perishable, the distortion is reversed and vertical price ceilings or inventory penalties coordinate incentives.

*Sauder School of Business, University of British Columbia; harish.krishnan@sauder.ubc.ca; ralph.winter@sauder.ubc.ca. We thank Yigal Gerchak and Julie Mortimer for helpful discussions about this research. We also thank seminar participants at the University of Illinois, University of Southern California, University of Western Ontario, Washington University at St. Louis, University of Wisconsin at Madison, and Southern Methodist University for constructive comments. We gratefully acknowledge support from the Social Sciences and Humanities Research Council and the Natural Sciences and Engineering Research Council of Canada.
1 Introduction

Price and inventory, the most basic of managerial decisions, are inextricably linked, and the control of pricing and inventory decisions is a central problem in any supply chain. Car manufacturers care about the price and inventory levels chosen by their dealers; movie studios care about the price and availability of their products in rental outlets; and manufacturers in all product categories care that their products are priced and stocked appropriately at retail stores. The coordination of price and inventory incentives along a supply chain is a central problem of management science, not only in the theoretical literature but in practice.

The literature on supply chain incentives is built almost entirely on the assumption that inventory is completely perishable and that the traditional, static, newsvendor model applies (see Cachon 2003 for a review). This assumption places a strong limitation on the existing research. In practice, the ability to carry inventory over time is obvious in most businesses. And in the academic literature, dynamics have always been central to inventory theory but are almost entirely missing from the recent work on the coordination of supply chain incentives.

In the model developed in this paper, a manufacturer distributes a product through downstream retailers facing uncertain demand. The retailers compete on both price and fill-rates. The key characteristic distinguishing a dynamic inventory model from a static model is, of course, inventory durability. In investigating the impact of dynamics on incentive coordination, the natural focus is therefore on changes in the durability of inventory. How do such changes affect incentive distortions and their resolution through coordinating contracts? We find that incentive distortions along the supply chain and their contractual resolutions vary dramatically with changes in inventory durability.

If the product is highly perishable, competing downstream retailers are biased towards excessive

\footnote{Fill-rate competition reflects the fact that inventory often plays a strategic role by attracting customers who value the greater probability of finding their preferred product in stock. As Dana (2001) points out, “car dealers, video rental chains, department stores, mail order suppliers, and appliance stores” regularly advertise availability. For example, in 1998, the video rental chain Blockbuster launched the successful “Go Home Happy” marketing campaign highlighting the increased availability of popular movies in its stores (Dana and Spier 2001). Ioannou et. al. (2008) estimate the effect of inventory levels on rentals in the U.S. video rental industry. The retail store Casual Male, which sells clothing to men who wear larger sizes, has advertised greater availability of popular lines of clothing (Speer 2006). Bernstein and Federgruen (2004) provide additional examples.}
price competition and away from competing on inventory or adequacy of stocks. A price floor or a subsidy for holding inventory can serve to eliminate this distortion in retailer incentives: the price floor and the inventory subsidy both prevent retailers from competing intensively on price and at the same time add to each retailer’s marginal benefit of carrying inventory. When the product is less perishable, however, the distortion in retailers’ competitive strategies can be reversed. The manufacturer may then optimally impose a price ceiling on retailers, thus lowering retail prices and dampening competition in the inventory dimension. A penalty on inventories achieves the same effect.

Our insights into the impact of dynamics on optimal contracts can be traced ultimately to a single effect. As shown in Krishnan and Winter (2007), the static model that has been at the core of the supply chain coordination literature does not allow for perhaps the most fundamental of vertical incentive distortions - the double-mark-up effect on prices: under a wholesale price or two-part pricing contract, once inventory is sold, the upstream manufacturer has no interest in the retail price. The upstream revenues are already determined. Recognizing the dynamics of real-world inventory decisions resurrects the double mark-up effect on prices; with the carry-over of inventory, the manufacturer now has a direct interest in the retailer’s price decision in any period because this decision influences how much the retailer buys in future periods. A dynamic theory - in which inventory is carried over to future periods - is therefore not only realistic but essential in understanding the incentive and contracting issues that arise in supply chains.

In the next section of this paper, we review the related literature on vertical contracts. After setting out our dynamic model, we analyze the impact of inventory durability on retailer incentives. While adopting the simplest possible dynamic model, we also set out a series of extensions that must be addressed in a full analysis of incentive coordination in a dynamic model.

2 Related Literature

Cachon (2003) reviews the extensive literature dealing with supply chain incentives. The economics literature on vertical restraints (see Katz 1989) analyzes the role of price restraints in resolving incentive conflicts. Papers in both literatures analyze price and inventory incentives in a static setting;

In a dynamic setting, Bernstein and Federgruen (2004) analyze oligopoly price and inventory competition but do not analyze the vertical coordination of downstream decisions. The paper closest to ours is Bernstein and Federgruen (2007). Assuming that unsatisfied demand is backlogged, i.e. satisfied in the next period, they show that the first-best solution can be achieved when each retailer is offered a (unique) wholesale price combined with a “backlogging penalty” per unit of backlogged demand. Wang and Gerchak (2001) analyze a static model with deterministic demand where decentralized retailers choose inventory levels that influence demand.

None of the papers above has identified the role that product durability plays in shaping firms’ incentives. The optimal management of perishable inventories by a single firm has been extensively studied in the inventory theory literature (see Nahmias (1982) for a well-cited review). The literature leaves open, however, the question addressed in this paper: how product durability (or perishability) affects supply chain incentive coordination.

A key assumption in our paper, that customers are attracted to firms with high fill-rates, is not standard in inventory theory. Inventory is usually assumed to play a purely operational role, i.e. the values of high inventories is simply that they allow demand to be satisfied in more states of the world. But the importance of inventory availability to customers and firms is well documented (see Corsten and Gruen (2004) and Anderson et. al. (2006)). Several researchers have analyzed the impact of this strategic effect of inventory on a single firm’s decisions (Wang and Gerchak (1994) and Balakrishnan et.al. (2004), Dana and Petruzzi (2001)) and oligopoly models (Li (1992), Li and Lee (1994), Deneckere and Peck (1995), Hall and Porteus (2000), and Dana (2001)). With the exception of Dana (2001), these papers assume – as we do – that availability is observable to consumers. Bernstein and Federgruen (2004, 2007) do analyze price and fill-rate competition in a dynamic setting but, as noted above, do not consider how inventory durability affects incentives.
3 Model

The theory on incentive coordination even within the traditional static models can be quite complex, and dynamic inventory models, even in a centralized setting, can also be difficult to analyze. Therefore, in asking how inventory dynamics impact incentive coordination it is essential to adopt the simplest possible dynamic model.

We consider a monopolist distributing a single product, in discrete time, through two competing retailers. The only link between periods is the inventory that is carried over. In combination with a stationarity assumption on demand, this will yield a recursive, stationary structure in which optimal contracts can be characterized as the solutions to simple, myopic problems.

3.1 Demand

Demand in each selling period is uncertain and depends on the price and inventory level chosen by each retailer $i$. In each period $t$, at retail prices $p_t = (p_{1t}, p_{2t})$ and inventory levels $y_t = (y_{1t}, y_{2t})$, the demand at retailer $i$ is $q_{it}(p_t, y_t, \phi_t)$, where $\phi_t$ is a random variable representing demand uncertainty. The number of transactions or sales by firm 1 in period $t$ is then given by $T_{i1}(p_t, y_t, \phi_t) = \max(q_{i1}(p_t, y_t, \phi_t), y_{i1})$. The uncertainty parameter $\phi_t$ is independent and identically distributed over time.

