PRICING VS SLOT POLICIES
WHEN AIRPORT PROFITS MATTER

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Abstract: This paper analyzes pricing and slot-allocation mechanisms to manage airport capacity when profits are important to an airport, owing to budget constraints or profit maximization. We find that congestion pricing and slot trading/slot auctioning do not lead to the same results. Total traffic is higher under slot auctions than under congestion pricing. Furthermore, if airport profits matter just marginally, then slot auctions will outperform congestion pricing in terms of achieving a higher objective-function value. On the other hand, if airport profits matter sufficiently highly, which mechanism is better is then very much dependent on parameter values. In particular, congestion pricing may be strongly preferred over slot auctions for certain parameter values. The impact of congestion remedy mechanisms on individual carriers is also examined.

Keywords: congestion pricing, slot auction, slot trading, airport pricing
1. Introduction

Traffic growth has outpaced capacity increases at major airports around the world. For the last several years airlines and passengers have increasingly been suffering from flight delays; it is then no surprise that congestion delays have become a major public policy issue.\(^1\) Other than capacity expansions, perhaps the most suggested and discussed congestion remedy has been the pricing mechanism, where the social planner works to correct congestion externalities by setting congestion tolls. In the context of airports, recent literature has recognized the importance of market power that air carriers may have at a given airport: with non-atomistic airlines optimal Pigouvian tolls would only charge a carrier for the congestion it imposed on other carriers, owing to the internalization of flight congestion. In particular, a higher toll should be imposed on a smaller carrier, since it internalizes a smaller amount of congestion than a larger carrier,\(^2\) something that is potentially controversial in implementation as it would be politically sensitive to establish differentiated tolls (Brueckner, 2002; Morrison and Winston, 2007).

In addition to pricing, other widely discussed congestion remedies are related to control and management of airport slots, including slot sales, slot trading and slot auctions.\(^3\) Proponents of these solutions have argued that by setting a number of slots, congestion problems would be obviously solved while, by allowing trading of slots or by auctioning them, it would be ensured that each slot goes to the airline which values it most. Therefore, these mechanisms would circumvent the problem of perceived unfairness that congestion pricing has, while using *secondary markets* to achieve efficiency.

An obvious question that arises is whether these two mechanisms, pricing vs. slot policies, are equivalent. Recently, Verhoef (2008) and Brueckner (2009) compared analytically the pricing and slot-allocation remedies. They found that the fist-best congestion pricing and slot trading/slot auctioning are equivalent in terms of both the amount of traffic generated and total social welfare, as long as the fixed number of slots is optimally chosen. On the other hand, slot sales – which is actually treated as a uniform congestion toll – is inferior to congestion pricing and slot trading/slot auctions except when the airlines are symmetric in size. Czerny (2008), on the other hand, provided a graphical analysis of the pricing and slot-based approaches to airport congestion when there is uncertainty on congestion costs. He found that with uncertainty, congestion pricing may be more preferable than slot constraints.\(^4\)

This paper generalizes the analysis of Verhoef (2008) and Brueckner (2009) – in which the uncertainty issue is abstracted away– by considering a situation where airport revenues are important, that is, they are not simply transfers from one type of agent to another. In other words, we explicitly consider the role of airport profits in the comparison

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1. Figures and facts regarding airport congestion can be found, for example, in Kirby (2008).
3. See, e.g., Jones, Viehoff and Marks (1993) and Forsyth and Niemeier (2008) for general discussions of the economics of airport slots.
4. In a more recent study, Czerny (2009) shows analytically that airport networks (as opposed to the case of single airports) increase the relative welfare benefits.
of pricing vs. slot approaches to airport management. This emphasis on airport behavior is different from the approach of Verhoef (2008) and Brueckner (2009) in which the airport, while it may derive profits from its operation, is not considered in the welfare analysis. Their approach is appropriate if, for example, the airport as a decision maker is public owned and has no budget restrictions, and so it maximizes airlines’ and passengers’ surplus. In that case, airline payments for the use of airport services are in fact just transfers between the airlines and airport.

Airport profits can be important for a number of reasons. First, it would be of central interest for any private airport and note that, following the privatization of the British Airports Authority (BAA) in Britain in 1987, more and more countries have decided to partially or completely privatize their airports. While most of these privatized airports remain price regulated, some have been deregulated and others have been subjected to price monitoring only rather than formal (ex ante) regulations (e.g., Forsyth, 1997; Starkie, 2001; Productivity Commission, 2002). Second, with the airport corporatization (or commercialization) movement in recent years, even public airports have been under growing pressure from governments to be more financially self-sufficient and less reliant on government support. In other words, profits can also matter to a public airport that is subject to a certain degree of cost recovery (e.g., Zhang and Zhang, 2003).

We will first show that, while achieving the social optimum, congestion pricing, slot trading and slot auctions do generate different amounts of revenue to an airport, with slot auctions generating the highest revenues. We then investigate two questions: first, if the objective function to be maximized is a weighted function of airport profit and airlines’ and passengers’ surplus, is congestion pricing still equivalent to slot trading or slot auctions in terms of traffic and overall objective-function value? Second, which mechanism will yield higher traffic volumes and airport revenues as well as higher overall objective-function values?

We find that when profits matter to an airport, pricing and slot trading/slot auctioning do not lead to the same results. First, total traffic is higher under slot auctions than under congestion pricing. Second, if the importance of airport profits is not too large, then slot auctions will always outperform congestion pricing in terms of achieving a higher objective-function value. Third, if the importance of airport profits matters enough, which mechanism is better is then very much dependent on parameter values. In particular, pricing might be strongly preferred over slot auctions for some parameter combinations.

