Phelps Centre for the Study of Government and Business

Working Paper
2006 – 06

Vertical Control, Dynamics and the Strategic Role of Inventories

Harish Krishnan
Operations and Logistic Division
Sauder School of Business
University of British Columbia

and

Ralph Winter
Strategy and Business Economics
Sauder School of Business
University of British Columbia

April 17, 2006

Phelps Centre for the Study of Government and Business
Sauder School of Business
University of British Columbia
2053 Main Mall
Vancouver, BC V6T 1Z2
Tel : 604 822 8399 or e-mail: phelps_centre@sauder.ubc.ca
Web: http://csgb.ubc.ca/working_papers.html
Vertical Control, Dynamics, and the Strategic Role of Inventories

Harish Krishnan and Ralph A. Winter*

April 17, 2006

Abstract

When consumers select retail outlets on the basis of both price and the “fill rate” (the probability of the desired product being available) inventory has an *ex ante*, demand-enhancing, effect. Greater inventory becomes a competitive strategy rather than just a means of satisfying random demand. We consider the coordination of inventory and pricing incentives in a distribution system when inventory has this ex ante effect on the demand facing each retailer. The key characteristic in predicting the nature of incentive distortions and their contractual resolutions is the degree of perishability of the product. In a static “newsvendor” model or with sufficiently high perishability of the product, downstream retailers are biased towards excessive price competition and inadequate inventories. Vertical price *floors* can coordinate incentives in both pricing and inventories. In a dynamic setting, where the product is less perishable, the distortion is reversed and vertical price *ceilings* coordinate incentives.

---

* Sauder School of Business, University of British Columbia. harish.krishnan@sauder.ubc.ca; ralph.winter@sauder.ubc.ca. We gratefully acknowledge support from the Social Sciences and Humanities Research Council and the Natural Sciences and Engineering Research Council.
1 Introduction

An extensive and recent literature recognizes that the choice of optimal inventory is not just a single agent decision problem but rather involves the alignment of incentives all along a vertical supply chain. This literature focuses on what we refer to as the *ex post* role of inventory, in that greater inventory allows a firm to satisfy random demand in a larger set of states.

Inventory in reality is not just a means of satisfying demand. It is also an instrument of competitive strategy because an increase in inventory will attract demand. Consumers who incur transactions costs in shopping are attracted to stores which offer a high fill rate, i.e., a greater probability of finding the item in stock (see Dana and Petruzzi 2001). We refer to this as the *ex ante* effect or *strategic* effect of inventory. When inventory has this effect, greater inventory not only allows a firm to satisfy demand in a larger set of states, it also increases demand in all states.

This paper analyzes the impact that the strategic effect of inventories has on incentive coordination in a supply chain. We consider a manufacturer selling to retailers who are competing in prices and inventories. We focus entirely on the strategic effect of inventory decisions and address the following questions: can inventory and pricing decisions be decentralized in a distribution system? What are the sources of potential incentive distortions in these decisions, and what kinds of contracts can bring the incentives of individual retailers in line with the manufacturer’s interests?

Surprisingly, the impact that this strategic inventory effect has on incentives depends upon what is perhaps the most basic product characteristic in an inventory model: the perishability of the product. In other words, vertical control in a distribution channel depends on inventory dynamics. If the product is completely perishable retailers are biased towards excessive price competition and away from competing on inventory or adequacy of stocks. Resale price maintenance (a price *floor*) can serve as an instrument to eliminate this distortion in retailer incentives: the price floor prevents retailers from competing intensively on price and at the same time by protecting the retail margin.
adds to each retailer's marginal benefit of carrying inventory. When the product is less perishable, however, the distortion in retailers’ competitive strategies is reversed. The manufacturer optimally imposes a price ceiling on retailers, thus lowering retail prices and dampening competition in the inventory dimension.

Our approach differs from the existing literature in three ways. First, because the incentive distortions depend on the perishability of the product, our model is set in a dynamic context. In the supply chain contacting literature static models are more common. Second, we highlight the role of price restraints, distinguishing between price floors versus price ceilings. The incentive distortions giving rise to the simplest use of these two instruments are diametrically opposite: price floors respond to a bias in retailers’ competitive strategy towards excessive price competition relative to inventory competition. Price ceilings respond to excessive prices and excessive inventory competition. Because the availability of each restraint under the law differs (and because the incentive distortions giving rise to each restraint are opposite) it is important to understand which price restraint is profitable. Third, we assume that the contract signed between the manufacturer and a retailer maximizes the combined profits of the contracting parties, i.e., fixed transfers between the manufacturer and retailers is allowed. In other words, if parties see a contracting opportunity that increases their combined wealth, they will take it and find some way of dividing the gains. Because incentive distortions arise due to the sum of vertical (double markup) and horizontal (competitive) externalities, the assumption of joint wealth maximization does not trivialize the problem. Rather, it provides a fundamental starting point in allowing us to identify the sources and resolutions of incentive problems.¹

¹In much of the supply chain literature, the manufacturer, in designing contracts, is assumed to be limited to linear pricing and therefore must often compromise between the goals of maximizing collective profit and extracting the largest share possible of this profit. These competing goals add substantially to the complexity of the models. Moreover, the restriction is not explained endogenously in the models. It is argued in the literature that ruling out fixed fees reflects the fact that they are rare in reality. But if one is going to incorporate various aspects of real markets that are not explained within the model, then surely it is important to recognize that manufacturers actually care about the downstream profitability of carrying their product. The more profitable a product, the greater the set
In a companion paper (Krishnan and Winter 2006) we analyze the economic foundations underlying the coordination of incentives in a distribution system with (ex post) inventory interaction. The foundations are based on two principles. First, an organization faces an incentive problem when an agent within the organization does not appropriate the full collective benefits of decisions taken. Second, an incentive problem is resolved when some of the agent’s actions are constrained at the optimal levels and prices or reward systems internalize the externalities imposed by the remaining actions on other agents within the organization. We apply the same two fundamental principles here.

