Measuring the True Harm from Price-Fixing to Both Direct and Indirect Purchasers

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Draft: April 25, 2007
ABSTRACT

Legal actions by direct and indirect purchasers to recover damages as a result of price-fixing by suppliers have been common in the United States for many years and are now beginning in a number of other countries including Australia and Canada. This paper argues that traditional measures of harm used in establishing damages as a result of price-fixing may be conceptually flawed and that they will often significantly understate the true harm suffered by downstream purchasers. The largest errors are associated with circumstances in which downstream markets are less than perfectly competitive. The paper provides measures of the degree of understatement of the true harm when traditional approaches are used and shows how the size of the error depends on various model parameters that relate to the degree of competitiveness of downstream markets, the number of competitors downstream and the degree of product homogeneity downstream. The paper also provides measures of distribution of the true harm between direct and indirect purchasers.
1. INTRODUCTION

In a number of countries it is possible for individuals and businesses harmed by the price-fixing activities of sellers to launch legal actions against those sellers in an attempt to recover damages.¹ Many of these actions are “class actions” that combine the (frequently) small individual damage claims of numerous plaintiffs into one large action. Nowhere has there been more such activity than in the United States which has allowed private actions for damages under the antitrust laws since 1914. As class action procedures have developed in the U.S. and elsewhere in recent years, the number of these kinds of actions has grown significantly. In addition to the United States, class actions for damages as a result of price-fixing are now permitted in Canada, Australia and some European countries. Recent years have witnessed many very large actions, frequently involving international cartels as defendants. For example, the vitamin price-fixing conspiracies of the 1990s have led to damage awards or settlements in class actions of $1.17 billion in the United States, CDN$140 million in Canada and AUS$30.5 in Australia.²


² Sources: U.S. case -- Wall Street Journal (Eastern Edition), New York, NY, November 4, 1999, p. A.3; Canadian case -- National Post, Don Mills, ON, June 1, 2005, pg. FP2; Australian case -- Wall Street Journal (Eastern Edition), New York, NY, July 18, 2006, p. B8. The American total cited here is based on the original settlement. Subsequently many class members opted out to pursue their own actions, presumably because they believed they could secure even larger damage awards. In fact, none of these numbers should be taken to represent the sum off all damage awards in each county. There are frequently multiple lawsuits launched by different sets of plaintiffs. It is common, for example, for large direct purchasers to pursue their own private damage actions separate from class actions being led on behalf of smaller direct and indirect purchasers. Therefore these amounts simply represent the largest public settlements of which we are aware. For details on the vitamin conspiracy, see Connor [2007, chapters 10-12] who also has information about damage awards in other cases (chapter 15).
Apart from establishing that the defendants were indeed responsible for the harm to plaintiffs – often accomplished by simple reference to earlier convictions or guilty pleas from actions prosecuted by antitrust agencies – the most important step in these proceedings is establishing what harm was done to plaintiffs as a way to establish the appropriate level of damages to be paid.

In this paper we argue that the standard measures of harm used in these proceedings may produce substantial underestimates of the true harm suffered by direct purchasers (those who bought directly from a cartel member) and indirect purchasers (those who bought from direct purchasers or from those further down the distribution chain). Specifically, we address two questions: (i) how much greater is the true total harm from price-fixing than that estimated by traditional methods and what determines the size of this error?; and (ii) what determines the share of this harm borne by direct purchasers as opposed to indirect purchasers? While related to it, by taking into account all factors that affect downstream agents’ surpluses (e.g. quantities sold before and after price-fixing as well as prices paid and received) this second question is importantly different from the question of the degree of “pass-through” of cartel price increases considered in a number of papers. We take up the relationship between these “price pass-through” and “harm-sharing” problems below.

Our results suggest that traditional measures may capture less than half the true harm done by price-fixing when downstream markets are not perfectly competitive. We also show that the fraction of the harm borne by direct purchasers will depend to a considerable degree on factors that influence the level of competition downstream and can range in our model from zero to 67%.
Throughout this paper we will be careful to distinguish between what we shall call “harm”, which involves losses in economic surplus to direct and indirect purchasers as a result of the price-fixing and its consequent downstream effects, and “damages”, which we take to be a legal term representing payments defendants must make to plaintiffs as a result of the harms suffered. In some cases the law (and courts) may equate damages with proven harm, but in other cases they are not exactly the same thing. For example, the law may not recognize that some harms are recoverable as damages\(^3\) and in some cases the law may make the damages some multiple of the actual harm.\(^4\) Our focus then is in getting the measure of harm correct, we are not making arguments about how these measures should be translated into damage awards. This said, we recognize that most measures of damages will be based to some extent on the true harm suffered, and therefore there is value in getting harm measured correctly.\(^5\) We also note the potential usefulness of correct measures of harm caused to be used in the determination of appropriate fines by antitrust courts and authorities.

2. TRADITIONAL MEASURES OF HARM

Because private actions to recover damages from price-fixing have become so numerous (particularly in the United States) there is now an extensive literature on how one measures this harm.\(^6\) The emphasis in most of the work done by economists is on

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\(^3\) For example, as a result of the decision in the *Illinois Brick* case discussed below, harm to indirect purchasers are not recoverable as damages in Federal price-fixing class actions in the United States.

\(^4\) Under U.S. antitrust law, successful plaintiffs are entitled to damages equivalent to three times the harm established.

\(^5\) Black’s Law Dictionary [1990] defines “damages” as: “pecuniary compensation or indemnity, which may be recovered in courts by any person who has suffered loss, detriment, or injury, whether to his person, property, or rights, through the unlawful act or omission or negligence of another.”

providing techniques to determine what price would have existed “but for” the defendants’ price-fixing activity. The most common approach to measuring harm takes the difference between the cartel’s price and the but-for price (i.e. the per-unit overcharge) and multiplies it by the number of units purchased by direct purchasers.

This measure is illustrated below in Figure 1. Let $w^1$ be the collusive price and $w^0$ be the price that would have obtained without (or “but for”) the price-fixing. The amount of harm suffered is then calculated as the per-unit overcharge $(w^1 - w^0)$ multiplied by the amount of the product sold during the period of collusion ($X^1$). This would be area $R$ (for “Rectangle”) in Figure 1. The damage suffered by any one buyer is then the overcharge times the quantity that buyer purchased during the period of collusion.

There are two reasons to believe that area $R$ may be a significant underestimate of the harm done to direct and indirect purchasers, even if the but-for price is correctly

Rubinfeld and Steiner [1984] and White [2001]. Some of these provide general advice and others discuss particular case applications.
estimated. First of all, it is well-recognized that area T (“Triangle”) above also represents lost surplus – these would be gains to buyers for units they would have purchased at the lower but-for price but which are not purchased at the cartel price.\(^7\) Area T is typically not estimated and included in the measure of harm for two main reasons: it requires additional information (the but-for quantity) and it is in any case often viewed as small relative to area R. We would argue that in many cases estimating T – using well-established techniques – is not necessarily so difficult and that it may often be big enough to be worth the effort. For example, Brander and Ross [2006] provide a simple example in which T is half as large as R, meaning that R itself captures only 2/3 of the harm represented by R+T.