The paper is organized as follows. Section 2 sets up the basic model and examines the effect of congestion remedies (pricing vs. slot allocations) on airport profits. Section 3 investigates the equivalence result when airport profits matter, and Section 4 further compares the alternative mechanisms in terms of traffic, airport revenue and overall objective-function value. Section 5 contains the concluding remarks.

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2. The Model and Effect of Congestion Remedies on Airport Revenue

One of the main results on airport congestion pricing is that the optimal toll to be charged to an airline has two components (see Pels and Verhoef, 2004, Zhang and Zhang, 2006, and Basso, 2008). The congestion component charges for un-internalized congestion and is therefore aimed at reducing an airline’s production, as is the usual case of Pigouvian taxes. Yet, since airlines are non-atomistic (i.e., each produces more than one flight), this charge considers only the congestion imposed on other airlines’ flights but not on own flights. The second component of the toll is actually a subsidy, aimed at increasing an airline’s production. This is optimal because ceteris paribus, airlines reduce production levels below what is optimal given that they usually have market power. Thus, whether the final toll will be positive or negative depends on which of the components dominates. Importantly, if the market power effect dominates, this implies that in the absence of tolls there is no problem of congestion but a problem of not enough flights.

As indicated above, the two theoretical studies finding equivalence between pricing and slot trading/slot auctions are Verhoef (2008) and Brueckner (2009). In the former study, airlines offer a homogenous product but differ in costs, and it is assumed that the congestion effect dominates the “market power” effect. In the optimum then, only one airline operates. Brueckner (2009) instead focused on the case of perfectly elastic demands so that the market-power effect is suppressed, which allows a “pure” comparison to be made between congestion pricing and slot allocation policies. Airlines offer independent products –yet congestion links airlines’ cost functions– which implies that at the optimum the two airlines usually operate.

We will follow the setup of Brueckner (2009); the model is as follows. There are two airlines serving a congested airport. With perfectly elastic demands, passengers of airlines 1 and 2 are willing to pay “full prices” –i.e. sum of ticket plus congestion costs– $P_1$ and $P_2$, respectively, for travel in and out of the airport. The two airlines may be asymmetric in the sense that, without loss of generality, $P_1 \geq P_2$ and therefore airline 1 serves the higher-price market and is a larger carrier if $P_1 > P_2$, while the two carriers are symmetric if $P_1 = P_2$. Let $f_i$ denote the flight volume of carrier $i$, $T(f_i)$ the carrier’s marginal cost function, and $\Phi_i$ the airport (runway) charges paid by carrier $i$. Then each airline’s profit function is given by:

$$\pi_i(f_i, f_j) = [P_i - T(f_i)]f_i - c(f_i + f_j)f_i - \Phi_i, \quad i = 1, 2$$

(1)

where $c(f_i + f_j)$ is the congestion cost function, which represents the sum of passengers’ and airlines’ costs caused by airport congestion (here, and below, if the indices $i$ and $j$ appear in the same expression, then it is to be understood that $i \neq j$). Following Brueckner (2009), it is assumed that $T(0) > 0$, $T'(\cdot) > 0$ and $T''(\cdot) \geq 0$, and that $c(0) = 0$, $c'(\cdot) \geq 0$ and $c''(\cdot) \geq 0$.\(^6\) Furthermore, in specification (1) the number of passengers per flight has,

\(^6\)Note that the assumption on $T$ implies that an airline operates under diseconomies of scale. As indicated in Brueckner (2009, p. 683), this assumption is needed to generate sensible results in the presence of perfectly
as is common in the literature, been assumed to be constant, which is further normalized to unity. Consequently, \( f_i \) represents both the number of flights operated by carrier \( i \) and the number of passengers carried by \( i \).

Consider first that a public airport chooses airport congestion management methods (pricing or slot policies) to maximize social welfare and let us denote the airport’s objective function as \( OF^{Pub} \). Since consumer surplus is zero under perfectly elastic demands, total surplus is just the sum of airline and airport profits. For simplicity, we assume that the airport’s marginal costs are constant and normalized to zero, and that there are no fixed costs. As a consequence, the airport’s profit equals its total revenue \( \Phi = \Phi_1 + \Phi_2 \), and \( OF^{Pub} \) is given by:

\[
OF^{Pub} = W = \Phi + (\pi_1 + \pi_2) = \left[P_1 - T(f_1)\right]f_1 + \left[P_2 - T(f_2)\right]f_2 - (f_1 + f_2) \cdot c(f_1 + f_2)
\]

(2)

where \( W \) denotes total surplus. As can be seen from (2), any airport charges, which make up airport revenues, will come from the airlines, and thus they cancel up in total surplus. As a result, \( W \) is just equal to the airlines’ pre-airport-payment profits, and the airport’s interests can be ignored in the welfare analysis— as done in Brueckner (2009) and Verhoef (2008)—or for that matter, in the analysis of behavior of a public airport with \( OF^{Pub} \). Maximizing \( OF^{Pub} \) with respect to \( f_1 \) and \( f_2 \), the following first-order conditions are obtained:

\[
P_i - T(f_i) - f_i T'(f_i) - c(f_i + f_j) - (f_i + f_j) \cdot c'(f_i + f_j) = 0, \quad i = 1, 2
\]

(3)

Assuming the second-order conditions to hold, equations (3) imply the “first best” (i.e., welfare-maximizing) traffic. As shown in Brueckner (2009), the first-best traffic is implementable through Pigouvian taxes, which will take different values for asymmetric airlines.