Consumers base their shopping decisions on the price and expected fill rate at each store. In any period, consumers shop at only one retail outlet and unsatisfied demand in each period is lost.\(^2\) We assume that in every period consumers observe the price and inventory choices of both retailers and can therefore infer fill rates: we thus follow the literature in abstracting completely from time delays in learning about inventories, assuming that reputation formation is instantaneous and that firms can costlessly and perfectly convey availability.\(^3\) In Section 7, we discuss how this assumption

\(^2\)In principle, our model can be extended to incorporate backlogging. Backlogging occurs, for example, when a customer who does not find the product in stock has the product delivered after a delay (or is informed when the product arrives in stock). Under either assumption, what is important for our analysis is that the customer gets a higher utility when the product is immediately available. Our lost-sales assumption is that the consumer settles for some second-best alternative (which can include simply forgoing consumption) if the product is not in stock.

\(^3\)As Dana (2001) has pointed out, in reality consumers may not be able to observe firms’ inventory choices. Firms can, however, communicate this information to consumers in two broad ways. First, stores build reputations for good fill rates. Customers can obtain this information by learning through experience (over multiple periods) or through an information intermediary that collects information about a store’s fill-rate performance. Second, firms can signal their private information about current availability in a variety of ways. Dana (2001) argues that a high price can signal a high fill-rate. In practice, however, firms employ a broader set of signals. Car dealerships often advertise
may be relaxed.

Note that the decision of a consumer to shop at store \( i \) reduces the fill rate for all other consumers at that store. In this respect, the demand side is analogous to demand in a market with congestion externalities. Since consumers impose negative externalities on each other, the demand functions \( q_{it} \) are themselves determined as the outcome of a game among consumers. With enough structure on the model, which we shall provide in Section 5, the existence of an equilibrium in this demand-side game is assured.

Throughout the paper we assume that a firm’s demand function, and therefore its transaction function, is non-increasing in its own price and non-decreasing in the price charged by the other outlet.\(^4\) Similarly, demand and transactions are non-decreasing in a firm’s inventory level and non-increasing in the inventory level at the other outlet. We also assume that the partial derivatives \( \partial ET_{it}/\partial p_{it}, \partial ET_{it}/\partial y_{it}, \partial ET_{jt}/\partial p_{it} \) and \( \partial ET_{it}/\partial y_{it} \) exist. (This can be assured by, for example, assuming continuous probability distributions for the random variables.) In Section 5 we will also make assumptions to guarantee the existence of unique and symmetric solution for the centralized firm, and a symmetric pure strategy Nash equilibrium in the game between the retailers. In other words, we ensure that the transaction function \( T_{it} \) is sufficiently well behaved. For now, however, we simply assume the existence of a unique, symmetric, centralized solution and a symmetric pure strategy Nash equilibrium in the decentralized game. We abstract from a number of other features of inventory management. For instance, we assume that the lead time between a retailer placing and receiving an order is zero, and the manufacturer has sufficient capacity to satisfy the retailers’ orders. From the inventory theory literature, it is well known that capacity constraints and positive lead times (in the presence of lost sales) complicate the structure of the optimal policy.

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\(^4\)In Krishnan and Winter (2007), an increase in a rival’s price has two impacts on a firm’s demand: a direct effect of the price increase and an indirect effect that operates through inventory spillovers. The indirect effect may be negative. The contractual evidence in Krishnan and Winter indicates that the direct effect dominates. We focus here on the presumptive case where the direct effect dominates, but our analytical framework can accommodate the scenario where the indirect effect dominates.
3.2 Payoffs

3.2.1 Centralized firm

The manufacturer’s marginal production cost is \( c \) per unit. The only inventory costs incorporated in the model are due to perishability and discounting.\(^5\) Since we assume that consumer demand depends on both price and inventory levels, the penalty cost, a standard feature of many inventory models, is endogenously accounted for.\(^6\)

Consider first the payoffs of a centralized (vertically-integrated) firm that chooses price and inventory to maximize total profits. Consider initially a single period. The profit of the centralized firm in this static model (hence the superscript \( s \)) is obtained by choosing \((p, y)\) to maximize:

\[
E\Pi^s(p, y, \phi) = p_1 ET_1(p_t, y_t, \phi) + p_2 ET_2(p, y, \phi) - cy_1 - cy_2
\]

In a dynamic model, however, excess inventory is carried forward. Define \((S_1, S_2)\) as the starting inventory available at each outlet in any period. In our model, the starting inventory is the only component of history relevant for current payoffs i.e. \((S_1, S_2)\) is the state in any period. Let \(EV_t(S_1, S_2)\) denote the steady-state expected discounted profits in period \( t \) given the state \((S_1, S_2)\). Let \(\delta\) be the discount rate and \(\rho\) be the exponential rate of inventory decay.\(^7\) The dynamic programming recursion is:

\[
EV_t(S_1, S_2) = \max_{p_t \geq 0; y_{1t} \geq S_1; y_{2t} \geq S_2} E\Pi^s(p_t, y_t, \phi_t) + c \sum_{i=1}^{2} S_i \nonumber
\]

\[
+ \delta EV_{t+1}[(1 - \rho)(y_{1t} - T_{1t}(p, y, \phi_t)), (1 - \rho)(y_{2t} - T_{2t}(p, y, \phi_t))]
\]

In (2), \( c \sum_{i=1}^{2} S_i \) represents the cost saved because inventory is carried over into the current period.\(^8\) The single period payoff in the current period, therefore, is \(E\Pi^s + c \sum_{i=1}^{2} S_i\). Rather than accounting for the carried-over inventory in the current period, we account for this inventory in the

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\(^5\)A per unit linear holding cost, a standard feature of many inventory models, is redundant here because the discount factor captures this.

\(^6\)Bernstein and Federgruen (2004) provide a review of the literature on “endogenous” penalty costs.

\(^7\)The “exponential decay” model of inventory perishability is often used in the inventory literature (see Nahmias 1982).

\(^8\)As long as the optimal order upto levels, \(y_{it}\), in each period exceeds the starting inventories, \(S_{it}\), the full cost savings are a benefit. We will show that this condition holds in our stationary framework. It will not necessarily hold in a non-stationary framework.
previous period. Specifically, any inventory carried over into the current period is worth $c \sum_{i=1}^{2} S_i$ in this period, or equivalently, is worth $\delta(1-\rho)c \sum_{i=1}^{2} S_i$ in the previous period. Let $\gamma \equiv \delta(1-\rho)$; we refer to $\gamma$ as the “durability factor” as a high value of $\gamma$ indicates that the product maintains its value well into the future. We now define:

$$E\Pi^d(p, y, \phi) = E\Pi^s(p, y, \phi) + \gamma[c[y_i - ET_i(p, y, \phi) + y_j - ET_j(p, y, \phi)]]$$  \hfill (3)$$

The function $E\Pi^d$ is well defined because we have assumed sufficient conditions for $E\Pi^s$ to be well defined. As a result, it is clear that the dynamic program (2) satisfies the four sufficient conditions for the existence of a myopic optimum (see Heyman and Sobel (2004), pp. 83-85).

Since the payoff to a centralized firm in a dynamic model with inventory carry-over is myopically optimal, the stationary solution is found by choosing $(p, y)$ to maximize the modified single period payoff function $E\Pi^d$. The aim of contractual solutions to incentive problems is (to the extent possible) to duplicate this joint profit solution. We next analyze the fully decentralized solution in order to identify incentive distortions. Since we are interested in the feasibility of decentralizing downstream price and inventory decisions, we set aside the complete contract where the manufacturer simply specifies both the price and inventory decisions downstream.