We review the literature in the next section. In Section 3, we set out the game in the case of complete perishability (a static model) and then generalize to a dynamic model. Using a more structured model in Section 4 we characterize the incentive distortions in the dynamic model.

2 Related Literature

The supply chain contracting literature studies the incentive for firms to carry inventory in anticipation of uncertain demand. Cachon (2003) reviews this extensive literature and surveys dozens of models that vary along dimensions such as: (1) market structure: one manufacturer and one retailer; one manufacturer and oligopolistic retailers; one manufacturer with a competitive retail sector; (2) decisions on inventory versus decisions on inventory and prices; (3) the possibility of demand “spillovers” (from one downstream outlet to another) in the events of stock-outs; (4) the flexibility to set prices ex post; (5) decisions on retailer effort, in addition to inventory decisions; (6) the opportunity for replenishment of inventory; (7) static versus dynamic frameworks; (8) symmetric versus asymmetric information. In each of the various cross-products of these assumptions, the literature has considered contracts such as buy-back contracts (Pasternack 1985), revenue-sharing

of retailers that will carry the product (and indeed in many resale price maintenance cases manufacturers refer to the practice as protecting a wide distribution system). Also, any non-linear pricing scheme (such as quantity discounts) can replace fixed fees in our model.
contracts (Cachon and Lariviere 2005), quantity-flexibility contracts (Tsay 1999), sales rebates and markdown allowances (Taylor 2002, Krishnan et.al. 2004), and so on.

The economics literature on vertical restraints (see Mathewson and Winter 1984 and Katz 1989) has considered the role of price restraints in resolving incentive conflicts in a vertical structure. A number of papers in this literature have analyzed inventory incentives; including Deneckere, Marvel and Peck (1996, 1997), Butz (1997), Dana and Spier (2001), and Krishnan and Winter (2006).

The above papers clearly recognize the importance of inventory but they do not incorporate the fact that inventory has an ex ante effect on demand. It is reasonable to assume that customers will be attracted to stores that have a greater probability of having the product in stock. Kelly et.al. (1991) provide survey evidence to show that customers anticipate stockouts. Incorporating this ex ante effect of inventory, both Wang and Gerchak (1994) and Balakrishnan et.al. (2004) characterize the structure of the optimal inventory control policy for a monopolist; Wang and Gerchak assume stochastic demand while Balakrishnan et.al. assume deterministic demand. Dana and Petruzzi (2001) study price and inventory decisions of a monopolist facing uncertain demand, and show that higher inventories, demand and profits will result from the ex ante effect. Deneckere and Peck (1995) study an oligopoly with price and inventory competition, and characterize the equilibrium solution.

Wang and Gerchak (2001) and Bernstein and Federgruen (2003) study incentive issues in a decentralized supply chain, where demand depends on retail inventory. Wang and Gerchak consider a static model with deterministic demand; decentralized retailers choose inventory levels which influences demand. They show that the manufacturer can coordinate the channel needs by offering an inventory holding cost subsidy to the retailer.2

2When multiple retailers compete, these authors make assumptions about how customers are allocated to retailers and, characterize retailers’ Nash equilibria in these models. In our paper, customer allocation to retailers is endogenously determined as a rational expectations equilibrium of a game among consumers.
Bernstein and Federgruen consider a dynamic model with a manufacturer selling to downstream newsvendors competing in price and fill rates, and they seek to find contracts that maximize supply chain profits. Unsatisfied demand is assumed to be “backlogged,” i.e., satisfied in the next period. Bernstein and Federgruen show that the first-best solution can be achieved when each retailer is offered a (unique) wholesale price combined with a “backlogging penalty” per unit of backlogged demand. In contrast, we find that simple price restraints can achieve the first best solution and we trace the optimal contract to underlying product characteristics.

3 The basic setting

A monopolist sells a single product to two retailers who then sell it to consumers. Two retailers $i = 1, 2$ choose prices $p_i$ and inventories $y_i$ before the realization of uncertain demand. In order to focus on the ex ante effect of inventory decisions, we assume that consumers have time to visit only one store, and cannot determine the availability of a product before leaving home. Instead, consumers will choose to visit the store that yields the highest expected consumer surplus, depending on the price and the fill rate.

In reality, stores gain reputations for good or poor fill rates. We suppress the reputation story, allowing consumers to know immediately the (equilibrium) fill rates. The simplest way to do this is to collapse this to an instantaneous learning process by assuming that consumers can observe the inventory decisions or fill rates of firms. (For example, a consumer information service publishes this information.) In short, we assume that consumers make the choice of whether and where to shop with knowledge of the prices and inventory choices, but no knowledge of the realized demand.