In this setup, slot trading (hereinafter, \( ST \)) and slot auctions (hereinafter, \( SA \)) achieve the same equilibrium as the Pigouvian taxes. In the case of \( ST \), airlines negotiate until they agree on the number of slots to be traded and the price per slot, starting from some initial number of slots given for free by the airport. In the second case, Brueckner (2009) assumes that the airlines bid for each incremental slot their marginal benefit, yet the process of auctioning is according to a uniform-price, i.e. at the end, there is only one price \( y \) to be paid for each of the slots. This price is chosen such that carriers bidding at least \( y \) for incremental slots receive them and a total of \( n \) slots is allocated.\(^7\) This last point is of vital importance: in both cases, \( ST \) and \( SA \), the total number of slots \( n \)—which is the same

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\(^{7}\) Of course, there are a number of different bidding mechanisms one can think of. The mechanism used by Brueckner (2009) —and here indeed— is just one possibility, which we deem to be a good starting point and that relies on the assumption of non-manipulative behavior from the part of the airlines; see Brueckner (2009) for further discussion.
as the total flight volume— is taken as parametric by the airlines when they make their
decisions. As a result, the equilibrium conditions for both SA and ST are:

\[ P_i - T(f_i) - f_i T'(f_i) - c(n) = y, \quad i = 1, 2 \]  

\[ f_i + f_j = n \]  

where \( y \) is the price of slot, which goes to the airport in the case of SA, or is exchanged
between the airlines in the case of ST. What Brueckner has pointed out is that in both the
first-best and ST/SA cases, the left-hand side of equation (4) is set to equal a common value,
independent of carrier \( i \). This point can, here, be seen by comparing (4)-(5) with (3).
Therefore, if in addition \( n \) is set to equal \( f_i^* + f_j^* \), where superscript * denotes the first-
best equilibrium, then the congestion pricing, ST and SA all lead to the same traffic levels
and, hence, also to the same level of social welfare: the three congestion remedies are
equivalent.

Here, we examine first a follow-up question: are these methods equivalent in the
amount of airport revenues generated? and if not, which mechanism delivers a larger airport
profit? Again using Brueckner’s setup and letting \( z_i \) be the price that airline \( i \) must pay
with congestion pricing, then \( z_i = f_i^* c'(f_i^* + f_j^*) \), i.e. un-internalized congestion. Thus,
airport revenues \( \Phi \) under the first-best congestion pricing solution are equal to:

\[ \Phi^{FB} = z_1 f_i^* + z_2 f_j^* = 2 f_i^* f_j^* c'(f_i^* + f_j^*) \]  

where \( FB \) stands for the “first best” congestion pricing. Under Brueckner’s assumption, \( ST \)
does not provide the airport with revenues. \( SA \) does, however, and is equal to
\( \Phi^{SA} = y \cdot (f_i^* + f_j^*) \), where \( y \) is given by either of the two expressions in (4) and must be
evaluated at the first-best frequencies, and \( n \) is given by \( f_i^* + f_j^* \). Thus, from (4) and (5)
we obtain

\[ y = P_i - T(f_i^*) - f_i^* T'(f_i^*) - c(f_i^* + f_j^*) \]

It follows from (3) that \( y = (f_i^* + f_j^*) c'(f_i^* + f_j^*) \). Therefore, airport revenues under \( SA \)
are given by:

\[ \Phi^{SA} = \left[ (f_i^*)^2 + (f_j^*)^2 \right] c'(f_i^* + f_j^*) + 2 f_i^* f_j^* c'(f_i^* + f_j^*) \]

\[ = \left[ (f_i^*)^2 + (f_j^*)^2 \right] c'(f_i^* + f_j^*) + \Phi^{FB} > \Phi^{FB} \]

where the second equality follows from the use of (6). The above analysis leads to:

\[^{8}\text{As argued in Brueckner (2009), the airport announces the slot total while allocating this fixed number of}
\text{total slots to individual air carriers, with the allocation achieved either by ST or through SA. As a}
\text{consequence, it is proper for the airlines to view the total as fixed when each decides on its flight level.}\]
Proposition 1. To achieve the social optimum, the three congestion remedies – namely, congestion pricing, slot trading and slot auctioning – generate different amounts of airport revenue. Slot auctioning generates greater revenue to the airport than congestion pricing, which in turn generates greater airport revenue than slot trading.

A variant of Proposition 1 is contained in Brueckner (2009, Proposition 6, p. 688) when he examines whether the airport revenues raised under the different price and slot policies cover the airport cost. The intuition for this result is as follows: in the slot case an airline knows that if it gets an extra slot, congestion will not worsen since an extra flight for one airline necessarily means a flight less for the other. Hence, its marginal willingness to pay for an extra flight (slot) is larger than in the pricing regime, where with each extra flight, congestion increases marginally. And since in the slot case, airlines pay according to their willingness to pay, revenues will be larger.

The differential airport revenues/profits are of no material consequence if the airport is a public/welfare-maximizing airport, as indicated earlier in (2). However, as pointed out in the introduction, profits are important, for example, for a private airport or for a public airport subject to cost recovery. In this context then, one would like to know which airport management mechanism performs better in terms of the objective of the airport, and what the consequences of choosing either mechanism are in terms of airport revenues, traffic and so on. Consequently, we consider the following general airport objective function:

\[ \text{OF}(\alpha) = \alpha \cdot \Phi + (\pi_1 + \pi_2), \quad \alpha \geq 1 \] (8)

Here, the parameter \( \alpha \) captures how important airport profits (revenues) are to the decision maker. More specifically, when \( \alpha = 1 \), \( \text{OF}(\alpha) \) reduces to \( \text{OF}^{\text{Pub}} \) in (2), or Brueckner’s welfare function. In this sense, formulation (8) nests Brueckner’s setting (recall consumer surplus continues to be zero).

