Credible Retaliatory Entry and Strategic Toe-Holds*

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Abstract

When contemplating entry into a new market, an entrant will consider how incumbents in that market will respond. One possibility, suggested by a number of cases and described in the strategy literature, is that the incumbent will respond with entry of its own into an important market in which the first entrant was already operating. This paper provides a simple, formal model in which threats of such “retaliatory entry” can in fact be credible. The key to the model is that all firms produce from facilities subject to increasing marginal costs or capacity constraints. This creates a link between a firm’s activities in one market and its payoffs from competing in another market. When retaliatory entry is a credible threat, it may prevent any entry at all from occurring. The paper also considers how firms, when retaliatory entry is not immediately credible because of the costs of entry, can make it credible through the establishment of toe-hold investments in other markets. Toe-holds in this model are valued not for the profits they earn themselves, but for their entry deterrence value.
I. Introduction

A prospective entrant typically faces a considerable amount of uncertainty when it contemplates moving into a new market, including uncertainty about how buyers will receive its product and uncertainty about the costs it will face establishing itself and maintaining its operations in that market. In markets that are not perfectly competitive another important source of uncertainty relates to the responses of incumbents to new entry. Will incumbents accommodate entry, or will they take actions that make the entrant less profitable in an attempt to limit the entrant’s scale of entry, or even possibly to induce a quick exit?

While much of the law and economics literatures on incumbents’ responses to new entry have focused on predatory pricing and other responses within the very market subject to entry, the strategy and international business literatures have recognized that in some cases the incumbent’s retaliation may come in a different market. For example, it has been argued that entry by firm A into a market dominated by firm B, might induce B to retaliate by entering into A’s original (“home” market) market. Should A anticipate this reaction, it might choose not to enter B’s market in the first place.

The challenge for economists seeking to model these strategies is in making such threats of retaliatory entry credible. If B can make profits in A’s market, A should expect B to enter whether or not A enters B’s market first. If B cannot make profits in A’s market, A should not expect entry whether or not it enters B’s market.

1 Important articles on predatory pricing and related strategies include Areeda and Turner [1975], Yamey [1972], McGee [1958] and Bolton et al. [2000], the last of which reviews the significant contributions made by a number of authors using more game-theoretic models incorporating imperfect information.
The purpose of this paper is to show how threats of retaliatory entry can indeed be credible and how they can thereby serve as a behavioral barrier to entry. To do this, we clearly need to connect these markets in some way so that the entry by A into B’s market changes B’s payoff from entry into A’s market. The link we employ derives from the way firms produce for multiple markets. Specifically, we assume that a firm’s production for all of its multiple markets takes place (at least in part) in a single facility subject to increasing marginal costs or limited capacity. In such a situation, a firm expanding its production to enter another market does two things: (i) it increases the marginal costs of serving its home market; and (ii) by taking sales away from its rival in the new market, it pushes that rival down its own marginal cost curve (gives it low marginal cost or excess capacity) changing the costs to that rival for expanding output to enter another market. These effects – weakening itself at home while strengthening its rival – combine to increase the probability that the rival will want to retaliate with its own entry into the first firm’s home market.

In our model, then, retaliatory entry can be made a credible threat with the result, under some conditions, that the firms do not enter into each other’s markets even when there are profit opportunities in those markets. This work builds on the important insights on multimarket competition developed in the influential paper by Bulow, Geanakoplos and Klemperer [1985] (hereafter “BGK”). BGK saw that play in one market could have implications for payoffs and subsequent play in other markets. Our purpose here is to embed a particular source of the market linkage (rising marginal costs) into a model to understand when retaliatory entry can be made a
2 BGK [1985, p 505] anticipated this possibility which they illustrated with a simple numerical example. In Chen and Ross [2005] we employed a model with rising marginal costs linking two markets to study the implications of multimarket contact. In that paper, we suggested that a similar model could be helpful in understanding retaliatory entry and our purpose here is to provide that formal analysis – as well as to extend it to study toe-hold entry.

credible threat.2

Once we recognize that retaliatory entry will be a credible and sufficient threat (under certain conditions) to deter the initial entry, it is natural to ask if those conditions can be created when they are not naturally occurring. That is, can firms take actions to make retaliatory entry credible when it might not otherwise have been? In our view, this may be one of purposes of toe-hold entry, that is entry on a small scale that may not even be profitable in its own right, but which may serve a greater (i.e. strategic) purpose. Here we model this role of toe-hold entry as a limited entry that serves to make retaliatory entry credible, either through pre-investment in the sunk fixed costs necessary to move into a new market or through small scale entry that reduces the subsequent marginal costs of expansion.

The following section briefly reviews the existing literature on retaliatory and toe-hold entry. Subsequent sections present and analyse the formal models. Section V offers our conclusions.

II. Past Work on Retaliatory Entry and Strategic Toe-Holds

As suggested above, in a number of strategic situations, firms may be reluctant to enter into new markets for fear that this entry will lead firms in those markets to enter into the first firm’s original market in retaliation. Concern for this kind of reaction has been described in the strategy literature by, for example, Porter [1980, pp. 84-85] who cites as examples battles
between Folgers and Maxwell House in regional coffee markets and Caterpillar and Deere in the earthmoving and farm equipment industries. Karnani and Wernerfelt [1985] use the term “multiple point competition” to describe situations in which firms compete against each other in multiple markets and consider the kinds of strategies available to a firm that has been “attacked” in one of its markets. In some cases, the term “reciprocal entry” is used to describe cases in which firms enter each other’s market. As Van Witteloostuijn [1993] explains, this could be the result of a simple prisoner’s dilemma game and not produce the effect we mean to describe here in which one firm deliberately holds off entry because of its fear of retaliation.

In addition, the international economics literature has, for some time, recognized that a move to penetrate a foreign market might lead firms in those markets to “reciprocate” and the term “reciprocal” entry is there used to describe cases in which firms penetrate each other’s domestic market. The credibility of such a retaliatory strategy needs to be addressed, however:

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3 Porter [1980, p. 84]: “When one firm initiates a move in one area and a competitor responds in a different area with one that affects the initiating firm, the situation can be called a cross-parry.”

4 Karnani and Wernerfelt [1985] also provide some additional examples, including the battle between Michelin and Goodyear in the North American and European tire markets around 1970 and that between BIC and Gillette in the disposable pen and razor markets.

5 Calem [1988] studies a related problem in which reciprocal entry is possible after two firms in different (e.g. geographic) markets have already determined how much in total each will produce. Reciprocal entry to sell some of their output is a possible equilibrium. Calem’s game involves simultaneous moves by both firms and so does not demonstrate that retaliatory entry can be a credible threat deterring entry by a first-mover. This does, however, highlight the connections between the current paper and the reciprocal dumping literature of Brander and Krugman [1983], among others.