The determination of the demand functions is itself the outcome of a game because of the negative externalities consumers impose on each other: the decision of a consumer to shop at store $i$
reduces the fill rate for all other consumers at that store.\textsuperscript{3} With enough structure on the model (which we provide in Section 4), the existence of a rational expectations equilibrium of the demand game is assured. Given prices $p = (p_1, p_2)$ and inventory levels $y = (y_1, y_2)$, the demand equilibrium results in demand functions $q_i(p, y, m)$, where $m$ is a random variable representing demand uncertainty. The number of transactions given $(p, y, m)$ is given by $T_i(p, y, m) = \max(q_i(p, y, m), y_i)$.

The manufacturer’s marginal production cost is $c$ per unit. Retailers bear no cost other than the payments to the manufacturer, a per unit wholesale price $w$ and a fixed fee $F$. We consider the game in which the manufacturer first sets a contract that specifies $(w, F)$ and possibly other restrictions (but cannot specify inventory directly) and the retailers then play a game in $(p, y)$ to maximize expected profits.

### 3.1 Payoffs in the static framework

Consider a single-period model (the classic newsvendor assumption), where inventory perishes at the end of the period. The profit of retailer $i$ in this static price/inventory game is given by

$$\pi_i^e(p, y, m) = p_i T_i(p, y, m) - w y_i - F$$

We are interested in the Nash equilibrium in $(p, y)$ given payoffs $E\pi_i(p, y, m)$.\textsuperscript{4} Upstream, the manufacturer earns profits (excluding any fixed fee transfers) given by $\pi_M(y) = (w - c)(y_1 + y_2)$.

We define $(p^*, y^*)$ as the decision that maximizes the collective profit $E\Pi^e(p, y, m)$, where

$$\Pi^e(p, y, m) = p_1 T_1(p, y, m) + p_2 T_2(p, y, m) - c(y_1 + y_2)$$

\textsuperscript{3}In this respect, the demand side is analogous to demand in a market with congestion externalities.

\textsuperscript{4}A natural question at this point is to determine conditions that ensure the existence of a Nash equilibrium. We set this question aside because the existence of equilibrium is assured in a structured model (which we provide in Section 4). We will also restrict attention to models which result in a symmetric equilibrium.
3.2 Payoffs in the Dynamic Framework

We assume in the dynamic extension (the general case) that there are an infinite number of periods and stationary demand. In each period, each outlet inherits a constant fraction $\delta$ of the stock left over from the previous period. (We interpret $\delta$ as the percentage of stock not perishing; $\delta$ can also incorporate a discount factor and a holding cost.) We now interpret the firms’ inventory choice $y_i$ as a base-stock level. In other words, $y_i$ refers to the inventory level at firm $i$ that is chosen to meet demand in each period. Since we are concerned with only steady state profits in this stationary model, the dynamic extension involves simply adding $\delta c$ times the realized excess inventory to the aggregate profit function (reflecting the value today of the realized excess inventory), and $\delta w$ times the realized excess inventory to an outlet’s profit by a similar argument.\footnote{Federgruen and Heching (1999) prove that, for a firm choosing price and inventory in a dynamic setting where inventory is carried into the future, the structure of the optimal policy is a stationary base-stock list price policy. A base stock list price policy is characterized by a base stock level and a price (the “list price”). Bernstein and Federgruen (2004) show that, under certain assumptions, it is a Nash equilibrium in an infinite-horizon retailer game for each retailer to choose a stationary price and a stationary base-stock level, when firms are maximizing their average per period expected profits.} We again define $(p^*, y^*)$ as the decision that maximizes the collective profit $E \Pi^d(p, y, m)$, where

$$\pi^d_i(p, y, m) = p_i T_i(p, y, m) - wy_i + \delta w(y_i - T_i(p, y, m)) - F$$

and

$$\Pi^d(p, y, m) = p_1 T_1(p, y, m) + p_2 T_2(p, y, m) - cy_1 - cy_2 + \delta c(y_1 - T_1(p, y, m)) + \delta c(y_2 - T_2(p, y, m))$$

3.3 The Incentive Distortions

The starting point to any contract design problem is to identify why agents’ incentives may be distorted. In our simple approach this involves just the comparison of first-order conditions for the individual retailer’s optimum with those of the collective optimum. In other words, at the value
\( (p^*, y^*) \) where the collective profit is maximized (and the collective first-order conditions satisfied), why might the individual retailer have a marginal incentive to deviate?

The framework yields two kinds of distortions between an individual retailer’s marginal gains and collective gains from increasing price or inventory. The first is the vertical externality whereby the retailer ignores the impact on the upstream manufacturer’s profits of a change in either decision. This impact flows through the wholesale markup, \((w - c)\) and is responsible for the standard double markup problem. The second source of distortion is the horizontal externality whereby the retailer ignores the impact on profits earned by its retailer rival (or by the upstream manufacturer from sales to the retailer rival). These externalities flow through the retailer markup of price over the opportunity cost of selling an additional unit today, \((p - \delta w)\) and \(\delta(w - c)\) which is the manufacturer’s markup of tomorrow’s wholesale price over the marginal cost resulting from a unit that is not sold today. The sum of the latter markups is \((p - \delta c)\).