Because the most important insights in this paper occur for \( \alpha > 1 \), we justify this modeling in three different ways. First, \( \alpha \) would be greater than one if the airport needs to cover costs, or to create revenues for the local city. In any of those cases, a financial constraint in the optimization problem would generate a Lagrange function where airport revenues are multiplied by 1 plus the (positive) Lagrange multiplier; this sum would be \( \alpha \). A second justification is that \( \alpha \) may capture the degree of commercialization/privatization, with a larger value indicating a higher degree of commercialization/privatization. Obviously, the limiting case \( \alpha \to \infty \) corresponds to a pure unregulated private airport. For example, in a case like this, \( \alpha \) may capture the result of bargaining over the objective function between a private airport and public parties (which may be modeled as the result of a Nash-bargaining process, where \( \alpha \) captures the degree of bargaining power of the airport with respect to the public parties involved). Finally, one can resort to a political economy explanation: most airports are still wholly or majority public owned and are managed by government departments, or their delegated agents who behave like government bureaucrats. One could then appeal to the literature on budget-maximizing
bureaucrats, which often portrays these individuals as maximizing a weighted sum of their budget and consumer utility (here, consumers of an airport are airlines as well as passengers), in the same spirit as our treatment of $\alpha$ being between 1 and infinity in the objective function (8).\footnote{See, e.g. Moene (1986; equation 9, in particular). We thank Jan Brueckner for pointing out this motivation to us.}

With the objective function in (8) at hand we can then attempt to answer our two principal research questions:

\textbf{Question 1}: Considering that the airport, or a regulator, chooses a flight-allocation mechanism to maximize $OF(\alpha)$ given in (8), is pricing still equivalent to slot trading or slot auctions in terms of traffic volumes $f$'s and the $OF$ value given by (8)?

\textbf{Question 2}: Which mechanism will yield higher traffic volumes and airport revenues as well as higher overall $OF$ values?

Before investigating these two questions in Sections 3 and 4 respectively, we note that for $\alpha > 1$ it may not be appropriate to call the pricing remedy the first best congestion pricing, as there is now market power from the part of the airport that starts to kick in and, therefore, prices may end up above social marginal cost.\footnote{A private road would charge, as part of its price, un-internalized congestion but will add to this a market power mark-up (see, e.g., Small and Verhoef, 2007). In a vertical structure though, a private airport would charge more than the un-internalized congestion of each carrier (see Basso, 2008).} Moreover, since even for $\alpha = 1$ asymmetric airlines would lead to different first-best congestion pricing tolls, as $\alpha$ increases the pricing remedy may be considered a price-discrimination mechanism. Hence, we will use $PR$ for the price mechanism while, for simplicity, we keep $\ast$ to denote its resulting values. Furthermore, note that since the slot-sales mechanism actually corresponds to uniform pricing, it simply precludes the possibility of price discrimination (i.e. the airport’s charging asymmetric airlines differentiated tolls) and hence represents a more restricted optimization. As a consequence, it will always be dominated by $PR$. For this reason, we will focus only on the comparison between $PR$ pricing and $ST/SA$ in the remainder of the paper.

\section{3. Equivalence of Pricing and Slot Policies When Airport Profits Matter}

This section examines whether congestion pricing and slot trading/slot auctions remain equivalent when airport profits matter. Consider first the equilibrium under $SA/ST$. It is quite obvious that for any given number of slots being traded or auctioned, the (conditional on $n$) traffic levels under $SA$ or $ST$ are still given by equating the two equations given in (4). That is, the ratio of traffic levels under $SA$ or $ST$ is, for all $\alpha$, given by

$$P_1 - T(f_1) - f_1T'(f_1) = P_2 - T(f_2) - f_2T'(f_2)$$

\section*{References}

\footnote{See, e.g. Moene (1986; equation 9, in particular). We thank Jan Brueckner for pointing out this motivation to us.}

\footnote{A private road would charge, as part of its price, un-internalized congestion but will add to this a market power mark-up (see, e.g., Small and Verhoef, 2007). In a vertical structure though, a private airport would charge more than the un-internalized congestion of each carrier (see Basso, 2008).}
In this sense, $\alpha$ can only play a role once we look for the optimal $n$. In other words, the change in the objective function does not affect airlines’ bidding or trading behavior, a quite reasonable outcome.

Next consider the equilibrium under $PR$. In this case, we consider that the airport will charge, per flight, $z_1$ and $z_2$ to airlines 1 and 2, respectively, and the airlines take these charges as given and decide on the number of flights in order to maximize their profits.\footnote{Note that this per-flight airport charge scheme has ruled out a two-part tariff scheme which, by having more instruments, obviously leads to higher objective function values. We will discuss the issue further in the concluding remarks.} The result of the Nash equilibrium are airport traffic levels conditional on airport charges; these are the derived demands that the airport faces (see, e.g., Basso and Zhang, 2008b for more discussion on derived demands within the vertical structure approach to airport pricing). The downstream airline equilibrium, obtained after computing the airlines’ first-order conditions, give us:

$$z_i = P_i - T(f_i) - f_i T'(f_i) - c(f_i + f_j) - f_i c'(f_i + f_j), \quad i = 1, 2 \quad (10)$$

The two equations in (10) define, implicitly, the inverse derived demands for the airport $z_1(f_i, f_2)$ and $z_2(f_1, f_2)$. Solving the system of equations, one can obtain the corresponding direct demands.