6 The international business literature has noted this as well, see, e.g. Yip [1989]. In a well-known and related piece, Knickerbocker [1973] argued that firms tend to follow each other in making foreign direct investments in other countries, but his focus was on “third country” investments – that is investments that are not in any of the investing companies’ domestic
if retaliatory entry is profitable for firm B after firm A enters its market, why was entry into A’s market not profitable before A’s entry? One possible answer is that firm A’s move represented a defection from a repeated-game cooperative equilibrium, and B’s response is part of a reversion to a non-cooperative equilibrium.\(^7\)

This paper suggests there may be a simpler answer, however – one that does not rely on repeated play in a supergame – if the markets are linked by rising marginal costs of a shared facility. By entering firm B’s market and taking market share from it, firm A lowers firm B’s marginal costs and increases its own, making counter-entry into A’s market profitable when it may not have been so previously.\(^8\)

Formal models of toe-hold entry are hard to find and none that we are aware of have focussed on the issues that concern us here.\(^9\) The strategy literature has recognized that to make a threat of rapid retaliatory entry credible, a firm may have to establish limited “foothold” operations in the other’s market. While Karnani and Wernerfelt [1985] argue that, in such a

markets. Head, Mayer and Ries [2002] have provided a theoretical explanation for this behavior that relies on uncertainty and investors’ risk aversion. Leahy and Pavelin [2003] offer an explanation that is based on the desire of the firms to support collusion in the domestic market.

\(^7\) Haag [2002] models reciprocal entry as a punishment this way. In the international context see, for example, Graham [1998] and many of the reference cited therein.

\(^8\) Some work in the contestable markets literature, while primarily focussed on the conditions under which the prospects for hit-and-run entry can hold prices to competitive levels, has considered questions related those take up here. For example, Cairns and Mahabir [1988] and van Wegberg and van Witteloostuijn [1992] recognize that a move by a firm to enter a “foreign” market might open up its “home” market to entry by other firms, including possibly reciprocal entry by firms from the foreign market.

\(^9\) Malueg and Schwartz [1991] introduce a model of toehold entry with a different purpose. In their single-market model, the entrant secures a temporary toehold in order to influence the future investment decisions of the incumbent, in effect encouraging the incumbent to expand less rapidly, leaving more room in the market for the entrant in later periods.
case, retaliation would take the form of a rapid expansion of the foothold operation, they do not offer a formal model to demonstrate the credibility of this threat. On the empirical side, Gimeno [1999] studies the effects of multimarket contact in U.S. airline markets when firms hold smaller positions in their rivals’ important markets. He finds that holding even small shares of a rival’s important market can deter that rival from aggressive competitive behavior in your important market.

III. Retaliatory Entry

We formally model the idea of retaliatory entry as follows. Suppose initially each of market 1 and market 2 is served by a monopolist, denoted by firm A and firm B, respectively. Suppose further that each of the two monopolists is contemplating the possibility of invading the other market and/or the possibility of its market being invaded. To be more specific, the two firms play the following three-stage game. At stage 1, firm A decides whether to enter market 2. At stage 2, firm B, after observing firm A’s choice, decides whether to enter market 1. At stage 3 they engage in Cournot quantity competition in markets where they are both present. Let $\pi_j^m$ be firm j’s profits when each firm is a monopolist in its home market, $\pi_j^{dk}$ firm j’s profits when market k is served by both firms but the other market remains served by the monopolist (duopoly in market k only), and $\pi_j^{db}$ firm j’s profits when both markets are served by both firms (duopoly

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10 See also, van Wegberg and van Witteloostuijn [1992].

11 In fact, it is not necessary that competition be of the Cournot-type, though we will model competition in this way when we work with a more specific model. Our key results depend on the signs of differences in profit levels across different market structures and these may well be generated by different types of competition (e.g. Bertrand competition with differentiated products).
We show below that both assumptions are satisfied for the case where the two markets have the same linear demand function and firms have quadratic cost function. We use two variations of this model to illustrate the possibility of retaliatory entry. In the first variation, a firm has to incur a fixed cost of entry in order to enter the other market. In the second variation, the firm does not have to incur entry cost but the sizes of the two markets are different. We will show that, in each instance, fear over retaliatory entry can deter firm A from entering firm B’s market.

The proofs of all propositions can be found in the appendix.

III.1. Fixed Entry Cost

Suppose firm j (j = A, B) has to incur an entry cost $F_j$ if it enters the other market. To make the analysis interesting, we consider the situation where $\pi_A^{d_2} - F_A > \pi_A^m$. That is, in the absence of fear over retaliatory entry, firm A would enter market 2. We make two additional assumptions, which relate to the shapes of the demand and cost curves. First, we assume that it is more profitable for each firm to be a monopolist in one market that for it to be a duopolist in both markets: $\pi_j^m > \pi_j^{db}$ (j = A, B).

Second, we assume that firm B’s incremental profit from entering market 1 in the case where firm A does not enter market 2 is lower than in the case where firm A does enter market 2: $\pi_B^{d_1} - \pi_B^m < \pi_B^{db} - \pi_B^{d_2}$. This assumption is justified by the fact that firms face rising marginal costs of serving the two markets. If firms had constant marginal cost, there would be no linkage across the two markets and, consequently, a firm’s incremental profits from entering a new market would be zero.

\[\text{12} \] We show below that both assumptions are satisfied for the case where the two markets have the same linear demand function and firms have quadratic cost function.
The market would be independent of the market structure in its own market. That is, we would have $\pi_B^{d1} - \pi_B^m = \pi_B^{db} - \pi_B^{d2}$. With rising marginal costs, however, a firm that faces competition in its own market will produce less output and therefore have lower marginal costs as it enters its new market. This makes it a stronger entrant. At the same time, when its rival is already operating in both markets the rival will have a larger rates of output and correspondingly higher marginal costs weakening its position when the entrant arrives.

With these assumptions made, we can establish how the nature of the equilibrium will depend on the fixed cost to B of entering market 1, $F_B$:

**Proposition 1.** The equilibrium entry decisions by firms A and B depend on the value of $F_B$, as follows:

(i) If $F_B > \pi_B^{db} - \pi_B^{d2}$, firm A sells in both market 1 and market 2 while firm B remains in market 2 only.

(ii) If $\pi_B^{d1} - \pi_B^m < F_B < \pi_B^{db} - \pi_B^{d2}$, firm A and firm B stay out of each other’s markets.

(iii) If $F_B < \pi_B^{d1} - \pi_B^m$, firm A and firm B enter each other’s markets.

Thus, we see that if the conditions in (ii) hold, firm A will decide not to enter into firm B’s market, not because it is not profitable on its own, but because it will induce firm B to (rationally) enter into A’s market with the net result that firm A’s total profits are lower.