### 3.2.2 Decentralized firms

We assume that in any period the manufacturer charges the retailers a per-unit wholesale price $w$ and a fixed fee $F$. In a stationary dynamic model, optimal $w$ and $F$ are functions of the state variable. In our model, however, constant marginal production cost yields optimal $w$ and $F$ as constants. We will also allow the manufacturer to specify various other contractual terms at the outset, such as vertical price restraints (floors or ceilings), inventory subsidies (e.g., buy-backs) and inventory penalties.

The best-response payoff function for each retailer can be defined using arguments similar to that used for the centralized firm. In the static model, the profit of retailer $i$, gross of the fixed fee $F$, is given by choosing $(p_i, y_i)$ to maximize:

$$E\pi_i^s(p_i, y_i; (p_j, y_j)) = p_i ET_i(p, y, \phi) - wy_i$$  \hfill (4)$$
In the dynamic case, retailer $i$’s best-response payoff function is:

$$E v_{it}(S_i) = \max_{p_t \geq 0; y_t \geq S_i} E \pi_i^s(p_t, y_t, \phi_t) + cS_i + \delta Ev_{i,t+1}[(1 - \rho)(y_t - T_{it}(p_t, y_t, \phi_t))]$$  (5)

This function also satisfies the conditions for a myopic optimum (using an argument analogous to the centralized case). Retailer $i$’s myopically optimal best-response function, again gross of the per-period fixed fee $F$, is given by:

$$E \pi_i^d(p_i, y_i, \phi; (p_j, y_j)) = E \pi_i^s(p, y, \phi) + \gamma w[y_i - ET_i(p, y, \phi)]$$  (6)

Note that the excess inventory is simply valued by the firm as a reduction in the necessary expenditure on inputs next period. For the decentralized retailer, this reduction in future costs depends on the wholesale price. For the centralized firm, the cost savings depend on the savings in the production cost. As we show, the difference in “salvage value” of unsold inventory (between the centralized and decentralized firms) plays a crucial role in the dynamics of the incentive distortions in the supply chain.

4 The Incentive Distortions

The starting point to any contract design problem is to identify why agents' incentives may be distorted relative to collectively optimal decisions. At the joint profit maximizing decision (where the centralized firm’s first-order conditions satisfied), why might a decentralized retailer have a marginal incentive to deviate?

We extend the static framework of Krishnan and Winter (2007) to the dynamic problem considered in this paper. In our simple approach this involves the comparison of first-order conditions for the individual retailer’s optimum with those of the collective optimum. This framework yields two kinds of distortions between an individual retailer’s marginal gains and collective gains from increasing price or inventory. The first is the *vertical externality* whereby the retailer ignores the impact on the upstream manufacturer’s profits of a change in either decision. The second source of distortion is the *horizontal externality* whereby the retailer ignores the impact on profits earned by its rival retailer, and the upstream manufacturer, from sales to the rival retailer.
For the dynamic problem, using (6) and (3), we can solve for the difference in first-order conditions and decompose this difference as follows:

\[
\frac{\partial E\pi_i}{\partial y_i} = \partial E\Pi^d - (w - c)(1 - \gamma(1 - \frac{\partial ET_i}{\partial y_i})) - (p_j - \gamma c)\frac{\partial ET_j}{\partial y_i}
\]

(7)

\[
\frac{\partial E\pi_i}{\partial p_i} = \partial E\Pi^d - \gamma(w - c)\frac{\partial ET_i}{\partial p_i} - (p_j - \gamma c)\frac{\partial ET_j}{\partial p_i}
\]

(8)

In (7), the first term represents the marginal flow of profits upstream from an additional unit of \(y_i\) (the vertical externality); the second term represents the marginal flow of profits to both the rival retailer and the upstream firm through the impact of \(y_i\) on the transactions at outlet \(j\). Similarly, in (8), the first term represents the marginal impact on the upstream firm’s profit from an increase in \(p_i\) and the second term represents the impact on profits of the other firms by the impact of an increase in \(p_i\).

Note that for each decision, \(y_i\) and \(p_i\), the two vertical and horizontal externalities are opposite in sign. The purchase of an additional unit of inventory, for example, increases upstream profits directly but reduces the rival’s profit and upstream profits resulting from the impact on the rival’s sales. Note in addition that the vertical externality is increasing in \(w\) but the horizontal externality is independent of \(w\). This means that for each action, inventory and price, there is a particular \(w\) that will leave the externalities exactly offsetting and the retailer’s incentives optimal, conditional upon the level of the other action.

Can a contract that specifies only a wholesale price \(w\) and a fixed fee \(F\), with no other contractual provisions, achieve the first-best price and inventory in a decentralized setting? This requires that the same \(w\) leaves the externalities offsetting in both equations. (Because of the availability of fixed fees, the wholesale price \(w\) is an instrument used entirely for eliciting the right incentives; it need not be used to collect rents.)

If the pairs of externalities are not offsetting at the same \(w\), then we can say that retailers are distorted towards excessive price competition or towards excessive inventory competition. For
example, suppose setting \( w \) equal to some value \( w_{\text{inv}} \) renders the pair of externalities in the inventory equation identical (and similarly define \( w_{\text{price}} \)). If, at \( w = w_{\text{inv}} \), the horizontal externality in the price equation exceeds (in absolute value) the vertical externality in that equation then the retailer’s incentive is distorted towards setting price too low, i.e. towards excessive price competition. Similarly, if the horizontal price externality is lower than the vertical price externality then the retailer’s incentive is to price too high, i.e. towards insufficient price competition. Eliciting the correct value of price will involve setting \( w = w_{\text{price}} < w_{\text{inv}} \) and this will lead to excessive inventories.

A special case: complete perishability

Consider the single-period model where inventory perishes at the end of the period. In other words, the inventory carried over is always zero or, equivalently, \( \gamma = 0 \). From equations (7) and (8), setting \( \gamma = 0 \) yields the following:

\[
\frac{\partial E\pi_s}{\partial y_i} = \frac{\partial E\Pi^s}{\partial y_i} - (w - c) - \frac{\partial ET_j}{\partial y_i} \quad \text{(9)}
\]

\[
\frac{\partial E\pi_i}{\partial p_i} = \frac{\partial E\Pi^s}{\partial p_i} - p \frac{\partial ET_j}{\partial p_i} \quad \text{(10)}
\]

The last equation captures the main feature of the decentralization of price and inventory decisions in the static model. For a fixed level of inventory, \textit{pricing decisions are not subject to a vertical externality}. In other words, given the inventory choice of an outlet, the manufacturer has no \textit{direct} interest in the price at which the inventory is resold. However, the manufacturer is not indifferent to the pricing decision. Rather, from the manufacturer’s perspective the downstream pricing decision is \textit{always} distorted.\(^9\)

In which direction is the retail price distorted? Note that the inventory decision is affected by both the horizontal and vertical externalities (see (9)) while only the horizontal externality is at work in distorting the pricing decision (see (10)). The horizontal pricing externality invariably depresses prices, and since there is no offsetting vertical externality, decentralized pricing is biased

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\(^9\)Since the horizontal externality reduces profits for each outlet, the fixed fee that the manufacturer can charge up front is reduced.
towards a price that is, at the margin, too low.

We now explore the implications of the fact that this insight is sensitive to the perishability of the product. In particular, we show that if the product is non-perishable, then decentralized retailers can be biased towards excessive inventory competition. The vertical contracts that coordinate the channel, therefore, also depend on product durability.

**The general case:**

Now consider the case where inventory is carried forward and \( \gamma \in (0, 1) \). Now the pricing decisions are subject to a vertical externality. The manufacturer has a direct interest in the retail price, as it affects the amount the retailer will purchase in future periods (see (8)).\(^\text{10}\)

The wholesale price \( w \), which is endogenous, affects the vertical externalities. Incentives are aligned, and coordination is achieved, if the manufacturer can choose \( w \) such that the horizontal and vertical externalities in both price and inventory decisions cancel out. (Recall that a fixed fee can be used to transfer profits from the retailers to the manufacturer.) But under what conditions will the same wholesale price correct the incentive distortions in both the inventory and price decisions?