The problem faced by the airport is, using (8), then:

$$\text{Max}_{f_1, f_2} \quad OF(\alpha) = \alpha \cdot (z_1 f_1 + z_2 f_2) + \sum_{i=1}^{2} [P_i - T(f_i) - z_i - c(f_i + f_j)]f_i \quad (11)$$

where $z_1$ and $z_2$ are given by (10). Note that we set the problem in terms of inverse demands for simplicity but obviously the results do not change if we use the direct demands. From (11), the first-order conditions can be derived in a straightforward manner: taking derivative with respect to $f_1$ one obtains:

$$\alpha P_i - \alpha c(f_i + f_j) - \alpha T(f_i) + 2 f_i c'(f_i + f_2) - 3 f_i \alpha c'(f_i + f_2) - f_2 \alpha c'(f_i + f_2)$$

$$+ 2 f_i T'(f_i) - 3 f_i \alpha T'(f_i) + f_i^2 c''(f_i + f_2) + f_i^2 c'(f_i + f_2) - f_2^2 \alpha c''(f_i + f_2)$$

$$+ f_i^2 T''(f_i) - f_i^2 \alpha T''(f_i) = 0$$

Then, reorganizing terms we get
\[
\alpha [P_1 - T(f_1) - f_1 T'(f_1) - c(f_1 + f_2)] - (f_1 + f_2) c'(f_1 + f_2) \\
+ (1 - \alpha) \left[ 2T'(f_1) + f_1 T''(f_1) \right] f_1 \\
+ (1 - \alpha) \left[ 3 f_1 + f_2 \right] c'(f_1 + f_2) + (f_1^2 + f_2^2) c''(f_1 + f_2) = 0 \tag{12}
\]

and the first-order condition for \( f_2 \) is given by the analogous expression with the 1 and 2 subscripts interchanged. From these two equations, the ratio of traffic levels under \( PR \) can then be obtained from:

\[
\alpha [P_1 - T(f_1) - f_1 T'(f_1)] + (1 - \alpha) \left[ (2T'(f_1) + f_1 T''(f_1)) f_1 + (3 f_1 + f_2) c'(f_1 + f_2) \right] \\
= \alpha [P_2 - T(f_2) - f_2 T'(f_2)] + (1 - \alpha) \left[ (2T'(f_2) + f_2 T''(f_2)) f_2 + (3 f_2 + f_1) c'(f_1 + f_2) \right] \tag{13}
\]

A comparison between (13) and (9) gives us a hint that traffic levels will be different if a pricing regime is used by the airport, than if a slot auction/slot trading regime is used, which implies that the flight allocation mechanisms would be no longer equivalent. A proposition summarizing this non-equivalence result and its proof follows.

**Proposition 2.** (i) If airport profits matter so that a flight-allocation mechanism is chosen to maximize \( OF(\alpha) \), then sufficient conditions for the pricing and slot trading/slot auctioning mechanisms to lead to different results are \( \alpha > 1 \), \( P_1 \neq P_2 \) and that \( T'(\cdot) \) is linear or not ‘too’ concave. (ii) Pricing and slot trading/slot auctioning are equivalent when \( \alpha = 1 \) and then \( n \) is set to \( f_1^* + f_2^* \) (i.e. the first-best total traffic level). In that case, since airline payments are just transfers between the airlines and airport, it would be also true that \( OF^{CP}(\alpha = 1) = OF^{ST/S}(\alpha = 1) \).

**Proof:** (i) Traffic levels under \( PR \) fulfill equation (13). Traffic levels under \( SA \) or \( ST \), on the other hand, fulfill equation (9) leading to

\[
P_1 - P_2 = T(f_1^{SA}) + f_1^{SA} T'(f_1^{SA}) - T(f_2^{SA}) - f_2^{SA} T'(f_2^{SA}) \]

And since \( T'(\cdot) > 0 \) and \( T''(\cdot) \geq 0 \), then \( T(x) + x T'(x) \) is increasing in \( x \) and it follows that \( P_1 > P_2 \Rightarrow f_1^{SA} > f_2^{SA} \). We now assume –without loss of generality– that \( P_1 > P_2 \) and therefore \( f_1^{SA} > f_2^{SA} \), and show that assuming \( f_1^{SA} = f_1^* \), \( f_2^{SA} = f_2^* \) leads to a contradiction. First, if the SA and PR traffic levels are equal, then from (9) and (13) it would be true that:

\[
\left( 2T'(f_1^{SA}) + f_1^{SA} T''(f_1^{SA}) \right) f_1^{SA} + (3 f_1^{SA} + f_2^{SA}) c'(f_1^{SA} + f_2^{SA}) \\
= \left( 2T'(f_2^{SA}) + f_2^{SA} T''(f_2^{SA}) \right) f_2^{SA} + (3 f_2^{SA} + f_1^{SA}) c'(f_1^{SA} + f_2^{SA})
\]
which is equivalent to
\[
2(f_1^{SA} - f_2^{SA})c'(f_1^{SA} + f_2^{SA}) = (2T'(f_2^{SA}) + f_2^{SA}T''(f_2^{SA}))f_2^{SA} - (2T'(f_1^{SA}) + f_1^{SA}T''(f_1^{SA}))f_1^{SA}
\]

Next, if \( T' \) is linear (and so \( T''(x) = 0 \)) or is not too concave—such that \( T''(x) \) is not too negative—then \( (2T'(x) + xT''(x))x \) is increasing in \( x \), and since the left-hand side of the last equation is positive by \( P_1 > P_2 \), the only way that the right-hand side can be positive is if \( f_2^{SA} > f_1^{SA} \), which is a contradiction.

(ii) The proof of the second part of the proposition is in Brueckner (2009) and uses the definition of \( OF(\alpha) \).

Note that \( P_1 = P_2 \) does not imply that the mechanisms are equivalent; but the non-equivalence in that case has to refer to the actual number of slots \( n \) chosen, something that we cannot analyze in general (i.e. for the case of \( T''(x) \geq 0 \)). In the next section however, we show that if \( T' \) is linear (and so \( T''(x) = 0 \)), then the non-equivalence holds even if \( P_1 = P_2 \).