Consider the special case where the demand and marginal cost curves are linear. To be specific, suppose the demand function in market i ($i = 1, 2$) is $P_i = a - b(x_{Ai} + x_{Bi})$ and the cost
function of firm j (j = A, B) is \( C_j = c(x_{j1} + x_{j2}) + (d/2)(x_{j1} + x_{j2})^2 \). The marginal cost function of firm j is then \( MC_j = c + d(x_{j1} + x_{j2}) \). It is straightforward to derive firm B’s profits under different market structures:

\[
\pi_B^m = \frac{(a-c)^2}{2(2b+d)}; \quad \pi_B^{d1} = \frac{[2(b+d)(b+2d)(13b+8d)-d(5b+4d)^2]((a-c)^2}{2(2b+d)^2(3b+4d)^2} \\
\pi_B^{d2} = \frac{2(b+2d)^2(a-c)^2}{(2b+d)(3b+4d)^2}; \quad \pi_B^{db} = \frac{2(b+d)(a-c)^2}{(3b+2d)^2}.
\]

(1)

(2)

It can be verified that \( \pi_B^{d1} - \pi_B^m = \pi_B^{db} - \pi_B^{d2} \) if \( d = 0 \), and \( \pi_B^{d1} - \pi_B^m < \pi_B^{db} - \pi_B^{d2} \) if \( d > 0 \).

Therefore, rising marginal cost is a necessary condition for the possibility of entry deterrence via retaliatory entry.

The range of \( F_B \) over which entry deterrence occurs in equilibrium will depend on the magnitudes of the slopes of the demand (b) and marginal cost curves (d). To see this, define \( E \) as the length of the range of \( F_B \) over which entry deterrence will occur, i.e. \( E = [\pi_B^{db} - \pi_B^{d2}] - [\pi_B^{d1} - \pi_B^m] \). With the aid of a computer program (Maple), we conduct comparative statics to see how changes in b and d affect the size of E. The results are presented in the first three columns of Table 1. As the marginal cost curve becomes steeper (i.e. a larger d), the range E expands if \( d/b < 0.556 \) but it contracts if \( d/b > 0.557 \). On the other hand, as the demand curve becomes steeper (i.e. a larger b), the range E contracts if \( d/b < 2.545 \) but expands if \( d/b > 2.546 \). Thus, neither parameter has a monotonic affect on the range over which entry deterrence will be part of the
equilibrium. The reason for the non-monotonicity lies in the fact that while $d$, the slope of the marginal cost curve, determines to a considerable extent how “linked” are the two markets, $d$ and $b$ also determine (together with parameter “$a$”) the size of the market; so increasing either of them effectively reduces the profit potential in all scenarios. Thus, for example, as $d$ grows from 0 to some small positive amount, $E$ will increase above 0 as there is some connection between the markets. However, as $d$ continues to grow, profits in all competitive scenarios will fall towards 0 so that eventually $E$ must fall as well.

III.2. Different Market Sizes

A second variant of the above model can be used to show that fear over retaliatory entry can prevent entry even in the absence of entry costs. Suppose, instead of entry costs, the sizes of markets A and B are different. To be more specific, suppose the inverse demand functions are $P_i = S_i P(X_i)$ for $i = 1$ and 2. On the production side, firm j’s ($j = A, B$) total cost function is $C_j = C(X_j)$ where $X_j$ is the total output of firm j. We assume that $C’ > 0$ and $C” > 0$. Therefore, each firm faces a rising marginal cost curve.

We continue to use superscripts $m$, $dk$ and $db$ to denote equilibria where there is a monopoly in each market, there is duopoly in market k but monopoly in the other market, and duopoly in both markets, respectively. Starting from the initial situation where each firm is a monopolist in its own market, $S_2 P(x_{b2}^m)$ represents firm A’s marginal revenue from selling the first unit in market 2 and $C’(x_{A1}^m)$ is the marginal cost of selling this unit. Since $S_1 P(x_{A1}^m) + x_{A1}^m S_1 P’(x_{A1}^m) = C’(x_{A1}^m)$, we assume

$$S_1 P(x_{A1}^m) + x_{A1}^m S_1 P’(x_{A1}^m) < S_2 P(x_{b2}^m) \quad (3)$$
to ensure that firm A would enter market 2 in the absence of fear over retaliatory entry.\(^\text{13}\) As in
the first variant, we also assume that \(\pi_A^m > \pi_A^{db}\), that is, firm A’s profits are higher being a
monopolist in a single market than being a duopolist in both markets. If this latter assumption
were not true, firm A would not be deterred by the concern over retaliatory entry and would thus
want to enter market 2 independent of firm B’s reaction.

To simplify the presentation we define:\(^\text{14}\)

\[
L = \frac{P(x_{A2} + x_{B2}) + x_{B2}^d P/(x_{A2} + x_{B2})}{P(x_{A1})}; \quad M = \frac{P(x_{B2}^m + x_{B2}^d P/(x_{B2}^m)}{P(x_{A1})}; \quad Q = \frac{P(x_{B2}^m)}{P(x_{A1}^m + x_{A1}^d P/(x_{A1}^m)}.
\]

Inequality (3) then implies that \(S_1/S_2 < Q\). It is easy to see that \(M < Q\) because \(P' < 0\) implies that
the numerator of \(M\) is less than that of \(Q\) and the denominator of \(M\) is greater than that of \(Q\). It is
also true that \(L < M\). To see this, consider firm B’s profit-maximization condition in “case m”
(monopoly in each market) and “case d2” (duopoly in market 2 only):

\[
S_2 P(x_{B2}^m) + x_{B2}^d S_2 P/(x_{B2}^m) = C'(x_{B2}^m).
\]

\(^{13}\) While this assumption does put some conditions on the relative sizes of the two
markets, it is still possible that either market is larger than the other.

\(^{14}\) To get a sense of what these terms are, recognize that if the markets were the same
size each of \(L, M\) and \(Q\) would represent the ratio of the marginal revenue a firm would get
selling one more unit in its own market to the revenue it would get selling that unit in the other
market. For example, in \(L\) the numerator is marginal revenue firm B would realize from selling
one more unit in a duopolized market 2 and the denominator what it would earn if it chose to sell
one unit in the previously monopolized market 1 (which is actually just the market price in
market 1).
Since $x_{A2} > 0$, we have $x_{B2} < x_{B2}^m$ (i.e. firm B’s output in market 2 is smaller as a duopolist than as a monopolist). The assumption of rising marginal cost then implies that $C'(x_{B2}^d) < C'(x_{B2}^m)$. From (5) and (6) we conclude

$$P(x_{A2}^d + x_{B2}^d) + x_{B2}^d P'(x_{A2}^d + x_{B2}^d) < P(x_{B2}^m) + x_{B2}^m P'(x_{B2}^m).$$

The assumption of rising marginal cost also implies $x_{A1}^d < x_{A1}^m$, which entails $P(x_{A1}^d) > P(x_{A1}^m)$. In other words, the numerator of L is less than that of M, and the denominator of L is greater than that of M. Hence, $L < M$.