The second reason why R is a poor measure of the total harm is our main concern in this paper. While standard discussions of the measuring of harm implicitly or explicitly assume the first (direct) purchasers are also the final consumers, this is seldom the case. Most often cartels raise prices to direct purchasers who distribute and resell the product or who use the product as an input into the production of some other product.\(^8\) When this happens we would typically expect the cartel’s price increase to eventually lead to higher prices all the way downstream to final consumers. Along the way there is the potential for additional harm to be created as each stage suffers from some reduction in its consumers’ surplus. This is particularly true when the downstream stages are less than perfectly competitive. An example in Brander and Ross [2006] illustrates this most clearly for the case in which final demand is linear and the downstream market is a

\(^7\) See, e.g. Connor [2007, pp. 88-90], Clark et al [2004] and Brander and Ross [2006].

\(^8\) Consider for example the samples of cartel cases reviewed in Connor [2007] and in Levenstein and Suslow [2006] which include cartels related to the production of vitamins, lysine, sorbates, aluminum, copper, steel, sugar, sulfur and tin.
monopoly. In that case, the area R+T would be an accurate measure of the harm suffered by the downstream monopolist only, but it would ignore the harm to final consumers from the higher downstream prices that would inevitably result from the higher input prices.\(^9\)

Here we develop a simple two-stage model of a vertical chain to address two important questions:

(i) How good or poor an estimate of total harm are the measures used traditionally in price-fixing damage actions and on what factors does this accuracy or lack thereof depend? In addressing this question, we will consider both the error associated with using simply area R -- as is most common -- and also the error that would be associated with using areas R+T as the measure of harm.

(ii) Once we know how much the true total harm is, what fraction of that harm would reside with direct purchasers and what fraction would fall to the indirect purchasers (here all final consumers)?

Our research builds on a literature that seeks to establish the correspondence between input and output market Marshallian surpluses. For example, Jacobsen [1979] studies input price changes and derives the relationship between changes in input market surplus, downstream firm profits and downstream consumer’s surplus for the cases in which the downstream industry is either perfectly competitive or completely monopolized. Quirmbach [1984] extends these results by considering a homogeneous product oligopoly downstream.\(^10\) Basso [2006] provides a unifying framework that nests and generalizes the models from these two papers and provides a starting point for our analysis here.

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\(^9\) Hellwig [2006] shows this as well.

\(^10\) Quirmbach’s analysis is restricted to changes in a single price only, however. Other references in this literature include Anderson [1976], Carlton [1979] and Schmalensee [1976].
We further extend this work by considering more general models of competition that include product differentiation and, in parts, more general cost functions. We then apply these results to the questions posed above regarding the accuracy of traditional estimates of harm to others due to price-fixing. Importantly, this means we are focusing not on overall welfare losses from price-fixing but only on the effects on direct and indirect purchasers.\textsuperscript{11}

3. THE MODEL

As indicated, we use a simple model in which the cartelized industry is selling to a downstream market of buyers who in turn resell the product or embed it in a new product for sale to final consumers. There is therefore, after the direct purchasers, only one tier of indirect consumers.

To begin, we define a system of linear and symmetric $n$ (possibly) differentiated final good demands given by:

\[
p^i(q) = a - bq_i - e \sum_{j \neq i} q_j
\]

Where $b \geq e$. If $b > e$ then products are differentiated and the system can be inverted to obtain:

\[
q^i(p) = A - B p_i + E \sum_{j \neq i} p_j
\]

Where $a, b, e, A, B$ and $E$ are positive parameters.\textsuperscript{12}

\textsuperscript{11} That is, a full welfare analysis would have to consider the benefits of price-fixing to the cartel members.

\textsuperscript{12} Note the following relationships between the parameters:
The profits of the $i$-th downstream firm will be given by:

$$\pi^i(q) = p^i(q)q^i - c^i(q, w)$$  \hspace{1cm} (3)

where $\mathbf{w}$ is the vector of prices of $Z$ inputs, and $c^i$ is the cost function. Three observations about our approach are warranted here. First, we will nest a variety of competitive possibilities in our approach by modeling this as a quantity-setting game with “conjectural variations” that can effectively correspond to different models of downstream competition including Cournot, Bertrand and collusion.

Second, we will allow for changes in a vector of input prices, $\mathbf{w}$, rather than a just a single input price. This does not mean that all inputs have to be price-fixed, because $\mathbf{w}^i$ may be equal to $\mathbf{w}^0$ in all but one component $w_z, z \in \{1,\ldots,Z\}$. Considering many inputs, however, allow us to examine what happens when various inputs are price fixed.\(^{13}\) Of course, the simple single input case is nested within our model.

Third, our cost functions are, at the outset, quite general. They do not have to be identical across firms and we do not begin with a fixed proportions production assumption downstream. Stronger assumptions will be applied later in the paper to push the analysis further.

\[ a = \frac{A}{B - (n - 1)E} \quad b = \frac{B - (n - 2)E}{(B - (n - 1)E)(B + E)} \quad e = \frac{E}{(B - (n - 1)E)(B + E)} \]
\[ A = \frac{a}{b + (n - 1)e} \quad B = \frac{b + (n - 2)e}{(b + (n - 1)e)(b - e)} \quad E = \frac{e}{(b + (n - 1)e)(b - e)} \]

Thus, if $b > e$, then $B > (n - 1)E$, a condition sometimes referred to as diagonal dominance.

\(^{13}\) For example, the vitamin cartel manipulated prices on a variety of vitamin products.
On Conjectural Variations with Differentiated Demands

If, in a homogenous products framework in which firms choose quantities, $v = \partial \sum_{j \neq i} q_j / \partial q_i$ is the common conjecture that firms have about the change in rivals’ total output when they change their own output by one unit, it is well-known that:

- If $v = -1$, the outcome is Bertrand (perfect) competition
- If $v = 0$, the outcome is Cournot competition
- If the $n$ cost functions are identical and $v = n - 1$, the outcome is collusion. If, in addition, marginal costs are constant, the collusive and monopoly outcome coincide.

A conjectural variation approach for differentiated products however, has not been described in the literature, as far as we know. Yet it is straightforward to do so when demands are linear. First, consider again, as is usual, that $v = \partial \sum_{j \neq i} q_j / \partial q_i$ is the common conjecture firms hold. Letting $c_i^e \equiv \partial c_i^e (q_i, w) / \partial q_i$, the first-order conditions of (3) from the “conjectural variations game” are given by:

$$p_i + q_i \left( \frac{\partial p_i}{\partial q_i} - ev \right) - c_i^e = 0$$

(4)

It is easy to show that, as long as the firms’ cost functions are not ‘too concave’, a unique equilibrium exists for this game. Moreover, if the cost functions were identical, the equilibrium would be symmetric.\(^{14}\)

\(^{14}\) In this paper, the focus is on cost functions that are either linear or convex. Existence, uniqueness and symmetry of equilibrium under cost symmetry follow straightforward from Theorems (2.7), (2.8) and Remark (17) in Vives [1999, p.42-43]. Formal proofs are available from the authors upon request.
Quite clearly Cournot conjectures in a differentiated setting are going to be no different from in a homogenous product setting – in either case $v_{\text{Cournot}} = 0$, as each firm chooses output assuming its choice will not lead to changes in its rivals’ outputs. Similarly the collusive conjecture that $v_{\text{Collusion}} = n - 1$ will be the same with differentiated products are long as they are fully symmetric. Deriving the appropriate value for differentiated product Bertrand competition is slightly more complicated. Note first that choosing prices in an environment of product differentiation will generate first-order conditions of the following form:

$$\frac{q_i}{\partial q_i} + p_i - c_i' = 0$$

(5)

We can then find which values of $v$ make the conjectural variations first-order conditions identical to the first-order conditions (5). For this, just equate equation (5) to equation (4) to obtain the Bertrand conjecture:

$$v_{\text{Bertrand}} = -\frac{e}{b}(n - 1)$$

(6)

For the case of Bertrand competition, it is easy to see that the conjecture fulfills the condition $-1 \leq v_{\text{Bertrand}} < 0$, because $0 < e \leq b$. In fact, the conjecture will be always larger than $-1$, except in the case in which $e = b$, which coincides with what we know from the homogenous case.