The answer depends entirely on the durability factor. In general, given \( \gamma \), a single wholesale price that cancels out horizontal and vertical externalities in both price and inventory decisions may not exist. However, there may exist a particular value of \( \gamma \), \( \tilde{\gamma} \), at which a wholesale price \( \tilde{w} \) alone can coordinate incentives. To solve for the \( \tilde{\gamma} \) and \( \tilde{w} \) defined above, we set the externality terms in (8) and (7) to zero (when evaluated at \( (p^*, y^*) \)). By eliminating \( \tilde{w} \) we get,

\[
\tilde{\gamma} = \frac{\partial ET_j/\partial p_i}{(\partial ET_j/\partial y_i)(\partial ET_i/\partial p_i) + \partial ET_j/\partial p_i - (\partial ET_j/\partial p_i)(\partial ET_i/\partial y_i)} \bigg|_{(p^*, y^*)} \tag{11}
\]

Because of the signs of the partial derivatives, \( \tilde{\gamma} \) is always positive. In any real example, the durability factor \( \gamma \) must be less than or equal to 1. Accordingly, if \( \tilde{\gamma} \geq 1 \), then the externality terms in (8) and (7) do not sum to zero for any \( \gamma \) in the feasible range \((0, 1)\). From (11) it follows

\(^{10}\)In our model, with no inventory holding cost, the limit \( \gamma = 1 \) is uninteresting since there is no opportunity cost whatsoever to maintaining additional inventory and the firms would carry infinite inventory.

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that a necessary condition for $\tilde{\gamma} < 1$ is the following:

$$\frac{\epsilon_{pi}^t}{\epsilon_{pi}^c} < \frac{\epsilon_{yi}^t}{\epsilon_{yc}^t}$$

(12)

where $\epsilon_{pi}^t$ represents the price-elasticity of individual outlet transactions, and $\epsilon_{pi}^c$ represents the price-elasticity of combined (total) transactions, both evaluated at $(p^*, y^*)$. (Note that $p_i$ refers to the price charged by a decentralized retailer, and $p_c$ is the price charged by a centralized firm.

As the example in Section 5 shows, the market for the centralized firm and the market for each decentralized retailer are not identical; and so the price and inventory elasticities are different. The inventory-elasticities, $\epsilon_{yi}^t$ and $\epsilon_{yc}^t$, are defined similarly.\(^\text{11}\)

If (12), which we refer to as the elasticity ratio condition, is satisfied, and if $\gamma = \tilde{\gamma}$, then the wholesale price $w = \tilde{w}$ and a fixed fee can elicit the optimal price and inventory decision and can transfer rents between the two firms. But if $\gamma \neq \tilde{\gamma}$, then the same wholesale price cannot correct the incentive distortion in both price and inventory. If we choose $w$ to elicit the optimal inventory decision, then if $\gamma < \tilde{\gamma}$, the retail outlets will choose a price that is too low. (The last two terms in (8), $-\gamma(w - c)\partial ET_i/\partial p_i - (p_j - \gamma c)\partial ET_j/\partial p_i$, will sum to a negative quantity.) On the other hand, if $\gamma > \tilde{\gamma}$, then the retail outlets will choose a price that is too high. Another way of interpreting this statement is as follows: if $w$ is chosen to elicit the optimal price, then where $\gamma > \tilde{\gamma}$, the retail outlets choose an inventory level that is too high. We summarize the above arguments in the following proposition.

**Proposition 1**

(a) If $\frac{\epsilon_{pi}^t}{\epsilon_{pi}^c} < \frac{\epsilon_{yi}^t}{\epsilon_{yc}^t}$, then $\tilde{\gamma} < 1$ and:

(i) If $\gamma = \tilde{\gamma}$, then a wholesale price and fixed fee contract can elicit the optimal price and inventory decision.

(ii) If $\gamma < \tilde{\gamma}$, the outlets are excessively oriented towards price competition.

(iii) If $\gamma > \tilde{\gamma}$, the outlets are excessively oriented towards inventory competition.

\(^{11}\)Note that lowering prices and raising inventories are two ways of attracting demand. The idea is that contracts are necessary to fix the distortion in the “marginal rate of substitution” between these two instruments in attracting demand. The elasticity ratio is precisely the marginal rate of substitution.
(b) If $\frac{\epsilon_t}{\epsilon_{pc}} \geq \frac{\epsilon_y}{\epsilon_{yc}}$, then $\tilde{\gamma} \geq 1$ and the outlets are excessively oriented towards price competition for all values of $\gamma \in (0, 1)$.

This proposition describes the impact of dynamics – i.e. inventory durability – on supply chain incentive distortions in the case of a two-part pricing contract (with a per-unit wholesale price and a fixed fee). Note that as we move from the centralized firm to the individual decentralized firm, elasticities of demand with respect to price or inventory increase. According to Proposition 1(a), if price elasticity increases proportionately more, then retailers are always biased towards price competition and away from inventory competition relative to the collective optimum. If “inventory elasticity” increases proportionately more, then retailers may be biased away from price competition if the durability factor $\gamma$ is sufficiently high. If inventories are perishable, then the vertical externality on price is not strong enough to offset the horizontal externality of price and retailers are biased towards price competition. (In the extreme, static, case, the vertical externality on price is “missing.”) As inventories become less expensive to hold, however, the vertical externality on price can be strong enough to more than offset the horizontal price externality.

But what determines whether the pivotal elasticity ratio condition is satisfied? Clearly this depends on the specific form of the transaction function $T$. We must impose additional structure including assumptions on consumer preferences in order to determine the direction of the inequality in the elasticity ratios. We impose the necessary structure by developing, in Section 5, a simple spatial model of consumer preferences. The stylized model allows us to predict, for a reasonable set of assumptions, the nature of incentive distortions and their contractual resolutions.

### 5 Structured Model

We assume a set $\Theta$ of customer types, indexed by $\theta$. A given realization of demand is a density of customers on $\Theta$. Demand, however, is itself random. For simplicity, the random density is perfectly correlated across $\theta$.\(^{12}\) We index the realization of demand by $\phi$ and so the number of potential consumers of any type $\theta$, is simply $\phi$. The random variable $\phi$ has density $g$ and distribution function

\(^{12}\)In other words, the realization of demand across consumer types is perfectly correlated, e.g., if demand doubles at one value of $\theta$, it doubles for all values of $\theta$. 

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Consumer type $\theta$ attaches a value $u^i_{\theta}$ to one unit of the product offered by firm $i$. Consumers will choose between two firms and have time to visit only one store in each period. They make this choice before realizing random demand. Each customer will therefore choose to visit the store that yields the highest expected consumer surplus, which depends on the price and the fill rate. Each consumer purchases: zero units, or one unit of the product from retailer 1, or one unit from retailer 2.

For concreteness, assume that the two retail outlets are located at two ends of a unit line segment. Each outlet competes over the customers who are uniformly distributed in the space between the two retailers, with each consumers location indexed by $s \in [0, 1]$. Consumers incur a travel cost of $t$ per unit distance traveled. For our model, it turns out to be essential to also allow consumers to vary in the extent to which they are attracted by a reliable inventory, i.e. a higher fill rate. We do this simply by assuming that the consumers vary not only in their location but also in their value of the product. Consumers with a high product value care more about the fill rate relative to the price, at the margin. Consumers valuation for the product is indexed by $v \in [0, \hat{v}]$. Therefore, consumers each occupy a point in the customer space ($\Theta$) illustrated by a rectangle in Figure 1, and each consumer type $\theta$ is indexed by their location and valuation, i.e., $\theta \equiv (s, v)$.