It is clear from the above discussion that the assumption of rising marginal cost plays a key role in ensuring $L < M$. If firms had constant marginal cost, we would have

$$P(x_{A2}^d + x_{B2}^d) + x_{B2}^d P'(x_{A2}^d + x_{B2}^d) = P(x_{B2}^m) + x_{B2}^m P'(x_{B2}^m)$$

and $x_{A1}^d = x_{A1}^m$, and thus $L = M$.

**Proposition 2.** The equilibrium entry decisions by firms A and B depend on the value of $S_1/S_2$ as follows:

(i) If $S_1/S_2 < L$, firm A sells in both market 1 and 2 while firm B remains in market 2 only.

(ii) If $L < S_1/S_2 < M$, firm A and firm B stay out of each other’s market. A
necessary condition for this situation to occur is that $S_1 < S_2$.

(iii) If $S_1/S_2 > M$ (and $S_1/S_2 < Q$), firm A and firm B enter each other’s market.

Propositions 1 and 2 illustrate two factors, fixed entry costs and the relative market sizes, that help determine whether there exists an equilibrium in which fear over retaliatory entry stops firm A from entering market 2. One factor relates to the cost of entry, the other to the benefits. In the first case, that of differing entry costs, these costs have to be low enough for firm B to make the threat of retaliatory entry credible. But they cannot be so low that would allow firm B to initiate an unprovoked invasion of market 1. Similarly, in the case of market sizes, market 1 must be sufficiently large to make retaliatory entry profitable for firm B. But it cannot be so large that firm B will want to enter market 1 independent of what firm A does.

IV. Toe-Hold Entry

The theoretical framework here is an extension of the one that we have used to study retaliatory entry in section III. Suppose, again, that initially firm A and firm B are monopolists in market 1 and market 2 respectively. We consider the same situation where firm A is interested in invading market 2. Our focus here is on the question whether firm B can use toe-hold entry to forestall the invasion by firm A. With this in mind, we add an additional stage, called stage 0, to the three-stage game in section III.1. At stage 0, firm B decides whether to invest in toe-hold entry into market 1. The rest of the game is the same as that in section III.1. That is, at stage 1 firm A decides whether to enter market 2; at stage 2 firm B decides whether to enter market 1 on a full scale; and at stage 3 the two firms sell their output.
As in section III, we will consider two variants of this model, the differences turning on how toe-hold entry affects the costs of the full-scale entry that might follow. In the first variation, we assume that toe-hold entry reduces the fixed cost of full-scale entry. The analysis of this simple model is useful for the illustration of the incentives for toe-hold entry. The second variation assumes that toe-hold entry reduces the variable cost of serving the new market.

**IV.1. Toe-Hold Entry Reduces Fixed Cost**

Suppose firm $j$ ($j = A, B$) has to incur a fixed sunk cost $F_j$ if it enters the other market. In this case, toe-hold entry by firm $B$ means that firm $B$ pays at stage 0 a portion of the fixed entry cost, $T$ where $T < F_B$. Here toe-hold means that firm $B$ establishes a commercial presence but does not (yet) sell any units in the market. Firm $B$ will sell a positive quantity only if it chooses full-scale entry.

We make the same assumptions on the profits of firms as in section III.1. That is, we assume that: (i) $\pi_{A}^{d2} - F_A > \pi_{A}^{m}$ (i.e. absent concerns over retaliation, firm $A$ would find entry into market 2 to be profitable); (ii) $\pi_{j}^{m} > \pi_{j}^{db}$ ($j = A, B$) (i.e. both firms would prefer to be monopolists in one market alone to being duopolists in both markets); and (iii) $\pi_{B}^{d1} - \pi_{B}^{m} < \pi_{B}^{db} - \pi_{B}^{d2}$ (i.e. there is a range of fixed costs of $B$’s entry into market 1 such that $A$ would not enter market 2 only because it would trigger retaliatory entry by $B$ into market 1). We note again that rising marginal cost is a necessary condition for the last inequality to hold.

Note that if firm $B$ does not invest in a toe-hold at stage 0, the subgame from stage 1 onward is identical to the game studied in section III.1. From Proposition 1, we know that the two firms would stay out of each other’s market if $\pi_{B}^{d1} - \pi_{B}^{m} < F_B < \pi_{B}^{db} - \pi_{B}^{d2}$, and they would
enter each other’s market if \( F_B < \pi_B^{d1} - \pi_B^m \). In both cases, giving firm B the additional option of investing in toe-hold entry will not change these equilibriums in any way. In the former case, firm B does not need a toe-hold to make the threat of retaliation credible. In the latter case, firm B will want to enter market 1 even without a toe-hold; establishing toe-hold entry at stage 0 would simply make firm B’s incentives to enter market 1 at stage 2 stronger.

Where a toe-hold entry can make a difference to the equilibrium is in the case \( F_B > \pi_B^{db} - \pi_B^{d2} \). Part (i) of Proposition 1 implies that without a toe-hold entry, firm B is not able to prevent firm A from enter market 2. But as the following proposition shows, the option of investing in toe-hold entry may give firm B an additional instrument to deter firm A’s entry.

**Proposition 3.** If \( F_B > \pi_B^{db} - \pi_B^{d2} \) but \( F_B < (\pi_B^{db} - \pi_B^{d2}) + (\pi_B^m - \pi_B^{d2}) \), the subgame perfect equilibrium is one where firm B establishes a toe-hold entry by paying \( T = F_B - (\pi_B^{db} - \pi_B^{d2}) \) and firms A and B remain monopolists their respective markets.

Under the conditions outlined in Proposition 3, firm B is able to deter entry by firm A into market 2 by pre-paying a portion of its own fixed entry cost at stage 0. Here the toe-hold entry makes the threat of future retaliatory entry by firm B into firm A’s market credible. In the absence of such toe-hold entry, such a threat would not have been credible given the large entry cost \( F_B \).