Hence, in general, it is still true that as $v$ increases, the degree of competition decreases. By having $v$ as a parameter throughout, we will be able to assess then how the degree of competition affects our results. And by using the different values of $v$, we will
be able to say what happens exactly when competition is of the Bertrand type, the Cournot type, or when there is a cartel downstream.

4. RESULTS

We are now in a position to be able to evaluate the differences between the common measures of harm in these proceedings, which for convenience we continue to refer to as R and R+T, and the true harm as measured by the loss of surplus by direct and indirect purchasers.

4.1 The Error Associated with Using R+T to Measure True Harm

The first question we want to address asks about the relative magnitude of the error associated with only the area under the cartel’s derived demand curve (areas R+T in Figure 1) as a measure of harm due to price-fixing.

Let \( x_{iz}(q, w) \) be firm \( i \)'s conditional demand for input \( z \in Z \), and \( X_z = \sum_{i=1}^{n} x_{iz} \) be input \( z \)'s total demand. This is the derived demand which, if integrated between \( w_0 \) and \( w_1 \), delivers area R+T as in Figure 1. Formally, when there are many inputs that were potentially price-fixed, we have:

\[
R + T = \int_{w_0}^{w_1} \sum_{z \in Z} X_z(q, w) dw_z
\]  

(7)

While Jacobsen [1979] and Quirmbach [1984] derived the relationship between R+T and true downstream surplus for certain specific cases, the model restrictions imposed preclude the use of their results in the more general model we are examining. To derive more general results, we build on the work of Basso [2006].

To begin, we note that Basso shows that, if there exists a differentiable function \( H(q, w) \) such that:
\[
\partial H(q^*, w) / \partial q_i = \partial \pi'(q^*, w) / \partial q_i, \quad \forall i \tag{8}
\]

\[
\partial H(q^*, w) / \partial w_z = -\sum_i \partial c^i(q^*, w) / \partial w_z, \quad \forall z \tag{9}
\]

then

\[
\int_{w_z}^{w_z'} \sum_{z \in Z} X_z(w) dw_z = -[H(q^*(w), w)]_{w_z}^{w_z'} \tag{10}
\]

where \(q^*(w)\) denotes an interior Nash-equilibrium of the downstream game as a function of \(w\). Thus, this result provides a useful way to describe what is captured by the input market surplus, that is, area \(R+T\). It suffices to find a function \(H\) fulfilling (8) and (9) above, which essentially means that we look for a function with critical values that coincide with the equilibria of the non-cooperative game we are studying.

As Basso [2006] goes on to show, there is a subset of the \(H\) functions above that fulfill the required conditions and that are particularly well-behaved: the exact potential functions of potential games (Monderer and Shapley [1996]). These functions fulfill a stronger condition than (8) – as the equality holds for every value of \(q\) and not only at equilibrium points, there is a well-defined way to calculate them, and they are defined up to an additive constant (and the constant is unimportant in (10)). As it happens, the game we have here is a potential game and, hence, an exact potential function exists. It is given by:15

\[
H(q, w) = \sum_{j=1}^{n} \left[ a - \left( b + \frac{eV}{2} \right) q_j \right] q_j - c^j(q, w) - e \sum_{k=l}^{n-1} \sum_{j=k+1}^{n} q_k q_j \tag{11}
\]

It is straightforward to verify that:

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15 See Basso [2006], Lemmas 1 and 2, for explanations on how to determine whether a game is potential or not, how to obtain the potential function when it exists, and for other properties of potential functions.
\[
\frac{\partial H}{\partial q_i} = a - q_i(2b + ev) - e \sum_{j \neq i} q_j - c_i = \frac{\partial \pi^i}{\partial q_i} \bigg|_{\text{Cond.Var.}}
\]

So condition (8) is fulfilled. It is also easy to see that,

\[
\frac{\partial H}{\partial w_x} = -\sum_j \frac{\partial c^i_j(q_j, w)}{\partial w_x}
\]

So that condition (9) is fulfilled. Hence, from (10) and Proposition (1) in Basso [2006] we obtain that:

\[
R + T = -\left[H(q^*, w)\right]_{w_0}^{w_1}
\] (12)

To make (12) useful, we need to relate \( H \) to downstream profits and (final) consumer surplus. For this, we first rewrite \( H \) as:

\[
H(q, w) = \sum_{j=1}^{n} \pi^j(q, w) + e \sum_{k=2}^{n} \sum_{j=1}^{k-1} q_k q_j - \sum_{j=1}^{n} e v \frac{q_j^2}{2}
\] (13)

Next, consumer surplus is given by the line integral \( \int_p \sum_q \pi(q)p dp \), which is straightforward to compute since direct demands exist (as long as \( b > e > 0 \)) and the solution is path independent. A linear integration path leads to

\[
CS = (b/2) \sum_k q_k^2 + e \sum_{k=2}^{n} \sum_{j=1}^{k-1} q_k q_j
\] (14)

Replacing (13) and (14) in (12) we finally obtain:

\[
R + T = -\left[\sum_{j=1}^{n} \pi^j(q^*, w) + CS(q^*, w) - \frac{b + ev}{2} \sum_{k=1}^{n} (q^*_k)^2\right]_{w_0}^{w_1}
\] (15)

Equation (15) shows that, unless \( b = e \) and \( v = -1 \), that is, homogenous products and Bertrand (perfect) competition, area R+T always underestimates the harm. The difference between R+T and the true harm is dependent on the difference between the
squares of downstream firms’ production in equilibrium at the fixed-prices and the but-
for prices.

Equation (15) is quite general, and allows us to assess what happens in a several cases
by simply replacing the various model parameters with candidate values. But to obtain
more clear-cut comparative statics from (15) is difficult for a number of reasons relating
to the possible opposing direct and indirect effects of parameter changes. For example, if
ν increases (i.e. less intense competition), the size of the error associated with the R+T
measure increases ceteris paribus, but, in fact, $q_k^*$ would decrease as well reducing both
the size of R+T and the size of the true harm. The same is true if competition is
effectively reduced by decreasing e.