The determination of the demand partitions is itself the outcome of a game because of the negative externalities consumers impose on each other: the decisions of consumers to shop at store $i$ reduce the fill rate for all consumers at that store. In the equilibrium of this demand-side game, consumers are partitioned into three sets as illustrated in Figure 1: those that purchase from 1; those that purchase from 2 and the no-purchase set.

### 5.1 Demand-side equilibrium

In each period, let $n = (n_1, n_2)$ be the measure of the set of consumer types that each consumer expects will shop at each outlet, given prices $p = (p_1, p_2)$ and inventory levels $y = (y_1, y_2)$. In other words, contingent upon the realization of $\phi$, the consumer expects $(\phi n_1, \phi n_2)$ shoppers. Define $\psi(n; (p, y)) = [\psi_1(n; (p, y)), \psi_2(n; (p, y))]$ as the measures of the sets of consumers that
would actually purchase at each retailer with the expectations \( n \). Then a fixed point \( \mathbf{n}(\mathbf{p}, \mathbf{y}) = [\tilde{n}_1(\mathbf{p}, \mathbf{y}), \tilde{n}_2(\mathbf{p}, \mathbf{y})] \) of \( \psi \) is a rational expectations equilibrium of the game. The demand under realization \( \phi \) is then \( q(\mathbf{p}, \mathbf{y}, \phi) = [q_1(\mathbf{p}, \mathbf{y}, \phi), q_2(\mathbf{p}, \mathbf{y}, \phi)] = [\phi \tilde{n}_1(\mathbf{p}, \mathbf{y}), \phi \tilde{n}_2(\mathbf{p}, \mathbf{y})] \).

We assume proportional rationing (see Tirole 1988; page 213) of inventory in the event of a stock-out; in other words, all customers who shop at a particular outlet have the same probability of getting the product.\(^\text{13}\) Hence, given \((\mathbf{p}, \mathbf{y})\) and expectations \( \mathbf{n} \), let \( H_i(y_i, n_i) \) represent the anticipated fill probability at retailer \( i \). Note that \( H_i(y_i, n_i) = \int \phi \min\{1, \frac{y_i}{\phi n_i}\} dG(\phi) = \int_0^{\frac{y_i}{\phi n_i}} dG(\phi) + \int_{\frac{y_i}{\phi n_i}}^{\infty} \frac{y_i}{\phi n_i} dG(\phi) \). The utilities for consumer type \( \theta = (s, v) \) from shopping at retail outlet 1 and 2 are:

\[
\begin{align*}
u_1^1 &= (v - p_1)H_1(y_1, n_1) - ts \\
u_2^2 &= (v - p_2)H_2(y_2, n_2) - t(1 - s)
\end{align*}
\]

and the consumers in the no purchase set derive zero utility. The demand partition is given by \( D_1(\mathbf{p}, \mathbf{y}, \mathbf{n}) = \{\theta | (u_\theta^1 \geq \max(u_\theta^2, 0)\} \) and similarly for \( D_2(\mathbf{p}, \mathbf{y}, \mathbf{n}) \) and the no-purchase set.

Given expectations \( \mathbf{n} \) and \((\mathbf{p}, \mathbf{y})\), \( \psi_i(\mathbf{n}; (\mathbf{p}, \mathbf{y})) = \int I_{D_i(\mathbf{p}, \mathbf{y}, \mathbf{n})}(\theta) d\theta \) where \( I_X(\theta) \equiv \{\theta \theta \in X\} \) is the

\(^{13}\)We rule out, for example, that consumers living near a store are first in line to get the product.
characteristic function of the set $X$. Note that

$$\int I_{D_1(p,y,n)}(\theta) d\theta = \int_{p_1}^{\hat{\theta}} s_1^1(v) dv + \int_{\hat{\theta}}^{\bar{v}} s_r(v) dv$$  \hspace{1cm} (13)$$

$$\int I_{D_2(p,y,n)}(\theta) d\theta = \int_{p_2}^{\hat{\theta}} s_2^2(v) dv + \int_{\hat{\theta}}^{\bar{v}} (1 - s_r(v)) dv$$  \hspace{1cm} (14)$$

where $s_1^1(v)$ refers to outlet 1’s customers on the product margin, $s_2^2(v)$ refers to outlet 2’s customers on the product margin, $s_r(v)$ refers to outlet 1’s customers on the inter-retailer margin, and $\hat{\theta} = (\hat{s}, \hat{v})$ is the point where the product margin intersects with the inter-retailer margin (see Figure 1). The following equations characterize these margins.

$$s_1^1(v) = \frac{(v - p_1)H_1(y_1, n_1)}{t} \hspace{1cm} (15)$$

$$s_2^2(v) = 1 - \frac{(v - p_2)H_2(y_2, n_2)}{t} \hspace{1cm} (16)$$

$$s_r(v) = \frac{(v - p_1)H_1(y_1, n_1) - (v - p_2)H_2(y_2, n_2)}{2t} + \frac{1}{2} \hspace{1cm} (17)$$

$$\hat{\theta} = \frac{t + p_1H_1(y_1, n_1) + p_2H_2(y_2, n_2)}{H_1(y_1, n_1) + H_2(y_2, n_2)} \hspace{1cm} (18)$$

$$\hat{s} = \frac{H_1(y_1, n_1)}{H_1(y_1, n_1) + H_2(y_2, n_2)} - \frac{(p_1 - p_2)H_1(y_1, n_1)H_2(y_2, n_2)}{t(H_1(y_1, n_1) + H_2(y_2, n_2))} \hspace{1cm} (19)$$

It is clear from (13) and (14) that the mapping $\psi$ is continuous and, since the set of $n$ is compact and convex, $\psi$ has a fixed point by Brouwer’s fixed point theorem. This fixed point must be unique for $q(p, y, \phi)$ to be well defined. The following Lemma is proved in the Appendix.

**Lemma 1** For all $\phi$, $q(p, y, \phi)$ is well defined and differentiable. $q_1(p, y, \phi)$ is decreasing in $p_1$ and $y_2$ and increasing in $p_2$ and $y_1$ over the range of $(p, y)$ where $D_1(p, y, n)$ and $D_2(p, y, n)$ are non-empty; similarly for $q_2(p, y, \phi)$.

### 5.2 Elasticity ratio comparison

Before determining the direction of the inequality in the elasticity ratio condition, we must characterize the Nash equilibrium in the game between retailers. Unfortunately, a difficulty in differentiated Bertrand models, as well as models in which firms choose quantities and prices, is that payoff functions are in general not quasi-concave. As a result, pure strategy equilibria may not exist (see d’Aspremont, Gabszewicz and Thisse 1979 and Friedman 1988).$^{14}$ This issue arises in the game

$^{14}$A mixed strategy equilibrium exists as long as payoff functions are continuous and strategy spaces are compact (Glicksberg 1952).
between retailers in the structured model described above.\textsuperscript{15} The standard approach in such models, which we adopt here, is to restrict exogenous parameters to ranges where the pure strategies do exist.\textsuperscript{16} Accordingly, we restrict consideration to the range of model parameters where a pure strategy equilibrium is guaranteed. We also restrict attention to the symmetric case, i.e. to retail markets that are sufficiently differentiated that (given the symmetry in demands) the centralized solution is also symmetric. The following proposition is proved in the Appendix.

**Proposition 2** Under the assumptions of the structured model, $\frac{\epsilon_i^{\pi}}{\epsilon_i^{pc}} < \frac{\epsilon_i^{y_i}}{\epsilon_i^{yc}}$. Therefore, $\tilde{\gamma} < 1$.