Note from Proposition 3 that the length of the interval of \( F_B \) over which toe-hold entry occurs is \( (\pi_B^m - \pi_B^{d2}) \), denoted by \( H \). We can say more about the property of \( H \) for the special
case of linear demand and quadratic cost functions, It can be shown that in this case,

\[ H = \pi_B^m - \pi_B^{d^2} = \frac{b(5b+8d)(a-c)^2}{2(2b+d)(3b+4d)^2}. \]  

(9)

Comparative statics analysis on (9) is conducted with respect to \( d \) and \( b \), the slopes of marginal cost and demand curves. As summarized in the last two columns of Table 1, the value of \( H \) decreases as \( b \) or \( d \) increases. In other words, the interval over which the toe-hold entry equilibrium occurs shrinks as the marginal cost or the demand curve becomes steeper.\(^{15}\)

**IV.2. Toe-Hold Entry Reduces Marginal Cost**

In this second variant, we suppose that, instead of a fixed entry cost, firm B has to incur a per unit retail cost when it sells in a new market, denoted by \( R_B \). Toe-hold entry takes the form of pre-shipment of output by firm B to market 1, and pre-shipment reduces the per unit retail cost. We think of these pre-shipped units as a simple way to model the opening up of distribution channels in the other market.\(^{16}\) Pre-shipped units are sold at stage 3 along with any additional units that firm B wants to sell. Let \( q_B \) denote the number of units that firm B pre-ships to market 1 at stage 0. We assume that \( R_B \) is related to \( q_B \) in the following way:

\[ R_B = f(q_B) \]

\(^{15}\) Essentially, \( H \) gives the region over which firm B is willing to pay the costs of toe-hold entry, assuming that it will work. A positive range for \( E \) (i.e. \( \pi_B^{d_1} - \pi_B^m < \pi_B^{d_2} - \pi_B^{d_2} \)) is needed to guarantee that entry by A into B’s market would increase B’s interest in entering market 1 (thereby giving reason for A to fear retaliatory entry.)

\(^{16}\) Another way to model the toe-hold might be to add more periods to the model so that what we call “pre-shipped” units are instead treated as actual sales in an earlier period which contribute to lower costs (via learning) in subsequent periods. We would not expect the results from such a model to be qualitatively different from those we derive with this simpler structure.
where $\alpha > r > 0$ and $\beta > 0$. Thus, with this toe-hold technology firm B can bring its retail cost in market 1 down to as low as $r$. Since we are not interested in the possibility of toe-hold entry by firm A, we assume that firm A’s retail cost in market 2 is constant at $r$ in order to simplify the analysis. The per unit retail cost of each firm in its own market is assumed to be $r$ as well.

As in section III.2., the inverse demand functions are of the form $P_i = S_i P(X_i)$ for $i = 1$ and 2, and firms’ cost functions remain $C_j = C(X_j)$ with $C' > 0$ and $C'' > 0$. We continue to assume that $S_1/S_2 < Q$ to ensure that firm A would find it profitable to enter market 2 in the absence of concern about retaliation by firm B.

We maintain the assumption that the profits from being a monopolist in a single market are higher than being a duopolist in two markets. To state this assumption more precisely, however, requires some additional notation. This is because a firm’s profits when there is a duopoly in both markets depends on firm B’s retail cost in market 1, $R_B$, which, in turns, depends on its size of pre-shipment $q_B$. As a result, we write firms’ quantities and profits as functions of $R_B$: $x_{ji}^{dh}(R_B)$ and $\pi_j^{dh}(R_B)$. We assume that $\pi_A^{m} > \pi_A^{dh}(r)$, that is, for firm A being a monopolist in a single market is better than being a duopolist in two markets when its competitor’s retail cost is $r$.

To simplify presentation, we focus our analysis on a situation where the size of firm B’s toe-hold needed to bring its retail cost in market 1 down to $r$ is less than the corresponding Cournot duopoly output, $x_{B1}^{dh}(r)$. Using (10) we can write this assumption as $\frac{(\alpha - r)}{\beta} < x_{B1}^{dh}(r)$. 

\[
R_B = R(q_B) = \begin{cases} \alpha - \beta q_B & \text{for } q_B \in [0, (\alpha - r)/\beta] \\ r & \text{for } q_B > (\alpha - r)/\beta \end{cases}
\]
In what follows we will discuss the equilibrium in this game only informally. Interested readers are invited to read the appendix where the subgame perfect equilibrium is analysed in detail.

At the outset, it is useful to recall that the game from stage 1 onward is qualitatively the same as the game studied in section III.2. Therefore, the analysis in section III.2 can be easily adapted for the current model. As in III.2 (see in particular Proposition 2), the equilibrium depends on the value of \( S_1/S_2 \) relative to \( L \) and \( M \). If \( S_1/S_2 < L \), firm B would not find it profitable to enter market 1 in response to firm A’s invasion of market 2, even if firm B has the lowest possible retail cost \( (r) \) in market 1. As a result, in equilibrium firm A sells in both markets while firm B remains in market 2 only. In the case \( S_1/S_2 > M \) (and \( S_1/S_2 < Q \)), on the other hand, firm B will, irrespective of firm A’s entry decision, find it profitable to enter market 2 if its retail cost in this market is \( r \). Firm B, therefore, will invest in a toe-hold at stage 0 to bring its cost down to \( r \). Knowing that firm B will enter its market at stage 2, firm A will choose to enter market 1 at stage 1. Thus, the equilibrium in this case is such that firm A and firm B sell in both markets.

The case of particular interest to us is the one where \( S_1/S_2 \) is between \( L \) and \( M \). In this case, firm B will not want to enter market 1 if its own market is not invaded by firm A. If an invasion by firm A occurs, on the other hand, firm B will find it profitable to enter market 1 if its retail cost in market 2 is low enough (for example \( R_B = r \)). As in section III.2, the threat of retaliatory entry can deter firm A from entering market 2. What is new here, is that firm B may have to invest in a toe-hold in order to achieve the entry-deterrence effect.

To find out the conditions under which a toe-hold investment is made to deter entry in equilibrium, we define a critical value of retail cost, \( R^* \), such that, if firm B achieved \( R^* \) firm A
would find it equally profitable to compete in duopolies with B in both markets as to be only a monopolist in its own market, i.e., \( \pi^d_A(R^*) = \pi^m_A \). It is shown in Appendix that \( R^* > r \), and that \( \pi^d_A(R_B) < \pi^m_A \) if \( R_B < R^* \).

If the toe-hold technology described in equation (10) is such that \( \alpha < R^* \), firm B will retaliate against firm A’s entry, even in the absence of any prior toe-hold investment. Consequently, firm A stays out of firm B’s market for fear of retaliatory entry. This, in essence, is the retaliatory entry equilibrium described in part (ii) of Proposition 2.

If, on the other hand, \( \alpha > R^* \), investment in a toe-holder is needed if firm B wants to deter entry. The question, then, is whether firm B has the incentives to invest in a toe-hold at stage 0. As shown in appendix, firm B will indeed invest in a toe-hold to bring its retail cost down if the toe-hold technology is such that the value of \( \beta \) exceeds some critical value \( \beta^* \). The minimum toe-hold required to block A’s entry (\( q_{B^*} \)) will be given by the condition \( R^* = \alpha - \beta q_{B^*} \), where \( R^* \) (as defined above) does not depend on \( \alpha \) or \( \beta \). Thus, it is easy to see that the size of the toe-hold needed to deter entry will be positively related to \( \alpha \) (the marginal cost with no pre-shipments) and negatively related to \( \beta \) (the rate at which pre-shipments reduce marginal costs).