A simple way to derive more clear-cut comparative statics, and hence to see how the
parameters of the model affect the magnitude of the mistake, is to assume symmetry in
cost functions. This induces a symmetric downstream Nash equilibrium in which
$q_k^* = q \equiv Q / n \ \forall k$. Note that, despite this symmetry, the (common) cost function does
not require a specific functional form. With symmetry, consumer surplus in (14) is given
by:

\[
CS = \frac{Q^2(b + e(n - 1))}{2n}
\]

(16)

And the last term on the RHS of (15) is now \( \frac{b + ev}{2} \sum_{k=1}^{n} (q_k^*)^2 = \frac{Q^2(b + ev)}{2n} \). Hence, (15)
can be re-written as:

\[
R + T = \left[ \sum_{j=1}^{n} \pi_j \right] + \left[ CS \left( 1 - \frac{2n}{CS} \right) \right]_{w_1}
\]
or, substituting for $CS$:

$$R + T = -\sum_{j=1}^{n} \pi^j + CS \left[ \frac{e(n-1-v)}{b(1+\frac{e}{b}(n-1))} \right]^{\omega_1}_{\omega_0} \Delta$$

(17)

If the term labeled $\Delta$ in (17) were to equal 1, areas $R+T$ would exactly equal the reduction in downstream profits and consumers’ surplus. However, since $-1 \leq v \leq n-1$ and $e \leq b$, then $0 \leq \Delta \leq 1$. Thus, the areas $R+T$ will in general be less than the true total harm suffered by direct and indirect purchasers.

While the understatement of the true total harm is our focus here, it is also important to recognize that, as long as $\Delta > 0$, $R+T$ will overstate the harm to direct purchasers alone.

From (17) it is easy to see how the model parameters influence the magnitude of the mistake. Consider the following special cases:

Cases with respect to the value of $v$:

(i) If $v = n-1$ (downstream collusion), then $\Delta = 0$. In this case, the measure $R+T$ totally misses the harm to downstream consumers’ surplus.

(ii) If $v = 0$ (differentiated Cournot), $\Delta = \frac{e(n-1)}{1+\frac{e}{b}(n-1)} < 1$.

(iii) If $v = 0$ and $e = b$ (homogenous product Cournot competition), $\Delta = \frac{n-1}{n} < 1$.

(iv) If $v = -1$ and $e = b$ (homogenous product Bertrand competition), then $\Delta = 1$. In this case, if the downstream market produces homogenous products under conditions of perfect competition, the area under the input demand curve will be the same as the area under the final product demand curve.
If \( v = -\frac{e(n-1)}{b + e(n-2)} \) (differentiated Bertrand), then \( \Delta = \frac{e}{b} \frac{(n-1)}{1 + \frac{e}{b}(n-2)} \leq 1 \)

Note that \( \Delta_{\text{Bertrand}}^{\text{Diff}} > \Delta_{\text{Cournot}}^{\text{Diff}} \), which is consistent with the fact that the differentiated Bertrand game is more competitive (in terms of market power), than the differentiated Cournot game.\(^{16}\)

Cases with respect to the degree of differentiation \( e/b \):

- (vi) If \( e/b = 0 \) (each downstream firm is a monopolist), then \( \Delta = 0 \), just as in the collusion case.
- (vii) If \( e/b = 1 \), then \( \Delta = \frac{n-1-v}{n} \).

Cases with respect to the number of firms, \( n \):

- (viii) If \( n = 1 \) (monopoly), then \( v \) must be 0 because there are no rivals and thus, \( \Delta = 0 \), just as in the collusion case, or the \( n \) monopolies case.
- (ix) If \( n \to \infty \), then \( \Delta = 1 \), so the error goes to zero as the number of firms increases toward infinity.

From these observations, it is evident that equation (17) – and indeed equation (15) – generalizes the Jacobsen and Quirmbach results. Jacobsen’s results correspond to points (vi), (viii) and (ix), while Quirmbach’s results correspond to point (vii), if we were to consider a single input. Combining all these results we can state the following proposition:

\(^{16}\) See Vives [1999, Chapter 6].
**Proposition 1:** The following will decrease the value of $\Delta$, effectively increasing the error (as measured by the magnitude of $1-\Delta$) associated with using $R+T$ as a measure of the total harm produced by upstream collusive price increases: (i) decreasing the intensity of competition as captured by decreases in the value of the parameter $v$; (ii) reducing the degree of product substitutability as measured by $e/b$; and (iii) reducing the number of firms competing in the downstream market, $n$.

**Proof:** These results follow trivially from differentiating the expression for $\Delta$ in equation (17). We see immediately that

$$\frac{\partial \Delta}{\partial v} = -\frac{e}{b} < 0$$

$$\frac{\partial \Delta}{\partial (e/b)} = \frac{n-1-v}{\left[1 + \frac{e}{b(n-1)}\right]^2} > 0 \quad \text{(and also, } \frac{\partial \Delta}{\partial e} > 0 \text{ )}$$

$$\frac{\partial \Delta}{\partial n} = \frac{e(b + e v)}{b + e (n-1)^2} > 0$$

QED

These results tell a consistent story about the effects of downstream competition on the magnitude of the error from using $R+T$. Decreasing $v$, or increasing $e/b$ or $n$ are all ways to make the downstream market more competitive and here we see that in each case the size of the error is reduced as these parameters take on “more competitive” values.

Another point worth mentioning here -- because we believe it is often not appreciated when price-fixing damages are measured -- relates to the special case in which the downstream market is essentially monopolized. In the cases above this can happen when $n=1$, $e/b = 0$ and/or when $v=n-1$. In each of these cases, the harm measured by $R+T$ is exactly the loss of profits suffered by the firm(s) downstream. In response to higher input prices, these downstream firms will certainly raise their retail
prices and it is sometimes said that by so doing they are passing on some of the harm R+T to their customers. While downstream customers do indeed suffer from higher retail prices, these are harms in addition to the R+T harm suffered by the direct purchasers. Put another way, R+T is the true measure of the harm suffered by the direct purchasers in these cases – that is the harm they are left with after accounting for the higher prices they will themselves collect from their customers. To reduce the damages they collect below R+T on the theory that they passed on some of this harm would be incorrect.\(^{17}\)

Consider now a different measure of the error associated with the use of R+T, one that might be more appealing and useful. This measure compares measures the amount of the true harm “missed” by R+T and expresses it in percentage terms. In other words, it answers the question: if we use R+T as our measure of harm, what percentage of the true harm are we missing? Let \( \Phi \) be this percentage error made by using R+T. Then \( \Phi \) is given by:

\[
\Phi = 1 - \left( \frac{R+T}{-\sum_{j=1}^{n} \pi^j + CS_{w^*}} \right)
\]

Replacing \( R+T \) by its expression in (17) we get:

\[
\Phi = 1 - \left( \frac{\sum_{j=1}^{n} \pi^j + CS \left( \frac{e}{b} \frac{(n-1-v)}{1+e/b(n-1)} \right)}{\sum_{j=1}^{n} \pi^j + CS_{w^*}} \right)
\]

\(^{17}\) This result is certainly not new to us (see, e.g. Jacobsen [1979]) but we are not aware of it being acknowledged in price-fixing damage actions. Where the R+T area may overstate the harm that remained with direct purchasers is in cases in which there are rivals competing downstream such that each benefits (somewhat) indirectly through higher input prices because they induce rivals to raise their retail prices. Hellwig [2006] discusses this as well.
We know that profits are given by (3) and consumer surplus is given by (16). Yet, to have expressions that depend only on the parameter values, we need an explicit expression for $Q^*$, which cannot be obtained in general. What is needed is a more explicit cost function. We use the following:

$$c_i^i(q_i, w) = q_i(\alpha + w)$$

(19)

With this cost function we are now considering a single input price, $w$. Also, this cost function features the fixed-proportion property in which one unit of input is needed for one unit of output and imposes constant marginal costs for firms: $c_i^i(q_i, w) = \alpha + w$. 19

While this is the most straightforward functional form to use for our purposes, we have repeated the analysis that follows using a quadratic cost function that generates an upward-sloping marginal cost curve. The qualitative results are not much altered and the results are presented in the Appendix.