Using (3) and (4), we can solve for the difference in first-order conditions and decompose this difference as follows:

\[
\frac{\partial E \pi_i^d}{\partial y_i} = \frac{\partial E \Pi^d}{\partial y_i} - (w - c)(1 - \delta(1 - \frac{\partial ET_i}{\partial y_i})) - (p_j - \delta c) \frac{\partial ET_j}{\partial y_i} \tag{5}
\]

\[
\frac{\partial E \pi_i^d}{\partial p_i} = \frac{\partial E \Pi^d}{\partial p_i} - \delta(w - c) \frac{\partial ET_i}{\partial p_i} - (p_j - \delta c) \frac{\partial ET_j}{\partial p_i} \tag{6}
\]

Note that for each decision, \(y_i\) and \(p_i\), the two vertical and horizontal externalities are opposite in sign. The purchase of an additional unit of inventory, for example, increases upstream profits directly but reduces the rival’s profit and (upstream profits resulting from the impact on the rival’s sales). Note in addition that the vertical externality is increasing in \(w\) but the horizontal externality is independent of \(w\). This means that for each action, inventory and price, there is a particular \(w\)
that, if set by the manufacturer, would leave the externalities exactly offsetting and the retailer’s incentives optimal, conditional upon the level of the other action. Only when the identical \( w \) leaves the externalities offsetting in both equations can the manufacturer generate first-best profits with no restraints. (Because of the availability of fixed fees, the wholesale price \( w \) is an instrument used entirely for eliciting the right incentives; it need not be used to collect rents.)

If the pairs of externalities are not offsetting at the same \( w \), then we can say that retailers are distorted towards excessive price competition or towards excessive inventory competition. For example, suppose that at the value of \( w \) that renders the pair of externalities in the inventory equation identical, the horizontal externality in the price equation exceeds (in absolute value) the vertical externality in that equation. The retailer’s incentive is distorted towards setting price too low, i.e. towards excessive price competition. The manufacturer can respond with the following: raise \( w \) until the optimal price \( p^* \) is elicited on the part of retailers (at which point inventory will be too low). If the retailer’s payoffs are quasi-concave, the manufacturer can now set a price floor at \( p^* \), and reduce \( w \), thus raising the optimal inventory on the part of retailers, until \( y^* \) is reached. A price floor, in short, can be used as an instrument in addition to \( w \) to elicit \((p^*,y^*)\) when retailers are biased towards excessive price competition. By a similar argument, a price ceiling is an optimal response to an incentive distortion towards excessive inventory competition.

The characterization of optimal coordination contracts, focussing on price restraints, therefore, consists of identifying whether retailers are biased in their choice of competitive strategy and in which direction. The insights provided by this decomposition are found by considering the two limiting cases, \( \delta = 0 \) (complete perishability) and the opposite limiting case \( \delta \to 1 \).\(^6\) These allow us to formulate principles of optimal coordination in the cases where inventory is a sufficiently important concern (\( \delta \) is sufficiently small) and inventory is an unimportant concern (\( \delta \) is sufficiently close to

\(^6\)Note that the limit itself, \( \delta = 1 \), is uninteresting since this represents the case where there is no opportunity cost whatsoever to maintaining additional inventory. Inventory is not an issue.
1). Proposition 1 answers this immediately for the case of complete perishability, and Proposition 2 provides a condition that determines whether retailers are biased and in which direction.

**Proposition 1** For static case, the retail outlets are biased towards excessive price competition. If the outlets’ profits are quasi-concave, the manufacturer can use a price floor (in combination with a wholesale price and a fixed fee) to elicit the first-best solution.

To prove Proposition 1, note that for the static case (δ = 0) comparing the individual incentives and collective efficiency in price and inventory using (1) and (2) yields the following:

\[
\frac{\partial E\pi^e_i}{\partial y_i} = \frac{\partial E\Pi^e}{\partial y_i} - \frac{\partial E\Pi^e}{\partial y_i} \left( w - c \right) - \frac{\partial E\Pi^e}{\partial y_i} \left( w - c \right) - \frac{\partial E\Pi^e}{\partial y_i} \left( w - c \right) - \frac{\partial E\Pi^e}{\partial y_i} \left( w - c \right)
\]

\[
\frac{\partial E\pi^e_i}{\partial p_i} = \frac{\partial E\Pi^e}{\partial p_i} - \frac{\partial E\Pi^e}{\partial p_i} - \frac{\partial E\Pi^e}{\partial p_i} - \frac{\partial E\Pi^e}{\partial p_i}
\]

The last equation captures the main feature of the decentralization of price and inventory decisions in the static model. For a fixed level of inventory, *pricing decisions are not subject to a vertical externality.*\(^7\) In other words, given the inventory choice of an outlet, the manufacturer has no direct interest in the price at which the inventory is resold. However, the manufacturer is not indifferent to the pricing decision. Rather, from the manufacturer’s perspective the downstream pricing decision is *always* distorted.\(^8\) Because only the horizontal externality is at work, decentralized pricing is biased towards a price that is, at the margin, too low. Therefore, retailers are biased towards excessive price competition and the appropriate price restraint is a price floor.

---

\(^7\)The implications of this “missing externality” on supply chain contracts, is explored in more detail in Krishnan and Winter (2006).