From Proposition 1, we know that there is a range of durability factors for which retailers are biased towards price competition. Proposition 2 tells us that in the structured model, there is a range of values of the durability factor for which retailers are biased towards inventory competition. In other words, the nature of incentive distortions (and their contractual resolutions) depend upon the most fundamental product characteristic in any inventory model: the durability or perishability of the product.

The intuition for Proposition 2 flows from the balance of two effects. For perishable products, the “missing” vertical externality on price dominates, and the retailers are biased towards price competition. In the structured model, why is this overturned for durable products? The intuition is depicted in Figure 1. For any changes in price and inventory, the marginal impact on the centralized firm is determined entirely through the tastes of consumers on the product margin: the consumers on the border of the “no purchase set.” Since these consumers have relatively low valuation for the product, the cost to them of encountering a stock out is small and the marginal ex ante value that they attach to greater inventory is therefore low. The aggregate profits are maximized at a combination of low inventory and low price. A retail outlet, on other hand, chooses price and inventory to accommodate the tastes of consumers on its margin. This includes part of the product margin but also the inter-retailer margin. The consumers on the inter-retailer margin have a higher

\textsuperscript{15}Intuitively, when the products offered by the two outlets are very close substitutes (travel costs are low), then an outlet may attain one local optimum in price by competing intensively with its rival, and another local optimum by forgoing such competition, charging a near-monopoly price, and relying on the chance of a stock-out at its rival to generate revenue.

\textsuperscript{16}For example, Salop (1979) and the entire address model literature on product differentiation (see Eaton and Lipsey 1989).
valuation on average and therefore attach higher ex ante value to inventories. When inventory is not perishable, i.e. it retains its value well into the future, the analysis therefore points to a distortion towards excessive inventory competition and excessive pricing in a decentralized retail system. This distortion is traced directly to consumer heterogeneity in product valuation.

6 Coordinating Contracts

In the previous section we showed that a wholesale price alone can elicit optimal price and inventory decision only if $\gamma = \tilde{\gamma}$ and $\tilde{\gamma} < 1$. The nature of incentive distortions and, as we show in this section, their contractual resolutions, hinge on the durability of the product, i.e. on whether $\gamma < \tilde{\gamma}$ or $\gamma > \tilde{\gamma}$.

We focus on three types of contracts: price restraints (floors or ceilings) and contracts that subsidize or penalize retail inventories.

6.1 Price restraints

Suppose retailers are biased towards excessive price competition: if the manufacturer sets $w$ to elicit the optimal inventory level $y^*$ the resulting price is lower than $p^*$. If the retailer’s payoffs are quasi-concave, the manufacturer can correct the pricing distortion simply by setting a price floor at $p^*$ (lowering $w$ enough can elicit the optimal inventory). A price floor, in short, can be used as an instrument in addition to $w$ to elicit $(p^*, y^*)$ when retailers are biased towards excessive price competition. By a similar argument, a price ceiling is an optimal response to an incentive distortion towards excessive inventory competition. This reinforces the insight from Deneckere et.al. (1996) and Krishnan and Winter (2007) that price floors help provide incentives to hold inventory in markets where inventories are “important,” i.e. where inventories are relatively perishable or expensive to hold. But, interestingly, price ceilings coordinate the channel when inventory is less important – which mirrors the well-known optimality of price ceilings in particular price-only situations (e.g., to fix the double mark-up problem). In summary:

Proposition 3 Assume that the retailer i’s payoffs are quasi-concave in $(p_i, y_i)$ given $(p_j, y_j)$.
When $\gamma < \tilde{\gamma}$, a wholesale price, fixed fee, and a price floor can elicit the efficient price and inventory decisions. When $\gamma > \tilde{\gamma}$, a price ceiling achieves the efficient outcome.
6.2 Subsidies and penalties on inventory

In the static model of Krishnan and Winter (2007) we showed that a buy-back contract can elicit the optimal price and inventory. The key to this result is that a buy-back contract introduces a vertical externality in the price decision. The manufacturer cares about the retailer’s pricing decision once a buy-back is in place, because the manufacturer must pay the retailer an amount $b$ for each unsold unit and the number of unsold units depends on price.

In the dynamic case, we use a similar argument to show that an inventory subsidy can coordinate the channel when $\gamma$ is low. But when the product is less perishable ($\gamma > \tilde{\gamma}$), an inventory penalty is needed to align incentives. To see this, let $b$ represent the subsidy paid to the retailer (or penalty paid by the retailer if $b < 0$) per-unit of inventory at the end of each period. Joint profits are still given by $E\Pi^d$ as in equation (3). In the decentralized case, retailer $i$’s profit function, gross of the per-period fixed fee $F$, is given by:

$$E\pi_i(p_i, y_i; (p_j, y_j)) = p_iET_i(p, y, \phi) - wy_i - (\gamma w + b)[y_i - ET_i(p, y, \phi)]$$ (20)

With these instruments, the difference in first-order conditions can be represented as follows:

$$\frac{\partial E\pi_i}{\partial y_i} = \frac{\partial E\Pi^d}{\partial y_i} - \left(\gamma(w - c) + b\right)\frac{\partial ET_i}{\partial y_i}$$ (21)

Define $w^*$ and $b^*$ as the values of $w$ and $b$ that cancel out the vertical and horizontal externalities in price and inventory at $(p^*, y^*)$. From equations (21) and (22) it follows that:

$$w^* = c + (p^* - \gamma c)\left[\frac{\partial ET_i}{\partial p_i}(1 - \frac{\partial ET_i}{\partial y_i}) - \frac{\partial ET_i}{\partial y_i}\right]_{(p^*, y^*)}$$ (23)

$$b^* = (p^* - \gamma c)\left[\frac{\partial ET_i}{\partial p_i} - \gamma\left(\frac{\partial ET_i}{\partial p_i}(1 - \frac{\partial ET_i}{\partial y_i}) - \frac{\partial ET_j}{\partial y_i}\right)\right]_{(p^*, y^*)}$$ (24)
Note that $b$ is decreasing in $\gamma$. When $\gamma = 0$ the results from the static model immediately fall out as they must: a wholesale price $w^* > c$ and a buy-back price $b^* > 0$ fully align incentives.\textsuperscript{17} Further, it also falls out that when $\gamma = \tilde{\gamma}$, then the wholesale price $w^* = \tilde{w}$ alone can coordinate both price and inventory incentives; and $b^* = 0$. (This can be verified by substituting equation (11) into (24).) Finally, when $\gamma \in (0, \tilde{\gamma})$, then $b^* > 0$, and when $\gamma \in (\tilde{\gamma}, 1)$, then $b^* < 0$. In sum:

**Proposition 4** Assume that the retailer $i$’s payoffs are quasi-concave in $(p_i, y_i)$ given $(p_j, y_j)$. When $\gamma < \tilde{\gamma}$, a wholesale price, fixed fee, and an inventory subsidy (i.e. buy-back) can elicit the efficient price and inventory decisions. When $\gamma > \tilde{\gamma}$, an inventory penalty achieves the efficient outcome.

### 6.3 Practical considerations in the choice of contracts

An appropriately designed price restraint, inventory subsidy, or an inventory penalty can coordinate price and inventory decisions in our theory. This raises the practical question: which of these contracts should a manufacturer choose? The choice between alternative coordinating contracts is outside the formal theory and depends upon the practical considerations in implementing each contract. The contracts considered in this paper require monitoring of retail prices or inventories. Monitoring is not costless, of course, and the costs depend upon the specific product and industry.