4. Comparison of Traffic, Airport Revenue and Objective-Function Value

Given that pricing and slot policies are not equivalent when airport revenues (profits) matter, we now investigate the second research question: Which congestion-remedy mechanism will yield higher traffic volumes and airport revenues as well as higher overall \( OF \) values? Note, first, that while traffic volumes are not an objective in itself, it is still a useful comparison. This is because differences in individual carriers’ traffic volumes indicate the extent of carrier asymmetry, which is an important starting point of the investigation, whereas differences in total traffic reveal the extent of airport congestion and delays, which is a main performance indicator for alternative congestion management mechanisms. Second, because slot trading does not generate revenue for the airport under the modeling assumptions, we will focus on the comparison between pricing and slot auctions. Third, we assume from now on that both the congestion cost function and the airline marginal cost function are linear:

\[
c(n) = Bn, \quad T(x) = t + ax
\]

with \( B \geq 0, t > 0 \) and \( a > 0 \). The simplification is necessary to push the analysis further but, as it will become clearer, it is more than sufficient to show some interesting results which clearly do not hinge on these linearity assumptions.

**Pricing solution**

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Substituting (14) into the first-order conditions (12), the PR traffic levels are obtained by solving the resulting system of equations. Straightforward algebra leads to:

\[ f_1^* = \frac{(a + B)(2\alpha - 1)(P_1 - t) - B\alpha(P_2 - t)}{2(a^2 + 2aB)(2\alpha - 1)^2 + 2B(3\alpha - 1)(\alpha - 1)} \]  

(15)

\[ f_2^* = \frac{(a + B)(2\alpha - 1)(P_2 - t) - B\alpha(P_1 - t)}{2(a^2 + 2aB)(2\alpha - 1)^2 + 2B(3\alpha - 1)(\alpha - 1)} \]  

(16)

Since \((\alpha - 1)(3\alpha - 1) \geq 0\), the denominators in (15) and (16) are positive, indicating that the second-order conditions for the airport problem (11) hold. Further, given that \(2\alpha - 1 \geq \alpha\), \(P_1 \geq P_2\) and \(a + B > B\), it is clear that \(f_1^*\) is positive if \(P_1\) and \(P_2\) are greater than \(t\). Since \(P_i > t\) are required for the cost function to start below the full price, it follows that \(f_1^* > 0\). Whilst \(f_1^* > 0\) in the model, \(f_2^*\) is positive only if parameter \(a\) is sufficiently large. Specifically, \(f_2^* > 0\) if

\[ a > \frac{B[(\alpha - 1)t + \alpha P_1 + (1 - 2\alpha)P_2]}{(P_2 - t)(2\alpha - 1)} \equiv a_0(\alpha) \]  

(17)

From (17) we have \(a_0(1) = B(P_1 - P_2)/(P_2 - t) \geq 0\), and \(a_0'(\alpha) = Bt/(P_2 - t)(2\alpha - 1) > 0\), i.e. \(a_0\) rises as \(\alpha\) increases. Also notice that when the two airlines are identical (i.e. \(P_1 = P_2\)) then, as expected, \(f_2^* (= f_1^*)\) is always positive.

Total traffic in the PR case is easily obtained by adding (15) and (16):

\[ f_T^* = \frac{(P_1 + P_2 - 2t)\alpha}{2a(2\alpha - 1) + 2B(3\alpha - 1)} \]  

(18)

As can be seen, \(f_T^*\) is always positive. Further, taking the derivative of (18) with respect to \(\alpha\), it is easy to see that total traffic falls as \(\alpha\) increases: As expected, as airport revenues matter more and more, the airport will exercise, in an increasing fashion, its market power, inducing output contraction. This outcome is a manifestation of the classic “double marginalization” problem, which is typical of an uncoordinated vertical airport-airlines structure (e.g. Basso and Zhang, 2008a). Taking the limit with \(\alpha\) approaching to \(\infty\), one can see that a private airport would charge in such a way that it will induce total traffic given by \((P_1 + P_2 - 2t)/(4a + 6B)\).

**Slot-auctions solution**

First, we solve for traffic levels of the two airlines given an arbitrary number \(n\) of slots being auctioned. These traffic levels are obtained by solving the system given by equations (5) and (9), leading to:
\[ f_{1\text{SA}}^{\text{SA}}(n) = \frac{P_1 - P_2 + 2an}{4a} \]  \hspace{1cm} (19) \\
\[ f_{2\text{SA}}^{\text{SA}}(n) = \frac{P_2 - P_1 + 2an}{4a} \]  \hspace{1cm} (20)

As expected, both traffic levels increase with \( n \).

Next, we can look at how traffic levels would compare between mechanisms, if the number of slots is set to what is best under pricing (PR), something that is only optimal when \( \alpha = 1 \). Replacing \( n \) in (19) and (20) by \( f_T^* \) in (18), one can easily show that if \( \alpha > 1 \), then \( f_{1\text{SA}}^{\text{SA}}(f_T^*) > f_1^* \) and \( f_{2\text{SA}}^{\text{SA}}(f_T^*) < f_2^* \). Therefore, if for \( \alpha > 1 \), the airport sets – wrongly – the total number of slots to be auctioned to what is best under PR pricing, then slot auctioning would lead to a larger traffic volume for \( f_1 \) but a smaller \( f_2 \), than PR pricing. If \( \alpha = 1 \), then the \( f \)'s are equal as expected.