**Solving the downstream game**

With the cost function in (19) the first-order conditions become:

$$\frac{\partial \pi_i}{\partial q_i} = a - q_i(2b + ev) - c \sum_{j \neq i} q_j - (\alpha + w) = 0$$

Solving for $q_i$, and noting that, in equilibrium, $q_i^* = Q^* / n$, we get

$$Q^* = \frac{n(a - \alpha - w)}{2b + e(v + n - 1)}$$

(20)

---

18 Recall that expression (17) for R+T was obtained assuming symmetry of the cost functions, so we need to use the expression for CS that follows from this symmetry assumption, that is, equation (16). The general expression for CS in (14) cannot be used together with (17) or (18).

19 Of course, given the assumption of fixed proportions, the assumption that those proportions are 1:1 is without further loss of generality.
Next, in equilibrium, \( p_i^* = p^*, \forall i \). Dropping the \(^*\), and replacing \( Q \) from equation (20) in \( p = a - bQ / n - e(n - 1)Q / n \), we obtain:

\[
p = \frac{a(b + ev) + (b + e(n - 1))(\alpha + w)}{2b + e(v + n - 1)}
\]

(21)

Note that, given our fixed proportions assumption, we have actually obtained the derived demand for the input, that is, \( X(w) \equiv Q^*(w) \), where \( Q^* \) is given by (20).

Total downstream profits are then given by:

\[
\sum_{j=1}^{n} \pi^j = \frac{n(b + ev)(a - \alpha - w)^2}{[2b + e(v + n - 1)]^2}
\]

(22)

And the change in profits, induced by the price-fixing is given by:

\[
\left| \sum_{j=1}^{n} \pi^j \right|_{w^1}^{w^0} = \frac{n(b + ev)}{[2b + e(v + n - 1)]^2} \left[ (a - \alpha - w^1)^2 - (a - \alpha - w^0)^2 \right]
\]

(23)

As for consumer surplus, using its expression for the symmetric case, (16), and using (20), we get:

\[
CS = \frac{Q^2(b + e(n - 1))}{2n} = \frac{n(b + e(n - 1))(a - \alpha - w)^2}{2[2b + e(v + n - 1)]^2}
\]

(24)

And therefore, the change in consumer surplus induced by the price-fixing is given by

\[
|CS|_{w^1}^{w^0} = \frac{n(b + e(n - 1))}{2[2b + e(v + n - 1)]^2} \left[ (a - \alpha - w^1)^2 - (a - \alpha - w^0)^2 \right]
\]

(25)

With all this, we can calculate, for example, the area under the demand curve, using equation (17). We obtain:

\[
R + T = -\frac{n((a - \alpha - w^1)^2 - (a - \alpha - w^0)^2)}{2(2b + e(v + n - 1))}
\]

We can also calculate:
\[
\sum_{j=1}^{n} \pi^j + CS = \frac{(2n(b + ev) + n(b + e(n-1)))(a - \alpha - w^1)^2 - (a - \alpha - w^0)^2}{2(2b + e(v + n - 1))^2}
\]

So, now we are prepared to look at the percentage error, \( \Phi \). Replacing (23) and (25) in (18) we obtain:

\[
\Phi = \frac{1 + \frac{e}{b} \nu}{3 + \frac{e}{b}(2\nu + n - 1)}
\]

(26)

Notice one important feature of this measure of the error from using R+T: it does not depend on the fixed price, \( w^1 \), or the “but-for” price, \( w^0 \). It depends only on model parameter values. This is an extremely useful property that tells us that the magnitude of the error (in percentage terms) is not influenced by the extent of the success the cartel enjoyed at raising price.

By applying particular parameter values it is straightforward to show the value of \( \Phi \) in a number of special cases:

Cases for the degree of competitive rivalry as captured by \( \nu \):

(i) If \( \nu = n - 1 \) (downstream collusion), then \( \Phi = \frac{1}{3} \).

(ii) If \( \nu = 0 \) (differentiated product Cournot), then \( \Phi = \frac{1}{3 + \frac{e}{b}(n - 1)} < \frac{1}{3} \).

(iii) If \( \nu = 0 \) and \( e = b \) (homogenous product Cournot), then \( \Phi = \frac{1}{n + 2} < \frac{1}{3} \).

(iv) If \( \nu = -1 \) and \( e = b \) (homogenous product Bertrand), then \( \Phi = 0 \).
If \( v = -\frac{e(n-1)}{b + e(n-2)} \) (differentiated product Bertrand), then we have

\[
\Phi = \frac{1 - \frac{e}{b}}{3 + \frac{e}{b}(n-4)} \leq \frac{1}{3}
\]

Cases for the degree of substitutability \( e / b \):

(vi) If \( e / b = 0 \) (each downstream firm is a monopolist), then \( \Phi = \frac{1}{3} \), just as in the collusion case.

(vii) If \( e / b = 1 \), then \( \Phi = \frac{v + 1}{2v + n + 2} \).

Cases for the number of firms, \( n \):

(viii) If \( n = 1 \) (monopoly), then \( v \) must be 0 because there are no rivals and thus, \( \Phi = \frac{1}{3} \), just as in the collusion case, or the \( n \) monopolies case.

(ix) If \( n \to \infty \), then \( \Phi = 0 \).

Notice that in none of these cases does the error exceed \( 1/3 \) and it is again zero if the downstream market features homogeneous products and Bertrand competition (i.e. perfect competition).

We are now in a position to establish a number of comparative statics results with respect to the size of this error:

**Proposition 2:** The following will increase the value of \( \Phi \), effectively increasing the error associated with using \( R+T \) as a measure of the total harm produced by upstream collusive price increases: (i) decreasing the intensity of competition as captured by increases in the value of the parameter \( v \); (ii) reducing the degree of product
substitutability as measured by $e/b$; and (iii) reducing the number of firms competing in
the downstream market, $n$. Also, $\Phi$ is bounded above by 1/3.