Another consideration is anti-trust laws which, in the past, have constrained the use of both kinds of price restraints. At different times, price floors (resale price maintenance or RPM) have been per se illegal in the U.S. – most recently between the repeal of the McGuire and Miller-Tydings Acts in 1975 and *Leegin Creative Leather Prods. v. PSKS, Inc.*, 2007 WL 1835892 which, in June 2007, overruled *Dr. Miles Medical Co. v. John D. Park & Sons Co.*, 220 U.S. 373 (1911). But price floors were popular when they were legal, and even when they were per se illegal manufacturers were free to unilaterally adopt a plan to establish suggested resale prices in advance and lawfully terminate retailers who fail to adhere to those prices (*United States v. Colgate & Co.*, 250 U.S. 300, 39 Sup. Ct. 465, 7 A.L.R. 443). Vertical price ceilings have been legal in the United

\textsuperscript{17}The optimal $w^*$ and $b^*$ obtained here are identical to Proposition 4 of Krishnan and Winter (2007).
States since 1996 (State Oil Co. v. Khan, U.S.S.C. 96-871). In summary, recent developments have relaxed many of the legal restrictions on the use of price restraints in the United States. In addition, Gurnani and Xu (2006) point out that resale price maintenance is prevalent and has been unambiguously legal in other jurisdictions such as China and Hong Kong.

The successful implementation of price restraints, however, requires the monitoring of retail prices. In practice, the monitoring of price floors can effectively be decentralized because (in a typical resale price maintenance case) violations are initially reported by a competing retailer. Similarly, violations of price ceilings are typically reported by customers.

The use of inventory subsidies and inventory penalties have not been subject to legal restrictions but these instruments carry significant monitoring costs. Inventory subsidies and penalties require the monitoring of retail inventories. In many industries, enterprise resource planning (ERP) systems capture and store real-time data about retail inventories. The accuracy of inventory records is, however, often questionable (Raman et. al. 2001) and compromises the viability of contracts using this information. A moral hazard problem distorts the information flow in that a retailer may manipulate inventory records to receive subsidies or to avoid paying penalties.

In some cases, for example with completely perishable products, it may be relatively easy for the manufacturer to verify the number of unsold units. (A well known example is the case of bookstores sending the torn-off covers of paperback books and magazines to publishers to obtain credit.) When inventory is carried forward by the retailer for sale in future periods, verifying the number of unsold units carried forward may be more complicated.

In sum, monitoring and enforcement conditions which are necessarily outside the simplest formal model of supply chain coordination influence the choice of contractual responses to the incentive distortions that we have set out.

7 Extensions

7.1 Extension of theory to non-stationarity

This paper extends the theory of supply chain contracting to the simplest of dynamic models. Our dynamic model yields a stationary, recursive equilibrium in which both the sources of incentive
distortions and their contractual resolutions are clearly identified. The parallel between a static framework and a stationary equilibrium means that the methodology developed in Krishnan and Winter (2007) can be readily adapted. Incentive distortions are traced to vertical and horizontal externalities.

In the dynamic context, however, the framework must incorporate externalities on future payoffs as well as current payoffs. The distortions in future payoffs are summarized by the impact of the externalities on the next period’s value function. In this paper’s stationary setting, the externalities on future payoffs are channeled through the durability factor, $\gamma$. The static model is the limiting case of the dynamic model developed here, as $\gamma \rightarrow 0$.

In principle, this approach would be equally valid in various non-stationary settings. While a full development is beyond the scope of the paper, we outline here the structures that would allow for three natural extensions. In our model, the stationarity of the optimal policy for both the decentralized and the centralized programs depends on a standard set of four conditions (Heyman and Sobel (2004), pp. 83-85). Each of the three extensions outlined below departs from the foundations for a myopically optimal solution by violating one of these four conditions.

Non-stationary demand: Extending our model to non-stationary demand, for example with Markov-modulated demand, would capture an additional element of realism. In this more general setting, a drop in demand increases the likelihood of both excess inventory and low demand next period. In some states, therefore, the on-hand inventory will exceed the optimal base-stock level. Unlike the present setting, the optimal policy will then have a “base-stock list-price” structure; see Federgruen and Heching (1999) for a description of this policy. The optimal policy will no longer be stationary; nor would the coordinating contracts. The vertical and horizontal externalities are still at the heart of incentive distortions (and the need for complex contracts) in a non-stationary model but are not so simply characterized.\(^\text{18}\)

Inventory with Shelf-lives: The exponential decay of inventory in our model is another simplifying assumption, allowing a recursive model with a single state variable (excess inventory) at each outlet. This is the natural first step for a model of inventory perishability but, in reality, perishability is

\(^{18}\)Another natural extension, to a finite-horizon model, is a special case of non-stationary demand.
not so simple. In the literature, a commonly considered alternative is fixed (or random) shelf life. Incorporating this feature in our model would involve at least two state variables (assuming a two period shelf file). The fundamental sources of incentive distortions (vertical and horizontal) would be unchanged, but again their characterization would be more complex. Even the centralized solution could not be characterized independently of the state variables.

**Reputation Dynamics:** Our model incorporates consumers’ concerns over the availability of products - the fill rate - into the determination of demand in inventory models. We have extended the inventory literature on fill-rate dependent demand to deal with not just the determination of optimal inventory and pricing for a single firm, but with the design of contracts to elicit optimal incentives. But we have retained the near-universal assumption from the literature that consumer learning about fill rates is instantaneous - an assumption incorporated in our model via that the ability of consumers to observe firms’ inventory levels. An important open task in this entire literature is the integration of realistic reputational dynamics into the theory of inventory dynamics. Consumers clearly care about fill rates, and it is unrealistic to assume, following the literature (including this paper), that changes in fill rates are recognized immediately.

In the research on reputational dynamics from which one could draw upon in developing this extension, the classic articles are Shapiro (1983), Klein and Leffler (1981) and Holmström’s (1999) “career concerns” model. Of these the most promising framework is Holmström’s, in which there is both hidden action and hidden information. The hidden information aspect of Holmström’s framework acts to mitigate the hidden action (or moral hazard) element in that an agent works hard so that his higher output causes the market to identify him as a higher type and therefore as having a higher likely future output.\(^\text{19}\)

In the inventory context, in the case of a single firm, the natural hidden information assumption would be private information about firms’ unit costs of production (which are the costs of producing inventory). A stock-out and low fill-rate in one period would lead buyers to increase their (Bayesian-updated) probability with which the firm was high cost and therefore less likely to produce adequate inventory in the future. The firm’s “reputation” at any time would be perceived probability that a

\(^{19}\)The hidden information aspect of the model generates incentives to work even in a finite period model, which would not arise in a model with only hidden action.
firm was a high-cost type, conditional upon its history of stock-outs; and the firm would invest in inventory in the current period partly to avoid a drop in reputation resulting from a stock-out.

The full development of reputational dynamics would be of most value initially in a single-firm, optimal inventory model. But the integration of reputational dynamics, while complex, would also add insight the theory of supply chain coordination as developed here. A new issue that would emerge in a decentralized equilibrium would be reputational spillovers: when costs are correlated among outlets, the inference by consumers from a stock-out at one outlet would be that costs are likely higher at all outlets - and therefore the probability of a future stock-out higher at all outlets. The integration of reputational dynamics into inventory “availability” models would also be complicated by the fact that rational consumers would infer product availability not only from the history of fill rates but also from prices, as in Dana (2001).

7.2 Linking to contracting practice

The propositions in this paper provide testable implications. Specifically, for perishable products (including products with a high risk of obsolescence) we predict price floors or inventory subsidies. For durable products, on the other hand, we predict price ceilings or inventory penalties.

