Airport Congestion Pricing and Passenger Types

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Abstract
We consider a public and congested airport together with an oligopolistic airline market. Airlines serve two types of passengers, business and leisure, with business passengers having a greater value of time than leisure passengers. We find that in the absence of passenger-type-based price discrimination by airlines, it can be useful to increase airport charges in order to protect business passengers from excessive congestion caused by leisure passengers. As a result, the welfare optimal airport charge can be substantially greater than what we learned from the recent literature on congestion pricing with non-atomistic airlines.

Keywords: Airports, airlines, passenger types, value of time, congestion, pricing.
1 Introduction

Air transport has realized an impressive growth during the last few decades and is expected to grow at high rates in the future. This development is however not paralleled by a respective increase in airport capacity; airport congestion management is therefore of growing importance. One way to manage congestion is to increase the airport charge in order to reduce the capacity demand until the socially optimal level of congestion is reached, which we shall call “airport congestion pricing”. The optimal airport charge that achieves this objective is however not easy to identify. We concentrate on three reasons that complicate the identification of the optimal congestion charge in the case of airports. First, air transport markets include a vertical structure with airports in the upstream market and airlines in the downstream market. Second, airline markets are not perfectly competitive but oligopolistic. Third, passengers are not a homogenous group of individuals but differ with regard to their values of time in a situation with traffic delays. Regarding time valuation, Pels et al. (2003) presented an empirical study where they find that business passengers have a greater value of time than leisure passengers.

In this paper we consider a public, monopolistic and congested airport, an oligopolistic airline market and two passenger types, business and leisure, with different relative values of time. The market consists of \( n \) symmetric airlines, and airport and airline behavior is modeled as a two-stage game. In the first stage, the airport chooses the airport charge to maximize welfare (sum of consumer surplus and the airline and airport profits). In the second stage, airlines choose passenger quantities to maximize individual profits (i.e., airlines are in Cournot competition). Here we assume that price dis-
crimination based on passenger types is not possible so airlines charge all passengers with a single ticket price.

We find that the optimal airport charge depends on the number of airlines or, respectively, market shares (i.e., $1/n$). This is for two reasons. First, airlines partly internalize marginal congestion costs if they are self-imposed, which depends on market shares. This leads to a general rule that implies an inverse relationship between the optimal airport charge and market concentration. Second, there is a positive relationship between market concentration and equilibrium airfares. As a consequence, the optimal airport charge can be small or even negative to correct for market power in the airline industry. Note that both reasons imply a negative relationship between the optimal airport charge and market shares.

In contrast, we find that the effect of passenger types with different values of time can work into the opposite direction. In this situation, it can be useful to increase the airport charge in order to protect passengers with a great relative value of time from excessive congestion caused by passengers with a low relative value of time. Moreover, our theoretical results and numerical simulations demonstrate that the effect of passenger types can dominate the market power effect in oligopolistic airline markets. The relationship between the differences in time valuations on the optimal airport charge is however not clear-cut. This is because passengers with a great value of time should not be protected by greater airport charges if the benefits of flying net of congestion costs are small.

The contribution of this paper is to identify and explore the effect of passenger types with different values of time on the welfare optimal airport charge. The relationship between the internalization of marginal congestion costs and market shares has been found and investigated previously by
Daniel (1995), Brueckner (2002), Pels and Verhoef (2004), Zhang and Zhang (2006), Basso and Zhang (2007) and Brueckner and van Dender (2008). In contrast to the theoretical results, the empirical findings with regard to the relationship between internalization and market concentration are controversial. Brueckner (2002) and Mayer and Sinai (2003) find positive evidence for the internalization of self-imposed congestion costs whilst Daniel (1995) and Daniel and Harback (2008) reject this idea based on their empirical results. Morrison and Whinston (2008) conclude that the welfare loss would be small if, in the process of choosing the airport charges, airline market shares would be ignored (i.e., carriers were treated as atomistic even though they are oligopolistic). Pels and Verhoef (2004) are the first who pointed out that it can be useful to reduce airport charges to correct for market power in an oligopolistic airline market.

Note that the existence of passenger types with different values of time can provide a theoretical explanation for an empirical situation that seems to be inconsistent with self-internalization. Furthermore, our results indicate that the existence of passenger types with different values of time can reduce the welfare loss when the airport charge is not corrected for market shares, which is consistent with Morrison’s and Whinston’s conclusions that are based on their empirical considerations.