It is useful to recap the conditions under which firm B is able to deter firm A’s entry by investing in toe-holder entry in market 1. They are: (i) market 1 is smaller than market 2 so that, in the absence of strategic considerations, firm A would want to enter market 2 and firm B would not want to enter market 1 (as implied by \( S_1/S_2 < M < Q \)), but market 1 cannot be too small

17 We have already assumed that, when both firms have the same costs, they would both prefer monopoly in one market to duopoly in both markets. Here, however, we are saying that firm B may have higher costs than A, making it a weaker competitor. In this case, A may prefer to compete against a weaker competitor in both markets than to be restricted to be a monopolist in only one market.
(S_1/S_2 > L) to make retaliatory entry (with the assistance of toe-hold investment) by firm B unprofitable; and (ii) firm B’s retail cost in market 1 is high without pre-shipment (α > R*), but decreases rapidly with pre-shipment (β > β*).

Note that, in this equilibrium and the equilibrium in the case S_1/S_2 > M, firm B invests in toe-hold pre-shipment. The reasons for doing so, however, are different. In the former case, investment in pre-shipment is to deter entry. But in the latter case, the prospect of retaliation by firm B does not deter firm A’s entry. The purpose of firm B’s investment in pre-shipment, then, is not entry deterrence. Rather it is to strengthen its competitive position in the inevitable duopoly competition following firm A’s entry.

IV.3 Retaliatory and Toe-Hold Entry: A Comparison

The above analysis illustrates the relationship between retaliatory entry and toe-hold entry. The entry deterrence effect of a toe-hold strategy depends on the threat of retaliatory entry. The toe-hold strategy can be valuable when it makes such a threat credible, deterring entry in cases in which entry would otherwise have occurred.

For the case in which toe-hold entry reduces only the fixed cost of entry, the discussion above demonstrates that the magnitude of firm B’s fixed cost of entry into market 1 (F_B) plays a large role in determining the nature of the equilibrium. The various possibilities are illustrated in Figure 1. Recall that we assumed throughout that F_A is low enough that, absent any concerns over retaliation, firm A would enter market 2. If B’s fixed cost of entering market 1 is very low (i.e. less than π_{B}^{d1} - π_{B}^{m}), firm B will always want to enter market 1, therefore A has no reason not to enter market 2 and both firms will sell in both markets. This is region I in Figure 1. In
region II, $F_B$ is large enough that B will not enter unless A has entered first, with the result that firm A chooses not to enter. Thus, in this range, retaliatory entry is a credible and effective threat on its own. If $F_B$ is in the range between $[\pi_B^{db} - \pi_B^{d2}]$ and $[(\pi_B^{db} - \pi_B^{d2}) + (\pi_B^{m} - \pi_B^{d2})]$, labelled region III, the fixed costs of entry are too large to make retaliatory entry credible with no toe-hold. However, they are small enough that B can make the threat credible by pre-spending enough of those fixed costs establishing a toe-hold. Of course, this additional strategic advantage of toe-hold entry comes at a cost. In our models, the toe-hold investment, whether in the form of a portion of fixed entry cost or a pre-shipment, does not generate additional profits for firm B. The investment is profitable only because it deters entry. Finally, for even larger levels of $F_B$, region IV, the fixed costs of B’s entry are so large that it cannot profitably make retaliation credible, therefore it does not establish a toe-hold. In this region, firm A enters market 2 but B does not enter market 1.

V. Conclusions

The models presented above formalize two ideas that have been circulating in the strategy literature for some time. First, when its own market is threatened by entry, an incumbent may choose to retaliate by entering another market in which its new rival already operates. The model of retaliatory entry in section III showed how such threats of retaliation could in fact be credible, with the implication that the first entry may be deterred. The key element in the model that created the credibility was the incorporation of a rising marginal cost curve from some part of the production process that was necessary to serve all markets. Rising marginal costs imply that, by entering a new market, a firm weakens itself (by moving up its marginal cost curve) and
strengthens its rival (by stealing market share and pushing its rival down its own marginal cost curve).

Second, when the costs of entering a new market are too great to make retaliatory entry a credible threat and thereby deter the first entry, firms may have incentives to establish toe-hold positions in other markets. These toe-holds, though costly themselves, may reduce the costs of expansion enough to restore the credibility of the retaliatory entry threat. Thus, we have illustrated that toe-holds may serve a strategic purpose even if they never grow and are not, by themselves, profitable operations.
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Appendix

**Proof of Proposition 1.** In the event that firm A stays out of market 2, firm B will enter market 1 if and only if $F_B < \pi_B - \pi_B^m$. Meanwhile, in the event that firm A enters market 2, firm B will enter market 1 if and only if $F_B < \pi_B^h - \pi_B^d$. In case (i), entry by firm A into market 2 will not trigger entry by firm B and thus firm A enters. In case (ii), entry by firm A will trigger entry by firm B, and anticipating this firm A stays out of market 2. In case (iii) firm A enters market 2 because firm B will enter market 1 no matter what it does.

QED

**Proof of Proposition 2.** Note that in the event of firm A’s entry into market 2 it is profitable for firm B to enter market 1 if and only if $S_1P(x_{A1}^d) > C'(x_{B2}^d)$. Using firm B’s profit-maximization condition, $S_2[P(x_{A2}^d + x_{B2}^d) + x_{B2}^dP'(x_{A2}^d + x_{B2}^d)] = C'(x_{B2}^d)$, we can rewrite this condition as

$$S_1P(x_{A1}^d) > S_2[P(x_{A2}^d + x_{B2}^d) + x_{B2}^dP'(x_{A2}^d + x_{B2}^d)]$$

i.e., $S_1/S_2 > L$. Furthermore, firm B will stay out of market 1 in the absence of firm A’s entry into market 2 if and only if $S_1P(x_{A1}^m) < C'(x_{B2}^m)$. Using firm B’s profit-maximization condition,

$$S_2[P(x_{B2}^m + x_{B2}^mP'(x_{B2}^m)] = C'(x_{B2}^m),$$

we can rewrite this condition as

$$S_1P(x_{A1}^m) < S_2[P(x_{B2}^m + x_{B2}^mP'(x_{B2}^m)],$$

(A1)

i.e., $S_1/S_2 < M$.