\(^8\)Since the horizontal externality reduces profits for each outlet, the fixed fee that the manufacturer can charge up front is reduced.
For $\delta > 0$, however, note that the pricing decisions are subject to a vertical externality. This is because the manufacturer does care about the retail price, as it affects the amount the retailer will purchase in future periods (see (6)). Suppose there exists a $\tilde{\delta} > 0$ and a $\tilde{w} > c$ such that the last two terms of equation (6) sum to zero when the last two terms of equation (5) sum to zero (when both equations are evaluated at $(p^*, y^*)$). For $\tilde{\delta}$ and $\tilde{w}$, the wholesale price alone is then sufficient for the manufacturer to elicit the optimal price and inventory levels. By setting the last two terms of equation (6) and (5) to zero and by eliminating $\tilde{w}$ we get,

$$
\tilde{\delta} = \left. \frac{\partial ET_i}{\partial p_i} \right| \left. \frac{\partial ET_j}{\partial y_i} \right| \left. \frac{\partial ET_j}{\partial p_j} \right| \left. \frac{\partial ET_i}{\partial y_j} \right| (p^*, y^*)
$$

(9)

It is straightforward to verify that if $\delta < \tilde{\delta}$ then the retail outlets are always biased towards price competition, and if $\delta > \tilde{\delta}$ then the retail outlets are always biased towards inventory competition. Clearly, if $\tilde{\delta} \geq 1$, then for all $\delta \in (0, 1)$ the retailers are biased towards excessive price competition. However, if $\tilde{\delta} < 1$, then the optimal price restraint switches from a price floor to a price ceiling for above the “threshold” level of product perishability represented by $\tilde{\delta}$.

Therefore, the optimal price restraint depends on $\delta$ if and only if $\tilde{\delta} < 1$. The key question in determining whether a price floor is always the optimal restraint depends on the value of $\tilde{\delta}$. Rewriting (9), we get the following result.

**Proposition 2** Let $\epsilon_{p_i}^i$ and $\epsilon_{p_m}^i$ represent the price-elasticities of individual outlet transactions and market transaction evaluated at $(p^*, y^*)$, and define $\epsilon_{y_i}^i$ and $\epsilon_{y_m}^i$ similarly. If $\frac{\epsilon_{p_i}^i}{\epsilon_{p_m}^i} > \frac{\epsilon_{y_i}^i}{\epsilon_{y_m}^i}$ then the outlets are excessively oriented towards price competition for all values of $\delta \in (0, 1)$. If $\frac{\epsilon_{p_i}^i}{\epsilon_{p_m}^i} < \frac{\epsilon_{y_i}^i}{\epsilon_{y_m}^i}$, then for $\delta < \tilde{\delta}$ a price floor is the optimal restraint; for $\delta > \tilde{\delta}$ a price ceiling is optimal.

In summary, the optimal price restraint for the static case is a direct consequence of the missing
vertical externality in the pricing decision. The optimal price restraint for the opposite limiting case \((\delta \to 1)\) depends on a comparison of the two elasticity ratios in the proposition. As we move from the market to the individual firm, elasticities of demand with respect to price or inventory increase. If price elasticity increases proportionately more, then we can conclude that retailers are biased towards price competition and away from inventory competition relative to the collective optimum. In this case, a price floor is optimal to redirect retailer competition. Similarly, for the case where “inventory elasticity” increases proportionately more, retailers are biased away from price competition and a price ceiling is needed to redirect retailer competition.

What determines the pivotal elasticity ratio? To make this comparison requires additional structure and assumptions about consumer preferences and demand. In the structured model developed in Section 4 we show that if one relaxes the assumption of complete perishability, the ex ante effect comes into play in a way that reverses the incompatibility in incentives: retail outlets tend to invest excessively in inventory. A vertical price ceiling is then profitable as a means of suppressing this incentive by tightening the retail margin.

4 Structured Model

There is a set \(\Theta\) of consumers, indexed by \(\theta\). The density of consumers on \(\Theta\) is random and, for simplicity, perfectly correlated across \(\theta\). We index the realization of demand by \(m\) and so the number of potential consumers, conditional upon \(m\), is \(m\theta\). The random variable \(m\) has density \(g\) and distribution function \(G\).

Consumer \(\theta\) attaches a value \(u_{\theta}^i\) to one unit of the product offered by firm \(i\), and has a zero utility of not purchasing. Each consumer purchases: 0 units, or 1 unit of the product from retailer 1, or 1 unit from retailer 2. Assume that the two retail outlets are located at two ends of a unit

\footnote{This comparison of elasticity ratios is applied in a different context in Winter (1993).}
line segment. Each outlet competes over the customers who are uniformly distributed in the space between the two retailers, with each consumers location indexed by \( s \in [0,1] \). Consumers incur a travel cost of \( t \) per unit distance travelled. For our model, it turns out to be essential to also allow consumers to vary in the extent to which they are attracted by a reliable inventory, i.e. a higher fill rate. We do this simply by assuming that the buyers vary in a second dimension as well as location: their value of the product. Consumers with a high product value care more about the fill rate relative to the price, at the margin. Consumers valuation for the product is indexed by \( v \in [0,\bar{v}] \). Therefore, consumers each occupy a point in the customer space \((\Theta)\) illustrated by a rectangle in Figure 1, and each consumer type \( \theta \) is indexed by their location and valuation, i.e., \( \theta \equiv (s,v) \).