We now calculate the optimal number of slots to be auctioned. First, the price that each slot ends up yielding, as a function of \( n \), is given by (4), i.e.

\[ y(n) = P_1 - T(f_1(n)) - f_1(n)T'(f_1(n)) - c(n) = P_2 - T(f_2(n)) - f_2(n)T'(f_2(n)) - c(n) \]  \hspace{1cm} (21)

Then the objective function of the airport is:

\[ \text{Max}_{n} \alpha y(n)[f_1(n) + f_2(n)] + \sum_{i=1}^{2}[P_i - T(f_i(n)) - y(n) - c(n)]f_i(n) \]  \hspace{1cm} (22)

where \( f_1(n) \) and \( f_2(n) \) are given by (19) and (20). The first-order condition from (22) leads to:

\[ n^{\text{SA}} = \frac{(P_1 + P_2 - 2t)\alpha}{2a(2\alpha - 1) + 4B\alpha} \]  \hspace{1cm} (23)

From (23), total traffic under \( \text{SA} \) decreases with \( \alpha \), similar to \( f_T^* \) in the \( \text{PR} \) case.

**Comparisons between mechanisms**

As can be seen from (23), for a profit-maximizing airport (i.e. \( \alpha \rightarrow \infty \)) total traffic under \( \text{SA} \), \( n^{\text{SA}} \), converges to \((P_1 + P_2 - 2t)/4(a + B)\). This traffic level is greater than \((P_1 + P_2 - 2t)/(4a + 6B)\), the limiting traffic under pricing. Direct comparison of (23) and (18) further reveals that \( n^{\text{SA}} = f_T^* \) when \( \alpha = 1 \), but \( n^{\text{SA}} > f_T^* \) for all \( \alpha > 1 \). Thus, there will always be more airport delays under slot auctions than under pricing (differentiated tolls),
except when $\alpha=1$, the case in which they are equal. Furthermore, the difference between $n^{SA}$ and $f^*_r$ increases as $\alpha$ increases, reaching its maximum for a private, profit-maximizing airport.

We now compare the actual traffic levels of individual airlines under the two flight-allocation mechanisms. That is, we compare, for $\alpha > 1$, $f^{SA}_i(n^{SA})$ with $f^*_i$. First, since both $f_i(n)$ and $f^*_i(n)$ increase with $n$, it is always true that under slot auctions, both individual traffic volumes are greater when the optimal number of slots is used, than when the optimal $PR$ number of slots is used, i.e. $f^{SA}_1(n^{SA}) > f^*_1$ and $f^{SA}_2(n^{SA}) > f^*_2$. Second, by $f^{SA}_1(f^*_r) > f^*_1$ as shown above, it follows that $f^{SA}_1(n^{SA}) > f^*_1$ for all $\alpha > 1$. However, since $f^{SA}_2(f^*_r) < f^*_2$, there are two opposing effects for the case of airline 2. In effect, whether finally $f_2$ is larger under one mechanism or the other will depend on the model’s parameter values. Three numerical examples are given in Figure 1, where the horizontal line is set at $f^{SA}_2(n^{SA}) = f^*_2$. One example (the “$P_2 = 9.5$” curve) shows $f^{SA}_2(n^{SA}) > f^*_2$ for all $\alpha > 1$, whilst the other two examples show $f^{SA}_2(n^{SA}) < f^*_2$ for all $\alpha > 1$.

**** Figure 1 about here ****

Summarizing the above comparisons yields

**Proposition 3.** If airport profits matter so that a flight-allocation mechanism is chosen to maximize $OF(\alpha)$, then $n^{SA} = f^*_r$ for $\alpha = 1$; but for $\alpha > 1$, $n^{SA} > f^*_r$, i.e. total traffic and airport delays are larger under slot auctioning than under pricing; furthermore, the difference between the two increases as $\alpha$ increases. Also, when $\alpha > 1$, airline 1’s traffic is greater under slot auctions than under pricing, while airline 2’s traffic may or may not be greater under slot auctions than under pricing.

Now that we have shown that the mechanisms are equivalent only when $\alpha=1$, and that we have established that in general $SA$ leads to higher traffic levels, we would like to see which mechanism is better in terms of leading to a higher objective-function value. What we have known from Proposition 1 is that for $\alpha=1$, airport revenues are larger with $SA$ than with $PR$. It is clear, therefore, that for values of $\alpha$ slightly above 1, $SA$ will outperform $PR$. The interesting question then is whether this result will hold for larger values of $\alpha$.

To study this, it is sufficient to replace the expressions we have obtained above and calculate the resulting values of $OF(\alpha)$ in (11) and (22) respectively. We calculated the ratio of the evaluated objective functions but the resulting expression was too complex to obtain an analytical result. We turned to numerical methods and found, somewhat surprisingly, that even with our linearity assumptions about the functional forms, which mechanism performs better is very much dependent on parameter values. To show this
more clearly we provide two sets of examples, presented in Figures 2 and 3 respectively. In each of these cases we graph the ratio of $OF$'s against $\alpha$, showing different curves for different values of $P_2$ and $t$. As can be seen, for some parameter values, SA is preferred over PR and the difference in the objective-function values between SA and PR is up to 10%. For other parameter values, however, PR is favored by up to 10% after $\alpha$ reaches some threshold value.

**** Figure 2 about here ****

**** Figure 3 about here ****

We summarize these findings in the following proposition

**Proposition 4.** If airport profits matter so that a flight-allocation mechanism is chosen to maximize $OF(\alpha)$, then for values of $\alpha$ slightly above 1, slot auctions will outperform congestion pricing in terms of leading to a higher objective-function ($OF$) value. For larger values of $\alpha$, however, which mechanism is better is very much dependent on parameter values. In particular, for certain parameter values, pricing outperforms slot auctions by up to 10% after $\alpha$ reaches some threshold value.