**Proof:** As with Proposition 1 these results are easily established by differentiating the
expression for $\Phi$ given in (26) with respect to the various parameter values. This gives us:

\[
\frac{\partial \Phi}{\partial v} = \frac{e \left( e \left( n-1 \right) + 1 \right)}{b \left( 3 + \frac{e}{b} (2v + n - 1) \right)^2} > 0
\]

\[
\frac{\partial \Phi}{\partial (e/b)} = -\frac{(n-1-v)}{3 + \frac{e}{b} (2v + n - 1)} < 0 \quad \text{Also, clearly } \frac{\partial \Phi}{\partial e} < 0 \text{ and } \frac{\partial \Phi}{\partial b} > 0.
\]

\[
\frac{\partial \Phi}{\partial n} = -\frac{e \left( 1 + \frac{e}{b} v \right)}{3 + \frac{e}{b} (2v + n - 1)} < 0
\]

The first derivative establishes that higher levels of $v$ are associated with larger errors, the
second that higher levels of substitutability lead to smaller errors and the third that a
larger number of downstream competitors will also lead to smaller errors. Finally, given
the signs on these derivatives we can find the highest possible value for $\Phi$ by setting $v$ to
the highest possible level we consider (i.e. $v=n-1$) and $e/b$ and $n$ to their lowest possible
levels (i.e. 0 and 1, respectively). Applying these values in (26) we find that, in this case,
$\Phi = 1/3$. \[QED\]

Proposition 2 establishes for this measure $\Phi$, just as Proposition 1 did for the
earlier measure, that the errors associated with the use of R+T will be larger the less
competitive the downstream market is. Any parameter change that effectively decreases
the intensity of competition: increasing ν, or decreasing ε/b or n, will result in area R+T being a poorer estimate of the true harm from price-fixing.

4.2 The Error Associated with using R to Measure the True Harm

Since R+T already underestimates the true harm, it is clear that by looking at R alone the mistake must be greater. To assess the additional mistake here we first determine what fraction of R+T is missed by using only R -- that is, we calculate the ratio of T to R+T:

\[ \frac{T}{R + T} = \frac{w^1 - w^0}{2(a - \alpha) - (w^0 + w^1)} \]  

(27)

Notice that this fraction does depend on the two prices. Comparative statics reveal that:

\[ \frac{\partial [T/(R + T)]}{\partial a} = -\frac{2(w^1 - w^0)}{(2(a - \alpha) - (w^0 + w^1))^2} < 0 \]

\[ \frac{\partial [T/(R + T)]}{\partial \alpha} = \frac{2(w^1 - w^0)}{(2(a - \alpha) - (w^0 + w^1))^2} > 0 \]

\[ \frac{\partial [T/(R + T)]}{\partial w^1} = \frac{2(a - \alpha - w^0)}{(2(a - \alpha) - (w^0 + w^1))^2} > 0 \]

Hence, the relative size of T and R+T will depend on the marginal cost of downstream firms, the intercept of final consumer demands, and both the but-for and cartel prices. For example, if input providers had flat marginal cost, were to charge marginal cost under competition and the monopoly price under collusion, it can be easily shown that T would be half the R area, and thus a third of the combined R+T area.20 To see this, let \( w^0 = c \).

The demand for the input is given by:

---

20 This is the result demonstrated by Brander and Ross (2006) and referred to earlier.
\[ X(w) = \frac{n(a - \alpha - w)}{2b + e(v + n - 1)} = \frac{n(a - \alpha)}{G} - \frac{n}{G}w \]

And therefore, the monopoly price is:

\[ w^1 = \frac{n(a - \alpha)}{G} + \frac{n}{2G}c = \frac{a - \alpha + c}{2} \]

where \( G \equiv 2b + e(v+n-1) \). Replacing these prices in (33), leads to \( \frac{T}{R+T} = \frac{1}{3} \).

Under these conditions that give us \( \frac{T}{R+T} = \frac{1}{3} \), we can assess the overall error made by using R as an estimate of the total true harm. Since R is only 2/3 of R+T and we know that R+T can be as little as 2/3 of the true harm, the measure R may be as little as 4/9 of the true harm. Thus, the common practice of using area R to measure the total harm to direct and indirect purchasers can result in missing more than 50% of the harm.

### 4.3 How is the Harm Shared between Direct and Indirect Purchasers?

The second question we want to address relates to how the harm suffered downstream is shared by direct and indirect purchasers. The question of the degree to which direct purchasers “pass through” input price increases via higher output prices has become a somewhat controversial one. In the United States, the key Supreme Court decisions in the Hanover Shoe and Illinois Brick cases established that only direct purchasers should have the right to recover damages from price fixing.\(^{21}\) There were two key arguments

made in support of this approach: first that measuring pass-through is always going to be a difficult, time-consuming and uncertain process about which courts may ultimately not have confidence; and second that deterrence was better served by letting the direct purchasers sue for the full amount of damages suffered even if this meant they were overcompensated in some cases.\textsuperscript{22} However, in response to these decisions, a number of U.S. states have enacted what are frequently referred to as “Illinois Brick repealer laws” that restore the rights of residents of those states who are indirect purchasers to pursue actions to recover damages.\textsuperscript{23} Whether the Supreme Court was correct in \textit{Illinois Brick} in trying to limit the rights to damages to direct purchasers has become an active topic for debate.\textsuperscript{24} We do not wade into that debate here. And in contrast to much past work by economists in this area we do not focus directly on measures of “pass through” of input price increases into output price increases. While such cascading price changes are certainly happening in our model, our purpose here is really to look at the totality of harm (which depend on quantity changes as well as price changes) and to establish how imprecise (and biased downward) the current approach to measuring total harm can be -- and then to show how that quantum of harm breaks down between direct and indirect purchasers.

If the harm was correctly measured, the percentage $\Psi$ of that harm that will be suffered by direct purchasers would be:

\begin{itemize}
\item Recall that under U.S. law, plaintiffs are already entitled to treble damages already so there is no presumption in the law that damages cannot exceed actual harm suffered.
\item See, e.g. Kosicki and Cahill [2006] and the references on state laws cited therein.
\end{itemize}
Replacing (23) and (25) in (28) we obtain:

$$\Psi = \frac{\sum_{j=1}^{n} \pi_j^{w} w_j}{\sum_{j=1}^{n} \pi_j^{w} + CS}$$

We see immediately that $\Psi \equiv 2\Phi$, that is, the percentage of harm falling on direct purchasers is exactly two times the percentage error made by using $R+T$ as the measure of harm rather than the true harm.

As above, we can consider a number of special cases in which we can pin down the value of $\Psi$, but because $\Psi = 2\Phi$ we have essentially already done the work – in each special case the resulting value of $\Psi$ will simply be double what the value of $\Phi$ was in that case. Of course, this implies that $\Psi$ will take on a maximal value of $2/3$. The comparative statics results must therefore also be essentially the same, leading immediately to Proposition 3.

**Proposition 3:** The fraction of total harm that falls on direct purchasers increases with increasing levels of the conduct parameter $v$, and with decreasing levels of product substitutability and with decreasing numbers of firms. The maximal value of the fraction of harm falling on direct purchasers is $2/3$.

**Proof:** Follows directly from the proof of Proposition (2) and the fact that $\Psi = 2\Phi$.

QED

These results present another consistent pattern – as each of these parameters changes in a way that can be interpreted as intensifying competition, the share of the harm that
remains with direct purchasers falls. The fraction that direct purchasers face ranges from a low of zero (when they are effectively perfectly competitive) to a high of 2/3 (when they are effectively monopolies).

5. SOME NUMERICAL EXAMPLES

To this point we have provided a large number of cases in which different parameter values lead to different conclusions about the errors associated with the R+T and R measures of total harm and the share of the true harm that will fall on direct purchasers. To summarize these results in a way that we hope will be useful we provide a set of tables below that combine various assumptions about parameter values.