The paper is organized as follows. Section 2 presents the model. Section 3 introduces the concept of a full price (sum of airfare and average congestion cost) and investigates equilibrium demands of business and leisure passengers. Section 4 analyzes optimal passenger numbers and airfares. We turn to airport charges under the airport-airline vertical structure in Section 5. This section is separated into three subsections. The first subsection analyzes airline behavior for a given airport charge, the second the optimal airport
charge, and the third provides comparative static results in the business pas-
sengers’ demand and value of time. Section 6 presents numerical simulations
to illustrate the findings of the previous sections. We conclude in Section 7.

2 The Model

We start by describing the demand side. Passengers are of two types and
include business and leisure passengers. Let \( q_B \geq 0 \) denote the number
of business passengers and \( q_L \geq 0 \) denote the number of leisure passengers.
Setting aside congestion, benefits of business passengers are \( B_B(q_B) \geq 0 \) and
benefits of leisure passengers are \( B_L(q_L) \geq 0 \). Benefit functions are three
times continuously differentiable with \( B'_x > 0 \) and \( B''_x < 0 \) for all \( x \in \{B, L\} \).
Note that we leave the sign of the third derivative of \( B_x \), \( B'''_x \), unspecified.

There is a single public and congested airport. Average delays, which are
constant over all passengers, are denoted by \( C(q) \geq 0 \) with \( C' > 0 \), \( C'' \geq 0 \)
and \( q = q_B + q_L \). Passengers take average delays as given. Furthermore,
there are constant values of time that depend on passenger types. Denote
the business passengers’ value of time by \( v_B \) and the leisure passengers’ value
of time by \( v_L \) with \( v_B \geq v_L > 0 \). In this context, the inverse demands of
business or leisure passengers are

\[
P_x(q_B, q_L) = B'_x(q_x) - v_xC(q).
\] (1)
Turning to the supply side, \( n(\geq 1) \) airlines serve the airport. Let \( q_{iB}(\geq 0) \) denote the number of business passengers and \( q_{iL}(\geq 0) \) denote the number of leisure passengers served by airline \( i \) with \( i = 1, 2, \ldots, n \) and

\[
q_x = \sum_{i=1}^{n} q_{ix}
\]  

(2)

for all \( x \in \{B, L\} \). Airline services are homogenous, and airlines are in Cournot competition. The airport charges a price \( \tau \in \mathbb{R} \) per passenger to airlines. ¹ Other variable airline costs, including the airlines’ congestion costs, are constant and normalized to zero. This simplifying assumption on the airlines’ cost structure allows us to concentrate on the differences in passengers’ time valuation. In this context, airline profits are

\[
\Pi_i = \sum_x q_{ix} [P_x(q_{iB}, q_{iL}) - \tau],
\]

(3)

and social welfare is

\[
W = \sum_x [(B_x(q_x) - q_x v_x C(q))].
\]

(4)

Airport and airline behavior is modeled as a two-stage game. In the first stage, the airport chooses \( \tau \) to maximize welfare, \( W \), taking into account airline responses in stage 2. In the second stage, airlines take the airport charge \( \tau \) as given and choose \( q_{iB} \) and \( q_{iL} \) to maximize \( \Pi_i \). As indicated in the introduction, price discrimination is not possible; as a consequence, airlines charge business and leisure passengers with a single ticket price denoted by \( p(\geq 0) \).

¹Public subsidies are normally not available for airports nowadays, and therefore negative values of \( \tau \) may not be realistic. However, we also consider negative values of \( \tau \) to simplify the analysis.
3 ‘Full prices’ and Equilibrium Passenger Demands

In this section, we derive the demands of business and leisure passengers as a function of the ticket price $p$. This is not trivial because demands depend on the ticket price $p$ and average congestion costs $C$. To make this relationship more transparent, we use the concept of a ‘full price’ for airline services. Denote the full price for business passengers by $\rho_B$ and the full price for leisure passengers by $\rho_L$ with

$$\rho_x(q_B, q_L) = p + v_x C(q)$$

for all $x \in \{B, L\}$. Observe that the full price for business passengers is greater than the one for leisure passengers because the business passengers’ value of time is greater (i.e., $v_B > v_L \Rightarrow \rho_B > \rho_L$). This shows that the full price for airline services depends on the passenger type even though airlines charge a single airfare to business and leisure passengers.

Since all passengers are charged with a single airfare, the condition $p = P_B = P_L$ must be satisfied in equilibrium. Substituting $p$ for $P_x$ in the inverse demands in (1) and rearranging gives the equilibrium conditions

$$\rho_x(q_B, q_L) = B'_x(q_x).$$

In the remainder, we call the right hand side of these conditions the inverse demands of business or leisure passengers with respect to the full prices $\rho_x$. We obtain equilibrium demand functions depending on the airfare $p$, denoted by $D_B(p)(\geq 0)$ and $D_L(p)(\geq 0)$, by simultaneously solving these two equilibrium conditions for $q_B$ and $q_L$. 