The determination of the demand functions is itself the outcome of a game because of the negative externalities consumers impose on each other: the decisions of consumers to shop at store \( i \) reduce the fill rate for all consumers at that store.\(^{10}\) In equilibrium, the consumers are partitioned into three sets: those that purchase from 1; those that purchase from 2 and the no-purchase set.

### 4.1 Demand equilibrium

Let \( n = (n_1,n_2) \) be the measure of the set of consumer types that each consumer expects will shop at each outlet, given prices \( p = (p_1,p_2) \) and inventory levels \( y = (y_1,y_2) \). In other words, contingent upon the realization of \( m \), the consumer expects \((mn_1,mn_2)\) shoppers. Define \( \psi(n; (p,y)) = [\psi_1(n; (p,y)), \psi_2(n; (p,y))] \) as the measure of the set of consumers that would actually purchase with the expectations \( n \). Then a fixed point \( \bar{n}(p,y) = [\bar{n}_1(p,y), \bar{n}_2(p,y)] \) of \( \psi \) is a rational expectations equilibrium of the game. The demand under realization \( m \) is then \( q(p,y,m) = [q_1(p,y,m), q_2(p,y,m)] = [mn_1(p,y), mn_2(p,y)] \).

We assume proportional rationing of inventory in the event of a stock-out; in other words, all

---

\(^{10}\)In this respect, the demand side is analogous to demand in a market with congestion externalities.
customers who shop at a particular outlet have the same probability of getting the product. Hence, given \((p, y)\) and expectations \(n\), the anticipated fill probability at \(i\) is \(\text{prob}(mn_i \leq y_i) = \text{prob}(m \leq y_i/n_i) = G(y_i/n_i)\). The utility for consumer \(\theta\) from shopping at retail outlet 1 and 2 are:

\[
\begin{align*}
    u^1_\theta &= (v - p_1)G\left(\frac{y_1}{n_1}\right) - ts \\
    u^2_\theta &= (v - p_2)G\left(\frac{y_2}{n_2}\right) - t(1 - s)
\end{align*}
\]

and the consumers in the no purchase set derive zero utility. The demand partition is given by \(D_1(p, y, n) = \{\theta \mid (u^1_\theta \geq \max\{0, u^2_\theta\})\}\) and similarly for \(D_2(p, y, n)\) and the no-purchase set. Given expectations \(n\) and \((p, y)\), \(\psi_i(n; (p, y)) = \int I_{D_i}(p, y, n)(\theta)d\theta\) where \(I_X(\theta) = \{\theta \mid \theta \in X\}\) is the characteristic function of the set \(X\). Note that

\[
\begin{align*}
    \int I_{D_1}(p, y, n)(\theta)d\theta &= \int_{p_1}^{\hat{s}} s^1_p(v)dv + \int_{\hat{s}}^{\hat{v}} s_r(v)dv \quad (10) \\
    \int I_{D_2}(p, y, n)(\theta)d\theta &= \int_{p_2}^{\hat{s}} s^2_p(v)dv + \int_{\hat{s}}^{\hat{v}} (1 - s_r(v))dv \quad (11)
\end{align*}
\]

where \(s^1_p(v)\) refers to outlet 1’s customers on the product margin, \(s^2_p(v)\) refers to outlet 1’s customers on the product margin, \(s_r(v)\) refers to outlet 1’s customers on the inter-retailer margin, and \(\hat{\theta} = (\hat{s}, \hat{v})\) is the point where the two margins intersect (see Figure 1). The following equations characterize these margins.
\[ s^1_p(v) = \frac{(v - p_1)G(\frac{y_1}{n_1})}{t} \]  \hspace{2cm} (12)
\[ s^2_p(v) = 1 - \frac{(v - p_2)G(\frac{y_2}{n_2})}{t} \]  \hspace{2cm} (13)
\[ s_r(v) = \frac{(v - p_1)G(\frac{y_1}{n_1}) - (v - p_2)G(\frac{y_2}{n_2})}{2t} + \frac{1}{2} \]  \hspace{2cm} (14)
\[ \hat{\nu} = \frac{t + p_1G(\frac{y_1}{n_1}) + p_2G(\frac{y_2}{n_2})}{G(\frac{y_1}{n_1}) + G(\frac{y_2}{n_2})} \]  \hspace{2cm} (15)
\[ \hat{s} = \frac{G(\frac{y_1}{n_1})}{G(\frac{y_1}{n_1}) + G(\frac{y_2}{n_2})} \left( \frac{p_1 - p_2)G(\frac{y_1}{n_1})G(\frac{y_2}{n_2})}{t(G(\frac{y_1}{n_1}) + G(\frac{y_2}{n_1}))} \right) \]  \hspace{2cm} (16)

It is clear from (10) and (11) that the mapping \( \psi \) is continuous and, since the set of \( n \) is compact and convex, \( \psi \) has a fixed point by Brouwer’s fixed point theorem. This fixed point must be unique for \( q(p, y, m) \) to be well defined. The following Lemma is proved in the Appendix.