5.1 Error from using R+T to estimate the total true harm

*True Harm is ‘x’ times R+T where x are cell entries below:*

<table>
<thead>
<tr>
<th>e/b</th>
<th>0.3</th>
<th>0.6</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>n</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1.50</td>
<td>1.50</td>
<td>1.50</td>
</tr>
<tr>
<td>5</td>
<td>1.31</td>
<td>1.23</td>
<td>1.17</td>
</tr>
<tr>
<td>10</td>
<td>1.21</td>
<td>1.14</td>
<td>1.09</td>
</tr>
</tbody>
</table>

Table 1a: Cournot conjecture (ν = 0)

<table>
<thead>
<tr>
<th>e/b</th>
<th>0.3</th>
<th>0.6</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>n</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1.50</td>
<td>1.50</td>
<td>1.50</td>
</tr>
<tr>
<td>5</td>
<td>1.27</td>
<td>1.13</td>
<td>1.00</td>
</tr>
<tr>
<td>10</td>
<td>1.17</td>
<td>1.06</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Table 1b: Bertrand conjecture (ν given in 6)

These tables illustrate the error associated by the use of R+T to measure total harm by providing the appropriate multiplier to be applied to R+T to get a measure of the true total harm. For example, using the Bertrand table on the right, with 5 firms and a value of e/b = 0.3, the true harm will be 1.27 times the harm measured by R+T.
5.2 Error from using R to estimate total true harm

With the additional assumptions of flat marginal costs of input production; a but-for price equal to marginal cost; and a fixed price equal to the monopoly price, we can create the same kinds of tables for the error associated with using only R as a measure of harm.

Real harm is ‘x’ times R where x are cell entries below

Table 2a: Cournot conjecture (ν = 0)                        Table 2b: Bertrand conjecture (ν given in 6)

These tables help to illustrate just how very inaccurate the most commonly applied measure of harm, R, can be as a measure of the true total harm suffered by direct and indirect purchasers. Even with five competing firms, if e/b = 0.3, R will measure only about half of the total true harm in either Cournot or Bertrand cases.

5.3 Percentage of harm suffered by direct purchasers

Finally, we present below examples to illustrate how the share of true harm that is suffered by direct purchasers will depend on the values of the various parameters. Again this share falling on direct purchasers can range from zero to 2/3.

Cell entries correspond to percentage of true harm borne by direct purchasers:

Table 3a: Cournot conjecture (ν = 0)                        Table 3b: Bertrand conjecture (ν given in 6)
For example, if 5 direct purchasers compete in a Bertrand fashion and $e/b = 0.6$, just over 22% of the true harm will be borne by direct purchasers.

6. DISCUSSION AND CONCLUSIONS

The primary purpose of this paper has been to demonstrate just how large an error can be made when one measures the harm due to price-fixing using simple overcharge measures, as is typically done in private damage actions. Our results show that, as downstream markets get less competitive (measured in a number of ways), the error associated with simple overcharge methods get larger. We also use our model to provide some guidance as to how the true harm as a result of price-fixing will be allocated between direct and indirect purchasers. We show that this is also highly dependent on the degree of competition downstream.

As damage awards are likely to depend on measures of actual harm, we think it is important to get these measures right. There are a number of ways our results could possibly be used by courts. At a most basic level, our results simply point to the fact that overcharge estimates are understatements of true harm and this by itself could be helpful to courts (and settling parties) at coming to reasonable damage awards. More specifically, they provide a way to estimate the true total harm from information on the overcharge. Using the tables we provide (and more detailed tables are easily generated from our equations) a court could apply the appropriate multiplier to the overcharge to get a better estimate of the true total harm. This may be a preferred alternative to actually trying to estimate the degree of pass-through and the subsequent harm further downstream from the direct purchasers.
Secondly, our results provide some guidance as to how harm is shared between direct and indirect purchasers and so might be helpful in legal regimes in which both types of purchasers have rights to recover damages. This is most likely to be the case when damages to indirect purchasers are distributed on a *cy pres* basis. In such a situation, detailed information on individual indirect purchases would not be required in any case, and our allocation rules would simplify the process of determining how much of the damage award should go to the indirect purchaser group.

The advantages of using the approaches made possible here are probably greatest in settlement negotiations when the defendant parties have accepted their liability and both sides wish to come to a reasonable estimate of harm and of how that harm was distributed between direct and indirect purchasers without the need for detailed and expensive econometric inquiries. If the parties can agree on reasonable values for the key parameters here (and many of the results are not highly sensitive to modest changes in the values of the parameters), they need only estimate the traditional overcharge measure. The results we present above can then be used to adjust the overcharge number to create an estimate of the total true harm, and provide an estimate of the shares of that harm borne by the two groups of purchasers.

This paper is hardly the last word on these topics – there is a great deal more that could be done to refine and expand on these results. For example, the analysis above implicitly assumes that all price changes take place simultaneously and instantaneously, however, in many cases downstream prices will adjust to increasing prices of inputs only

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25 That is, the damage award is not given directly to indirect purchasers because that would be too difficult and costly – particularly if the individual harm is very small – rather, the monies are allocated to good causes, such as charities, trade associations or schools, that are in some way related to the harmed indirect purchasers.
with a lag. This is particularly likely to be the case when the price-fixed input represents a small fraction of the downstream output price. In such cases, an increase in the price of a small input may not be enough to trigger output price changes on its own (given that changing prices is not a costless exercise) and will only be reflected in new output prices when other events trigger the change.

Third, the results here on the division of harm between direct and indirect purchasers are built on the assumption that downstream marginal costs are flat. While this may be reasonable in many cases, there will be other circumstances in which rising marginal costs (particularly in the short run) are more likely. When this is the case, even under perfect competition direct purchasers will have some producers’ surplus to lose -- and some will be lost -- through higher input prices, hence they will bear a higher fraction of the total harm than what is suggested here for the competitive case.26 It would be useful, then, to incorporate the possibility of marginal cost curves with less than infinite elasticity and to see how this affects the degree to which direct purchasers keep the harm.27 As indicated above, we have begun work in this direction by considering the case of an upward sloping, linear marginal cost curve. These results are reported in the appendix. While the qualitative results are very similar, there certainly are some differences, including: (i) the error associated with using R+T will be somewhat smaller

26 This result is well-established in the literature on the incidence of commodity taxation. The degree to which input price increases translate into output price increases then depend on the ratio of the elasticities of demand and supply. This is shown in a number of price theory and public finance textbooks, such as Pindyck and Rubinfeld [2005, pp. 326-9]. The pass-through formulae are reproduced in a number of articles on pass-through of harm due to price-fixing. See, e.g., Harris and Sullivan [1979, pp. 281-209] who have a nice non-technical discussion of the role of demand and supply elasticities in determining pass-through. Kosicki and Cahill [2006] offer some useful extensions. See, also, Brander and Ross [2006, p. 359].

27 In a related way, subsequent work could explore the effects of other shapes for the demand curve, here assumed to be linear. There is a nice discussion of the effects of different types of demand curves in Kosicki and Cahill [2006].
with rising marginal costs; (ii) the error shrinks as the marginal cost curve becomes steeper; and (iii) the fraction of the harm falling on direct purchasers will be somewhat greater with rising marginal costs and will only be zero if the number of firms goes to infinity.