In general, data on vertical contracts are confidential. In addition, Ippolito (1991) has pointed out, when price floors (also known as resale price maintenance or RPM) were illegal under antitrust laws “direct attempts to collect data on the use of RPM through surveys or similar methods” was “futile.” While the per se illegality of RPM has been overturned in the U.S., and firms may be more willing to share information about these practices, no such study exists and collection of data through primary data sources is beyond the scope of this paper.

The evidence, however, is that price floors have been popular in the distribution of perishable products. As noted in Krishnan and Winter (2007), inventory perishability (or limited shelf life) may be the result of: (1) seasonality in demand (e.g. greeting cards, toys and other holiday gifts, sports equipment); (2) physical depreciation (e.g. pharmaceuticals); (3) product obsolescence (e.g. magazines, newspapers, etc.); and (4) fashion goods with a limited period of popularity (e.g. 20

Somewhat related ideas are explored in Olsen and Parker 2008 and references therein, but they do not capture reputation as a state variable.
clothing). When it was legal, RPM was popular in all these industries. The evidence also suggests that buy-backs (inventory subsidies) are used in many of these markets. No comparable evidence is available unfortunately, to test the link between price ceilings, or inventory penalties, and the distribution of durable products.

8 Conclusion

This paper integrates inventory dynamics into the analysis of price and inventory incentives in a supply chain. The paper adopts a methodology in which an individual retailers’ incentives for marginal changes in price and inventory are compared with the collective payoff of the supply chain in order to identify incentive distortions. The roles of various contractual strategies in resolving the incentive distortions are also immediately apparent in the framework. The central insight in our application to a dynamic setting is that the incentive distortions and optimal contractual responses hinge on the durability of the product.

Two restrictive features of our model warrant discussion. We adopt a first-order approach to characterizing incentive distortions, but apply it to an environment (price competition between differentiated firms downstream) that is well-known to yield non-concavities in payoff functions and possible nonexistence of equilibria. We set aside these technical problems, following much of the economics literature, with the benefit of clear characterization of both the sources of incentive distortions and the role of contractual strategies in resolving the distortions. Departing from the first-order approach to this problem requires particular functional forms, simulation, or restrictive economic assumptions to avoid non-convexities.21 We view our methodology as complementary to these alternative approaches in that it sets out clear intuition for the roles of various contracts in coordinating supply chain decisions.

The second arguably restrictive feature of our model is its stationarity. Stationarity is adopted by much of the inventory literature and is, we believe, valuable as a first step in integrating dynamics into the theory of supply chain coordination. Many aspects of inventory policy beyond a one-state-variable, stationary structure are important in reality and are addressed in the inventory literature.

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21 Note that very few papers in the literature have considered the coordination of retail decisions under price and inventory competition.
In these models, however, the structure of the optimal centralized policy is itself complex; the contracts required to coordinate incentives would be even more complex.

In this paper, we have outlined approaches to three extensions to our stationary model. Other extensions, including the incorporation of production lead times and manufacturer capacity constraints, could also be considered. The full development of these non-stationary models are open topics that are beyond the first steps taken here to integrate inventory dynamics and supply chain coordination of pricing and inventory decisions. We believe, however, that the methodology set out in this paper will be useful in studying these extensions.

References


Appendix

Lemma 1:

Proof. The following property of \( \psi \) is easily verified: \( \psi_1 \) is strictly decreasing in \( n_1 \) and strictly increasing in \( n_2 \) and similarly for \( \psi_2 \). Given \((p, y)\) suppose that there are two fixed points \( n^a = (n^a_1, n^a_2) \) and \( n^b = (n^b_1, n^b_2) \), with \( n^a \neq n^b \).

Case 1: Let \( n^a \succeq n^b \) (i.e., \( n^a_1 \geq n^b_1 \) and \( n^a_2 \geq n^b_2 \)). If \( n^a \succeq n^b \), it must be true that \( \psi(n^b|(p, y)) \leq \psi(n^a|(p, y)) \), which is a contradiction unless \( n^a = n^b \).

Case 2: Let \( n^a_1 \geq n^b_1 \) and \( n^a_2 \leq n^b_2 \). It must be true that \( \tilde{n}^a_1 \leq \tilde{n}^b_1 \) and \( \tilde{n}^a_2 \geq \tilde{n}^b_2 \), which is a contradiction unless \( n^a = n^b \).

Differentiability can be shown by using the system version of the implicit function theorem, applied to \( \Psi(n;p,y) = \psi(n;p,y) - n \). (Applying the system implicit function theorem involves verification that the determinant of the Jacobian matrix of \( \Psi(n;p,y) \), with respect to \( n \) is of full rank.) To show that \( q_1(p,y,\phi) \) is decreasing in \( p_1 \), note that if it were not, then a higher \( p_1 \) would be accompanied by a higher \( n_1 \) which would imply that more customers prefer retailer 1 despite the higher price and demand. This is a contradiction. The impact changes in \( y_1 \), \( p_2 \), and \( y_2 \) on \( q_1(p,y,\phi) \) can be shown similarly. ■

Proposition 2:

Proof. To prove that \( \check{\gamma} < 1 \), we need to show that \( \frac{\partial \check{\gamma}}{\partial p} < \frac{\partial \check{\gamma}}{\partial y} \). Define \( T(p,y,\phi) = \sum_i T_i(p,y,\phi) \) as the total transactions in each period (summing over transactions at both outlets). It is sufficient to show that

\[
\frac{\partial ET_i(p,y,\phi)}{\partial p} < \frac{\partial ET_i(p,y,\phi)}{\partial y}
\]

where the numerators represent the marginal effect of changes in retail price and inventory on the expected transactions at an outlet, and the denominators represent the marginal effect of changes in retail price and inventory (simultaneously at both outlets) on the expected total transactions. Because demand uncertainty is perfectly correlated across all consumer types, it is sufficient to prove that

\[
\frac{\partial q_i(p,y,\phi)}{\partial p} < \frac{\partial q_i(p,y,\phi)}{\partial y}
\]
where \( Q(p, y, \phi) = \sum_i q_i(p, y, \phi) \).

From Figure 1 note that

\[
\frac{\partial Q}{\partial p} = 2 \frac{\partial[\int p s_p(v)dv + (\bar{v} - \hat{v})]}{\partial p} 
\]

(25)

\[
\frac{\partial Q}{\partial y} = 2 \frac{\partial[\int p s_p(v)dv + (\bar{v} - \hat{v})]}{\partial y} 
\]

(26)

For retail outlet 1 (the argument for retail outlet 2 is analogous):

\[
\frac{\partial q_1}{\partial p_1} = \frac{\partial\left[\int p_1 \hat{s}(v)dv + \hat{t}(\bar{v} - \hat{v})\right]}{\partial p_1} = 1 \frac{\partial Q}{\partial p} - \frac{\partial(\hat{s}(\bar{v} - \hat{v}))}{\partial p} 
\]

(27)

\[
\frac{\partial q_1}{\partial y_1} = \frac{\partial\left[\int p_1 \hat{s}(v)dv + \int p \hat{s}(v)dv\right]}{\partial y_1} = 1 \frac{\partial Q}{\partial y} + \frac{\partial \int p \hat{s}(v)dv}{\partial y} 
\]

(28)

We need to show that \( \frac{(27)}{(25)} < \frac{(28)}{(26)} \), i.e.,

\[
\frac{1}{2} - \frac{\partial(\hat{s}(\bar{v} - \hat{v}))}{\partial p} < \frac{1}{2} + \frac{\partial \int p \hat{s}(v)dv}{\partial y} 
\]

The inequality is satisfied because both \( \frac{\partial(\hat{s}(\bar{v} - \hat{v}))}{\partial p} \) and \( \frac{\partial \int p \hat{s}(v)dv}{\partial y} \) are greater than 0. ■