Therefore, if $L < S_1/S_2 < M$, firm B will reciprocate firm A’s entry into market 2 by
entering market 1. In equilibrium firm A stays out of market 1 and each firm remains a monopolist in its own market (part ii of the Proposition). The above analysis also implies that if $S_1/S_2 < L$ firm A will enter market 2 but firm B will stay out of market 1 (part i of the Proposition). If $S_1/S_2 > M$, on the other hand, firm B will enter market 1 even in the absence of firm A’s invasion of market 2. Invasion by firm A pushes firm B down its marginal cost curve while driving firm A itself up its marginal cost curve, making firm B’s incentive to enter market 1 even stronger. Given that firm B will enter market 1 at stage 2 in any case, firm A has nothing to lose by entering market 2 at stage 1. Thus, in equilibrium both firms enter each other’s market.

Finally, we show that $S_1 < S_2$ is a necessary condition for the situation in part ii to arise. Note that at $S_1 = S_2$ (A1) does not hold because $x_{A1}^m = x_{B2}^m$ and $P' < 0$. Straightforward comparative statics on a monopolist’s profit-maximization condition shows that $S_1P(x_{A1}^m)$ increases with $S_1$. Then, starting from $S_1 = S_2$, an increase in $S_1$ will raise the left-hand side of (A1). As a result, (A1) does not hold for $S_1 \geq S_2$. Therefore, to satisfy (A1) it is necessary to have $S_1 < S_2$.

QED

**Proof of Proposition 3.** We use backward induction to derive the equilibrium. Consider first stage 2. In the event that firm A enters market 2, firm B will enter market 1 if and only if $F_B - T \leq \pi_B^{db} - \pi_B^{d2}$ or $T \geq F_B - (\pi_B^{db} - \pi_B^{d2})$. Then at stage 1, firm A will enter market 2 if $T < F_B - (\pi_B^{db} - \pi_B^{d2})$, but it will not enter if $T \geq F_B - (\pi_B^{db} - \pi_B^{d2})$. Finally, consider firm B’s decision at stage 0. If it sets $T < F_B - (\pi_B^{db} - \pi_B^{d2})$, firm A will enter market 2 at stage 1 but firm B will not enter market 1 at stage 2. Firm B’s profit in this case will be $\pi_B^{d2}$. On the other hand, if firm B invests
T ≥ F_B - (π_B^{db} - π_B^{d2}) at stage 0, firm A will be deterred from entering market 2. In this case, firm B’s payoff is π_B^m - T. Since this payoff is decreasing in T, firm B chooses T = F_B - (π_B^{db} - π_B^{d2}). Given the condition that F_B < (π_B^{db} - π_B^{d2}) + (π_B^m - π_B^{d2}), the resulting payoff is greater than π_B^{d2}. Thus, firm B chooses toe-hold entry of the size T = F_B - (π_B^{db} - π_B^{d2}) at stage 0.

QED

Toe-Hold Entry Reduces Marginal Cost: Technical Details

Here we demonstrate formally the various claims made in section IV.2. First, we prove that S_1/S_2 < Q implies that firm A would enter market 2 in the absence of concern over retaliation by firm B. Such a simple minded firm A will want to enter market 2 if the marginal revenue from selling the first unit in this market is greater than its marginal cost, that is, if S_2 P(x_{B2}^m) > C'(x_{A1}^m) + r. Using firm A’s profit-maximization condition,

\[ S_1 [P(x_{A1}^m) + P'(x_{A1}^m)x_{A1}^m] = C'(x_{A1}^m) + r, \]

we can rewrite this condition as S_1/S_2 < Q.

Second, we prove that if S_1/S_2 < L, firm B would not find it profitable to enter market 1 in response to firm A’s invasion of market 2, even if firm B’s retail cost in market 1 is r. Firm B does not want to enter market 1 in such a situation if S_1 P(x_{A1}^{d2}) < C'(x_{B2}^{d2}) + r. Rewriting this condition using firm B’s profit-maximization condition,

\[ S_2 [P(x_{A2}^{d2} x_{B2}^{d2}) + P'(x_{A2}^{d2} x_{B2}^{d2} x_{B2}^{d2}) x_{B2}^{d2}] = C'(x_{B2}^{d2}) + r, \]

we obtain S_1/S_2 < L.

Third, we prove that in the case S_1/S_2 > M, firm B will, irrespective of firm A’s entry decision, find it profitable to enter market 1 if its retail cost in this market is r. Using the same reasoning as above, we can show that in the absence of firm A’s entry into market 2, firm B
wants to enter market 1 if $S_1/S_2 > M$. Entry by firm A into market 2 will push firm B down its marginal cost curve and drive firm A itself up its marginal cost curve. This will strengthen firm B’s incentive to enter market 1. Therefore, firm B enters market 1 independent of whether firm A enters market 2.

Forth, we define $R^*$ and prove that $R^* > r$. To do so, note that in the scenario where there is duopoly in both markets, firms’ outputs and profits are functions of $R_B$. In particular, if $R_B$ is such that $S_1P(x_{A1}^{d_2}) = C'(x_{B2}^{d_2}) + R_B$, firm B is just indifferent between selling and not selling the first unit in market 1. In other words, $x_B^1 = 0$ at $R_B = S_1P(x_{A1}^{d_2}) - C'(x_{B2}^{d_2})$. This implies that $\pi_A^{db}(S_1P(x_{A1}^{d_2}) - C'(x_{B2}^{d_2})) = \pi_A^{d_2}$, which is greater than $\pi_A^{m}$. Since $\pi_A^{db}(r) < \pi_A^{m}$, by continuity there exists $R^*$ between $r$ and $S_1P(x_{A1}^{d_2}) - C'(x_{B2}^{d_2})$ such that $\pi_A^{db}(R^*) = \pi_A^{m}$.

Furthermore, $\pi_A^{db}(R_B) > \pi_A^{m}$ if $R_B > R^*$.

Finally, we derive the subgame perfect equilibrium for the case $L < S_1/S_2 < M$. We do so by proving a series of claims that deal with the equilibriums at various stages of the game. Note that the last inequality implies that $S_1 < S_2$, as shown in the proof of Proposition 2.

Claim 1. Suppose $L < S_1/S_2 < M$. Consider the subgame following the move that firm B does not invest in toe-hold pre-shipment at stage 0 (i.e. $q_B = 0$). The equilibrium in this subgame is such that firm A will enter market 2 if $R_B > R^*$, but it will not enter market 2 if $R_B \leq R^*$.

Proof. First, we consider firm B’s strategies at stage 2 in response to firm A’s entry at stage 1. If $R_B > S_1P(x_{A1}^{d_2}) - C'(x_{B2}^{d_2})$, firm B will not enter market 1 at stage 2. The converse is true if $R_B \leq S_1P(x_{A1}^{d_2}) - C'(x_{B2}^{d_2})$.