**Lemma 1** For all \( m \), \( q(p, y, m) \) is well defined and differentiable. \( q_1(p, y, m) \) is decreasing in \( p_1 \) and \( y_2 \) and increasing in \( p_2 \) and \( y_1 \) over the range of \( (p, y) \) where \( D_1(p, y, n) \) and \( D_2(p, y, n) \) are non-empty; similarly for \( q_2(p, y, m) \).

### 4.2 Elasticity ratio comparison

Before we can compare the elasticity ratios and determine the relationship between product perishability and price restraints, we first need to characterize the Nash equilibrium in the game between retailers. Unfortunately, a difficulty in differentiated Bertrand models, as well as models in which firms choose quantities and prices, is that payoff functions are not concave or even quasi-concave and the non-existence of pure strategy equilibria is common (see d’Aspremont, Gabszewicz and Thisse 1979 and Friedman 1988).\(^{11}\) This is true in the game between retailers in the structured

\(^{11}\)A mixed strategy equilibrium exists as long as payoff functions are continuous and strategy spaces are compact (Glicksberg 1952).
model described above.\textsuperscript{12} A common approach in such models is to restrict strategies or parameters to ranges where the pure strategies do exist.\textsuperscript{13} In our structured model, if the two outlets are sufficiently differentiated (i.e., travel costs are above a critical level), then the payoff functions are quasi-concave. Accordingly, we restrict consideration to the range of sufficiently high $t$. We also restrict attention to the symmetric case, i.e. to retail markets that are sufficiently differentiated such that the centralized solution is also symmetric. The following Proposition is proved in the Appendix.

**Proposition 3** *Within the structured model, $\delta < 1$, i.e. for $\delta > \delta$ a price ceiling is optimal.*

The intuition for the above result follows directly from Spence (1975) and can be explained with the help of Figure 1. Note that the impact on the collective optimum $(p^*, y^*)$ of the ex ante effect of inventory, is entirely through the tastes of consumers on the *product margin*: the consumers on the border of the “no purchase set.” Since these consumers have relatively low valuation for the product, the cost to them of encountering a stock out is small and marginal ex ante value that they attach to greater inventory is therefore low. The aggregate profits are maximized at a combination of low inventory and low price. A retail outlet, on other hand, chooses the mix of competitive instruments price and inventory, to accommodate the tastes of consumers on its margin, including not just part of the product margin but also the *inter-retailer margin*. The consumers on the latter margin have a higher valuation on average and therefore attach higher ex ante value to inventories on average. The analysis therefore points to a distortion towards excessive inventory and excessive pricing in a decentralized retail system to the extent that consumers vary in their valuation of

\textsuperscript{12}Intuitively, when the products offered by the two outlets are very close substitutes (travel costs are low), then an outlet may attain one local optimum in price by competing intensively with its rival, and another local optimum by forgoing such competition, charging a near-monopoly price, and relying on the chance of a stock-out at its rival to generate revenue.

\textsuperscript{13}For example, Salop (1979) and all of the address model literature on product differentiation (see Eaton and Lipsey 1989).
the product. The role for vertical price ceilings is clear when inventory plays the strategic role of attracting customers.

To complete this intuition, we need to explain the role of the condition $\delta > \tilde{\delta}$; in other words, the perishability of the product. If inventories are important (i.e., the product is perishable or expensive to hold), then the “missing externality” argument explains why price floors are optimal. Price ceiling become optimal only as inventories become less important ($\delta \to 1$). This reinforces the insight from Deneckere et.al. (1996) and Krishnan and Winter (2006) that price floors help provide incentives to hold inventory in markets where inventories are important. But interestingly, price ceilings coordinate the channel when inventory is less important – which mirrors the well-known optimality of price ceilings in particular price-only situations (e.g., to fix the double mark-up problem).

5 Discussion and Conclusion

This paper provides a framework for analyzing optimal managerial control of the price and inventory decisions of downstream supply chain partners. A key insight is that the form of the optimal vertical restraints hinges on whether the optimal mix of competitive strategies at the retail level mirrors the efficient mix. Price floors and price ceilings are used to address the opposite types of incentive problems: a downstream bias towards price competition necessitates a price floor and a bias towards holding excessive inventory necessitates a price ceiling.

At a general level, the aim of applied contract theory is to trace market conditions into predictions about optimal contracts. Interestingly, both the strategic and dynamic nature of inventory play a role in determining the optimal contract. Because inventory has a strategic or ex ante effect, retailers may focus excessively on providing high “fill-rates” as a means to attract customers away from each other. This is clear in the structured model of Section 4, where retailers appealing to
marginal customers accommodate the preferences of customers on the inter-retailer margin (who prefer high fill-rates) instead of focussing entirely on the preferences of customers at the product margin (who prefer lower prices). The *ex post* effect of inventory reinforces the opposite distortion. The resolution of the two offsetting distortions, and the optimal contract, depends on the *perishability* of the product.

A natural question that arises in supply chain contexts is the following: what contractual mechanisms can resolve the incentive distortions identified in this paper? In Krishnan and Winter (2006), we trace the price floor arising from the missing externality into a number of other contracts that can substitute for a price floor. A buy-back contract (and, equivalently, a per-unit revenue royalty contract) with a fixed fee achieve the desired effect by introducing a vertical externality into the retailers’ pricing decision.  

14 Interestingly, a buy-back contract combined with a price ceiling is optimal and robust to information asymmetry about demand.  