Figure 2 shows a picture where SA performs better –in terms of objective function value– the more alike the two airlines are, i.e. when $P_1$ and $P_2$ are close. This is an important point that allows one to provide intuition for a number of the results we have so far. What happens when airlines are very asymmetric is that the airport would like to choose different traffic levels for the two airlines. The airport can actually achieve these traffic levels through the pricing mechanism since it allows discrimination (i.e. different tolls for each airline). The slot auction mechanism however cannot achieve the same asymmetric outcome because there, the levels of traffic are achieved through the airlines bidding behaviour, which depends on the willingness to pay, something leading to different ratios of traffic (Proposition 2). In fact, as we have shown, SA will, in general, allow airline 1 to achieve higher traffic levels than through pricing. It is because of this, then, that under auctioning the airport needs to choose a number of slots that is larger than the total traffic under pricing (Proposition 3): it is its way to take into account the sub-optimal traffic ratios between the airlines.

On the other hand, and as discussed before, with auctioning the airlines’ willingness to pay for an extra slot is larger than under pricing because with a fixed number of slots to be auctioned, an extra flight for an airline causes no marginal congestion cost. Therefore, there are two opposing effects: pricing allows better control of the traffic ratios, but auctioning leads to an increase of the airlines’ willingness to pay, which are then captured

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12 In effect, when $P_1 = 10$ and $P_2 = 9$, SA is always better than PR. On the other hand, when $P_1 = 10$ and $P_2 = 4$, PR is better than SA for values of $\alpha$ that are above a certain threshold. Values of $P_2$ between 4 and 9 lead to curves that lie in between the two curves shown.
by the airport. Thus, when airlines are very symmetric, the effect of higher willingness to pay dominates. If airlines are very asymmetric, then pricing—with its good control over traffic ratios—will dominate.

Given these results, it is natural to ask what happens to airport revenues by themselves; but given the intuition we have built, it is also evident that which mechanism leads to larger airport revenues depends also on parameter values, as shown in Figures 4 and 5, and airport revenues are higher when airlines are more symmetric. What is new, is that the threshold values of $\alpha$ in the case of $OF$ do not coincide with the threshold values of $\alpha$ for airport revenues.

**** Figure 4 about here ****

**** Figure 5 about here ****

Overall, it is quite complex to know which mechanism will perform better as it would require a lot of information on parameter values. For some parameter values, one mechanism may lead to larger revenues for the airport, while at the same time reducing airlines’ profits and the value of $OF(\alpha)$. This may imply that if a regulator asks airport managers to maximize $OF(\alpha)$ but leaves the choice of a flight-allocation mechanism to the airport, there may be social welfare losses. It is interesting to note that all these results occur in a context which, for all $\alpha > 1$, $SA$ leads to larger total traffic volumes than $PR$.

5. Concluding Remarks

This paper analyzes pricing and slot-allocation mechanisms when profits are important to an airport, owing to budget constraints or profit maximization. We find that pricing and slot trading/slot auctioning do not lead to the same results. Total traffic is higher under slot auctions than under congestion pricing. Furthermore, if airport profits matter just marginally, then slot auctions will outperform pricing in terms of achieving a higher objective-function value. On the other hand, if airport profits matter sufficiently highly, which mechanism is better is then very much dependent on parameter values. In particular, pricing may be strongly preferred over slot auctions for certain parameter values (specially when airlines are very asymmetric).

Our analysis suggests that strategic behavior on the part of the airport that cares about its profit can have a significant bearing on the comparison of price vs. slot-based approaches to congestion management, depending on what is asked from the airport, and what matters to the airport. This is because, for some parameter values, one mechanism may lead to larger airport profits while at the same time reducing airlines’ profits and the value of the overall objective function. Hence, if a regulator asks airport managers to maximize a certain objective function, but the choice of a flight-allocation mechanism is left to the airport, there may be social welfare losses, with the extent of welfare losses depending in general on cost and demand parameters. Our results thus imply that, if airport profits matter, then there is no simple solution as to which mechanism should be employed or implemented.
The paper has also raised several other issues and avenues for future research. First, our pricing approach has constrained the fixed entry fee to be zero and thus ruled out the possibility of a lump-sum tax levied by airports (in addition to a per-flight charge). As pointed out by an anonymous referee, given the objective chosen (for $\alpha>1$) the best approach for an airport would be to charge first-best congestion prices per flight, and then levy a lump-sum tax on the airlines for the right to be present at the airport (the entry fee), with the tax being set such that airline profits are entirely skimmed off. While desirable for the airport, such a lump-sum tax is not commonly observed in actual airport charges, and the corresponding two-part tariff scheme is rarely examined in the airport pricing literature (an exception is Basso, 2008; see, Basso and Zhang, 2007 for a review of the literature). Nonetheless, it would be interesting to theoretically investigate the properties of the two-part tariff scheme under the objective function considered in this paper, and compare them with the properties of slot policies as well as the present “second best” pricing approach. Second, to facilitate the analysis and to focus on our main concern of demonstrating possible non-equivalence between pricing and slot policies when airport profits matter, we have followed Brueckner (2009) by assuming that demand is perfectly elastic. It is noted that the benchmark result in the present paper – that the three policies of pricing, slot auctions and slot trading produce the same flight levels – critically hinged on that assumption. We see comparison of price and slot policies under more realistic demand specifications as a natural extension of the analysis presented here, although beyond the scope of the present article.
Figure 1. Traffic comparison for airline 2 under \( PR \) and \( SA \)
Figure 2. Comparison of objective-function value under PR and SA: variable $P_2$

Figure 3. Comparison of objective-function value under PR and SA: variable $t$
Figure 4. Comparison of airport revenues under PR and S4: variable $P_2$

Figure 5. Comparison of airport revenues under PR and S4: variable $t$
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