At stage 1, firm A takes into consideration the above strategies of firm B when deciding
whether to enter market 2. Recall that \( R^* < S_1 P(x_{A1}^{d2}) - C'(x_{B2}^{d2}) \), and that \( \pi_{A}^{db}(R_B) > \pi_{A}^{m} \) for \( R_B > R^* \). If \( R_B > S_1 P(x_{A1}^{d2}) - C'(x_{B2}^{d2}) \), firm A chooses to enter market 2 because firm B will not enter market 1 following firm A’s entry. If \( R_B \in (R^*, S_1 P(x_{A1}^{d2}) - C'(x_{B2}^{d2})) \), firm A’s entry into market 2 will trigger entry by firm B into market 1. But in this case \( \pi_{A}^{db}(R_B) > \pi_{A}^{m} \); firm A’s profits from playing a duopoly game against such a high cost (i.e., large \( R_B \)) firm B is larger than its monopoly profits in a single market. Thus, firm A still chooses to enter market 1. Finally, if \( R_B \leq R^* \), firm A’s entry into market 2 will trigger entry by firm B and this retaliatory entry will hurt firm A’s profits. In this case, firm A chooses to stay out of market 2.

QED

Claim 2. Suppose \( L < S/S_2 < M \) and \( \alpha < R^* \). In the subgame perfect equilibrium firm B does not invest in toe-hold entry and the two firms stay out of each other’s market.

Proof. By (10) \( \alpha < R^* \) implies that \( R_B < R^* \). Claim 1 suggests that firm A chooses to stay out of market 2 even if firm B does not investment in toe-hold entry. Given that firm A does not enter market 2 at stage 1, there is no need for firm B to invest in pre-shipment at stage 0.

QED

Claim 3. Suppose \( L < S/S_2 < M \) and \( \alpha > R^* \). There exists \( \beta^* > 0 \) such that if \( \beta > \beta^* \), in the subgame perfect equilibrium firm B invests in toe-hold pre-shipment \( q_{B}^{†} \) at stage 0, firm A stays out of market 2 at stage 1, and firm B sells only the pre-shipped units in market 1 at stage 2. The size of toe-hold pre-shipment \( q_{B}^{†} \) lies in the range \( [(\alpha - R^*)/\beta, (\alpha - r)/\beta] \).

Proof. Let \( te \) denote the type of scenario described in Claim 3. The profits of the two firms in
this scenario are:

\[ \pi_A^{le}(q_B) = S_1 P(q_B + x_{A1}^{le}) x_{A1}^{le} - C(x_{A1}^{le}) - r x_{A1}^{le}. \]  \hspace{1cm} (A2) 

\[ \pi_B^{le}(q_B) = S_1 P(q_B + x_{A1}^{le}) q_B + S_2 P(x_{B2}^{le}) x_{B2}^{le} - C(x_{B2}^{le} + q_B) - r x_{B2}^{le} - (\alpha - \beta q_B) q_B. \]  \hspace{1cm} (A3) 

Note that at \( q_B = 0 \), \( \pi_A^{le}(0) = \pi_A^m \), \( \pi_B^{le}(0) = \pi_B^m \).

Now consider firm B’s decision at stage 2. Suppose firm B has chosen \( q_B^* \) at stage 0. If firm A has not entered at stage 1, firm B will not want to sell more than \( q_B^* \) units at stage 2 because \( S_1/S_2 < M \) implies that \textit{ex ante} sale in market 1 is not profitable for firm B. On the other hand, if firm A has entered market 2 at stage 1, firm B will enter market 1 at stage 2 because \( q_B^* \geq (\alpha - R^*)/\beta \) units of pre-shipment has reduced its retail cost to below \( R^* \).

At stage 1 firm A realizes that entry into market 2 will trigger retaliatory entry by firm B. Then firm A will not want to enter market 2 if \( \pi_A^{le}(q_{B^*}) \geq \pi_A^{db}(R(q_{B^*})) \). Since \( q_{B^*} \leq (\alpha - r)/\beta \), the value of \( q_{B^*} \) can be very small if \( \beta \) is sufficiently large. Since \( \pi_A^m > \pi_A^{db}(r) \) and \( \pi_A^{le}(0) = \pi_A^m \), continuity implies that if \( \beta \) is sufficiently large, there exists \( q_{B^*} \leq (\alpha - r)/\beta \) such that \( \pi_A^{le}(q_{B^*}) \geq \pi_A^{db}(R(q_{B^*})) \). Let \( \beta_a \) be the critical value of \( \beta \) for this to occur.

Now consider firm B’s choice of \( q_B \) at stage 0. Let \( q_B^* \) be the solution to

\[ \max_{q_B} \pi_B^{le}(q_B) \text{ subject to } \alpha - \beta q_B \leq R^*. \]  \hspace{1cm} (A4) 

It is easy to see that the solution satisfies \( q_{B^*} \leq (\alpha - r)/\beta \), because \( R_{B^*} \) is reduced to its lowest possible value, \( r \), when \( q_{B^*} = (\alpha - r)/\beta \) and consequently a larger \( q_B \) will not raise \( \pi_B^{le} \). On the
other hand, we cannot rule out the possibility that \( q_B^\dagger \) is strictly greater than \( (\alpha - R^*)/\beta \) because when \( \beta \) is large, \( \pi^{le}_{B} \) may not be monotonic for \( q_B \) in the range \( [(\alpha - R^*)/\beta, (\alpha - r)/\beta] \).

At stage 0 firm B will choose \( q_B^\dagger \) if \( \pi_{B}^{le}(q_B^\dagger) \geq \pi_{B}^{d2} \). Since \( \pi_{B}^{m} > \pi_{B}^{d2} \) and \( \pi_{B}^{le}(0) = \pi_{B}^{m} \), continuity implies that \( \pi_{B}^{le}(q_B^\dagger) \geq \pi_{B}^{d2} \) if \( \beta \) is sufficiently large (and hence \( q_B^\dagger \) is sufficiently small). Let \( \beta_b \) be the corresponding critical value of \( \beta \).

Let \( \beta^* = \max \{ \beta_a, \beta_b \} \). Then if \( \beta > \beta^* \), firm B will choose \( q_B^\dagger \) and firm A will stay out of market 2.

QED
Table 1. Comparative Statics in the Linear Case

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<th>Value of $d/b$</th>
<th>Entry Deterrence via Retaliatory Entry</th>
<th>Entry Deterrence via Toe-Hold Entry</th>
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<td>$\partial E/\partial b$</td>
</tr>
<tr>
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<td>+</td>
<td>-</td>
</tr>
<tr>
<td>(0.557, 2.545)</td>
<td>-</td>
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<td>+</td>
</tr>
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</table>

Figure 1. Equilibria for Various Levels of Firm B’s Fixed Entry Cost