Fourth, this paper uses a simplified distribution chain with really just three stages – essentially the cartel sells to retailers who in turn sell to final consumers. Quite clearly, some distribution chains have many more stages than this. Intuitively, adding more stages, if they are also imperfectly competitive, introduces the possibility of more damaging markups down the chain, further magnifying the harm due to the original price increase. In addition to increasing the quantum of harm, the share going to the indirect purchasers (added together) would be expected to increase. It would be valuable to determine to what extent the adjustments we provide here would need to be themselves adjusted to take the additional steps in the chain into account.

Finally, throughout this paper we have assumed that all firms employ simple linear pricing. To the extent that direct purchasers are able to employ non-linear pricing techniques such as two-part tariffs, the direct harm will not be as magnified downstream as we have it here. The allocation of harm between direct and indirect purchasers will also be affected. For example, if we assume that direct purchasers can use two-part tariffs to extract all final consumers’ surplus, it is not difficult to show that all harm from price fixing will fall to direct purchasers and that the entire harm will be captured by the area under the input demand curve, what we have referred to here as R+T. Therefore, for cases in which nonlinear pricing is practiced downstream, adjustments to the calculations provided here must clearly be made.
REFERENCES


APPENDIX

In this appendix we repeat much of the analysis of Section 4 using a quadratic cost function that gives us a rising marginal cost curve. Specifically, we use the following:

\[
c^i(q_i, w) = q_i(\alpha + w) + \frac{\beta}{2} q_i^2
\]  

(19’)

Therefore, marginal costs for firm \(i\) will be given by: \(c^i(q_i, w) = \alpha + w + \beta q_i\). With the cost function in (19’) the first-order conditions for profit-maximization become:

\[
\frac{\partial \pi^i}{\partial q_i} = a - q_i(2b + ev) - e \sum_{j \neq i} q_j - (\alpha + w) - \beta q_i = 0
\]

Solving for \(q_i\) and imposing \(q_i = Q/n\), we get total output and equilibrium price as:

\[
Q = \frac{n(a - \alpha - w)}{2b + \beta + e(v+n-1)}
\]

(20’)

\[
p = \frac{a(b + ev + \beta) + (b + e(n-1)) (\alpha + w)}{2b + \beta + e(v+n-1)}
\]

(21’)

Total downstream profits are then given by:

\[
\sum_{j=1}^{n} \pi^j = \frac{n(2b + 2ev + \beta)(a - \alpha - w)^2}{2[2b + \beta + e(v+n-1)]^2}
\]

(22’)

Consumer surplus, solved for as before but using these altered functions becomes:

\[
CS = \frac{Q^2 (b + e(n-1))}{2n} = \frac{n(b + e(n-1)) (a - \alpha - w)^2}{2[2b + \beta + e(v+n-1)]^2}
\]

(24’)

And therefore, the percentage error term, \(\Phi\), now becomes:

\[
\Phi = \frac{1 + \frac{e}{b}v}{3 + \frac{e}{b} (2v+n-1) + \frac{\beta}{b}}
\]

(26’)

38
If the harm was correctly measured, the percentage, $\Psi$, of that harm that will be fall on direct purchasers would be:

$$\Psi = \frac{2\left(1 + \frac{e}{b}\right) + \frac{\beta}{b}}{3 + \frac{e}{b}(2v + n - 1) + \frac{\beta}{b}}$$  \tag{29'}$$

Notice the following about these new formulae:

(i) It remains true that $\Phi$ and $\Psi$ do not depend on the level at which price was fixed, $w^1$, or the “but-for” price, $w^0$. They depend only on model parameter values.

(ii) It is no longer true that $\Psi \equiv 2\Phi$ when $\beta > 0$. Indeed, in general, $\Psi \geq 2\Phi$.

(iii) The values of $\Psi$ and $\Phi$ associated with particular values of $n$, $v$ and $e/b$ will change, of course. The signs of the comparative statics on $\Phi$ and $\Psi$ are the same, however. There are new comparative statics, which are straightforward to derive: $\partial \Phi / \partial \beta < 0$ and $\frac{\partial \Psi}{\partial \beta} = \frac{b + e(n - 1)}{[3b + e(2v + n - 1) + \beta^2]} > 0$. That is, increasing the steepness of the marginal cost curve will, ceteris paribus, reduce the error associated with using R+T to measure total harm, and it will increase the share of harm falling on direct purchasers.

(iv) With quadratic cost functions the percentage error in using R+T, $\Phi$, reaches $1/3$ only for monopolies ($e=0$, or $v=0$ and $n=1$) or downstream collusion ($v=n-1$) and when $\beta = 0$. In “perfect competition” cases (i.e. homogenous product Bertrand competition or when $n \to \infty$), the error, $\Phi$, is still zero. Note from (22') that profits are not zero in this case, however, unless $n \to \infty$.

(v) With quadratic cost functions, the upper limit percentage on the harm that falls on direct purchasers is no longer $2/3$ – it will be higher. The share is no longer zero under homogeneous product Bertrand competition. It is still zero when $n \to \infty$.

To illustrate the potential impact of rising marginal costs in this model, we present below the tables from Section 5 here amended to allow $\beta=b$. 
True Harm is ‘x’ times R+T where x are cell entries below:

Table 4a: Cournot conjecture (v = 0)                      Table 4b: Bertrand conjecture (v given in 6)

<table>
<thead>
<tr>
<th>e/b</th>
<th>0.3</th>
<th>0.6</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>n</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1.33</td>
<td>1.33</td>
<td>1.33</td>
</tr>
<tr>
<td>5</td>
<td>1.24</td>
<td>1.19</td>
<td>1.14</td>
</tr>
<tr>
<td>10</td>
<td>1.18</td>
<td>1.12</td>
<td>1.08</td>
</tr>
</tbody>
</table>

Real harm is ‘x’ times R where x are cell entries below:

Table 5a: Cournot conjecture (v = 0)                      Table 5b: Bertrand conjecture (v given in 6)

<table>
<thead>
<tr>
<th>e/b</th>
<th>0.3</th>
<th>0.6</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>n</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>2.00</td>
<td>2.00</td>
<td>2.00</td>
</tr>
<tr>
<td>5</td>
<td>1.86</td>
<td>1.78</td>
<td>1.71</td>
</tr>
<tr>
<td>10</td>
<td>1.76</td>
<td>1.68</td>
<td>1.63</td>
</tr>
</tbody>
</table>

In both of these first sets of tables we see that the errors in using R+T or R, though often still large, are both somewhat less when we have rising marginal costs. The differences are smaller when n is large or e/b is close to 1.

Cell entries correspond to percentage of true harm borne by direct purchasers:

Table 6a: Cournot conjecture (v = 0)                      Table 6b: Bertrand conjecture (v given in 6)

<table>
<thead>
<tr>
<th>e/b</th>
<th>0.3</th>
<th>0.6</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>n</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>75.00</td>
<td>75.00</td>
<td>75.00</td>
</tr>
<tr>
<td>5</td>
<td>57.69</td>
<td>46.88</td>
<td>37.50</td>
</tr>
<tr>
<td>10</td>
<td>44.78</td>
<td>31.91</td>
<td>23.08</td>
</tr>
</tbody>
</table>

When compared with their counterparts in Section 5, these last two tables show that the fraction of harm that remains with direct purchasers will be higher with rising marginal costs.