15 The choice of a full set of optimal contracts that respond to the combination of ex ante and ex post effects as analyzed in this model remains an open question.

---

14 A revenue royalty contract cannot achieve this.

15 The absence of a fixed fee under information asymmetry implies that the manufacturer needs a higher wholesale price to transfer rents. This necessitates a higher buy-back price to coordinate the inventory decision, and the high buy-back price dampens price competition to the extent that a price ceiling needs to be imposed.
References


20


Figure 1: Spatial Model for Section 4
Appendix

Lemma 1: Proof. The following property of $\psi$ is easily verified: $\psi_1$ is strictly decreasing in $n_1$ and strictly increasing in $n_2$ and similarly for $\psi_2$. Given $(p, y)$ suppose that there are two fixed points $n^a = (n^a_1, n^a_2)$ and $n^b = (n^b_1, n^b_2)$, with $n^a \neq n^b$.

Case 1: Let $n^a \geq n^b$ (i.e., $n^a_1 \geq n^b_1$ and $n^a_2 \geq n^b_2$). If $n^a \geq n^b$, it must be true that $\psi(n^b|(p, y)) \geq \psi(n^a|(p, y))$, which is a contradiction unless $n^a = n^b$.

Case 2: Let $n^a_1 \geq n^b_1$ and $n^a_2 \leq n^b_2$. It must be true that $(\tilde{n}^a_1) \leq (\tilde{n}^b_1)$ and $(\tilde{n}^a_2) \geq (\tilde{n}^b_2)$, which is a contradiction unless $n^a = n^b$.

Differentiability can be shown by using the system version of the implicit function theorem, applied to $\Psi(n;p,y) = \psi(n;p,y) - n$. (Applying the system implicit function theorem involves verification that the determinant of the Jacobian matrix of $\Psi(n;p,y)$, with respect to $n$ is of full rank.) To show that $q_1(p,y,m)$ is decreasing in $p_1$, note that if it were not, then a higher $p_1$ would be accompanied by a higher $n_1$ which would imply that more customers prefer retailer 1 despite the higher price and demand. This is a contradiction. The impact changes in $y_1$, $p_2$, and $y_2$ on $q_1(p,y,m)$ can be shown similarly. ■

Proposition 3: Proof.

To prove that $\tilde{\delta} < 1$, we need to show that $\frac{\delta_{p_1}}{\delta_{y_m}} < \frac{\delta_{y_m}}{\delta_{y_m}}$. Define $T(p,y,m) = \sum_i T_i(p,y,m)$ as the total transactions in each period (summing over transactions at both outlets). It is sufficient to show that

$$\frac{\partial ET_i(p,y,m)}{\partial p_1} < \frac{\partial ET_i(p,y,m)}{\partial y_m}$$

where the numerators represent the marginal effect of changes in retail price and inventory on the expected transactions at an outlet, and the denominators represent the marginal effect of changes in retail price and inventory (simultaneously at both outlets) on the expected total transactions.
Because demand uncertainty is perfectly correlated across all consumer types, it is sufficient to prove that
\[
\frac{\partial q_i(p,y,m)}{\partial p} < \frac{\partial q_i(p,y,m)}{\partial y},
\]
where \(Q(p,y,m) = \sum_i q_i(p,y,m)\).

From Figure 1 note that
\[
\frac{\partial Q}{\partial p} = 2 \frac{\partial \left[ \int_{v_1} \hat{v} s_p(v) dv + (\bar{v} - \hat{v}) \right]}{\partial p} \quad (17)
\]
\[
\frac{\partial Q}{\partial y} = 2 \frac{\partial \left[ \int_{v_1} \hat{v} s_p(v) dv + (\bar{v} - \hat{v}) \right]}{\partial y} \quad (18)
\]

For retail outlet 1 (the argument for retail outlet 2 is analogous):
\[
\frac{\partial q_1}{\partial p} = \frac{\partial \left[ \int_{v_1} \hat{v} s_p(v) dv + \hat{s}(\bar{v} - \hat{v}) \right]}{\partial p} = \frac{1}{2} \frac{\partial Q}{\partial p} - \frac{\partial (1 - \hat{s})(\bar{v} - \hat{v})}{\partial p} \quad (19)
\]
\[
\frac{\partial q_1}{\partial y} = \frac{\partial \left[ \int_{v_1} \hat{v} s_p(v) dv + \int_{v_1} \hat{s} s_r(v) dv \right]}{\partial y} = \frac{1}{2} \frac{\partial Q}{\partial y} + \frac{\partial \int_{v_1} \hat{s} s_r(v - \hat{s}) dv}{\partial y} \quad (20)
\]

We need to show that \((19) < (20)\), i.e.,
\[
\frac{1}{2} - \frac{\partial (1-\hat{s})(\bar{v}-\hat{v})}{\partial p} \frac{\partial Q}{\partial p} < \frac{1}{2} + \frac{\partial \int_{v_1} \hat{s} s_r(v - \hat{s}) dv}{\partial y} \quad (21)
\]

The inequality is satisfied because both \(\frac{\partial (1-\hat{s})(\bar{v}-\hat{v})}{\partial p}\) and \(\frac{\partial \int_{v_1} \hat{s} s_r(v - \hat{s}) dv}{\partial y}\) are greater than 0. ■