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The comparative statics of demand functions $D_B$ and $D_L$, average congestion costs $C$ as well as full prices $\rho_B$ and $\rho_L$ with regard to the ticket price $p$ are also non-trivial:

**Lemma 1** The sign of $D'_B$ is ambiguous but

$$D'_L < 0, D'_B + D'_L < 0 \quad \text{and} \quad \frac{dC(D_B(p) + D_L(p))}{dp} < 0. \quad (7)$$

Furthermore, the sign of

$$\frac{d\rho_B(D_B(p), D_L(p))}{dp} \quad (8)$$

is ambiguous but

$$\frac{d\rho_L(D_B(p), D_L(p))}{dp} > 0. \quad (9)$$

**Proof** See Appendix A.

Lemma 1 shows that the effect of the airfare on the total number of passengers is clear-cut and negative. Hence, an increase in $p$ will reduce the total number of passengers and, consequently, also average congestion, $C$. For this reason, the effect of a change in $p$ on quantities $q_x$ and full prices $\rho_x$, which are composed of the airfare and average congestion costs, is ambiguous in principal. It turns out however that, in our setting, the ambiguity exists only for the case of business passengers. In contrast, the relationship between the airfare and the number of leisure passengers is negative while the relationship between the airfare and the full price for leisure passengers is positive. These comparative-static results are intuitive because of the greater relative value of time of business passengers.
4 First-best Passenger Numbers and Airfares

In this section, we explore the first-best (‘welfare optimal’) passenger numbers and airfares. The first-best passenger numbers are

$$ (q_B^*, q_L^*) = \arg \max_{q_B, q_L} W(q_B, q_L), $$

(10)

and the associated first-order conditions are

$$ \frac{\partial W}{\partial q_x} = B'_x - v_x C(q) - (q_B^* v_B + q_L^* v_L) C' = 0. $$

(11)

The first two terms on the right-hand side of (11) give the inverse demand of passengers of type $x$ with regard to the airfare $p$ in (1). The last term gives the part of the marginal congestion costs that is uninternalized by passengers. The second partial derivatives of welfare in (4) are

$$ \frac{\partial^2 W}{\partial q_x^2} = B''_x - 2v_x C' - (q_B^* v_B + q_L^* v_L) C'' < 0 $$

(12)

for all $x \in \{B, L\}$. These second partial derivatives are negative, which are necessary conditions for a welfare maximum. Furthermore, we assume that the Hessian of the welfare function, denoted by $\Omega$, is strictly positive to ensure the existence of solutions for welfare maximization.

Substituting $D_x(p)$ for $q_x$ in (4) gives welfare as a function of the airfare $p$, $W(D_B(p), D_L(p))$. The welfare optimal airfare is

$$ p^* = \arg \max_{p} W(D_B(p), D_L(p)), $$

(13)
and the associated first-order condition is

\[
\frac{dW}{dp} = \sum_x (B'_x - v_x C - (D_B v_B + D_L v_L) C') D'_x = 0. \tag{14}
\]

Observe that this condition includes the two first-order conditions in (11), which are multiplied by \( D'_x \).

**Proposition 1** In a context with business passengers who have a greater value of time than leisure passengers, the welfare optimal airfare fully internalizes marginal congestion costs and yields first-best passenger numbers.

**Proof** Using inverse demand with regard to the airfare \( p \) in (1) and substituting \( P_B \) and \( P_L \) by \( p \), this first-order condition in (14) can be rewritten as

\[
\left( p^* - (D_B(p^*)v_B + D_L(p^*)v_L)C' \right) \cdot (D'_B + D'_L) = 0. \tag{15}
\]

Since \( D'_B + D'_L < 0 \) holds true (see Lemma 1),

\[
p^* = (D_B(p^*)v_B + D_L(p^*)v_L)C' \tag{16}
\]

directly follows. The welfare-optimal airfare, \( p^* \), therefore leads to a full internalization of marginal congestion costs to business and leisure passengers. Moreover, it leads to the first-best passenger numbers because \( p^* = P_B(D_B(p^*), D_L(p^*)) = P_L(D_B(p^*), D_L(p^*)) \) satisfies the first-order conditions in (11).

This proposition shows that in the presence of passengers with different values of time a single airfare, which fully internalizes marginal congestion costs, is required to reach the welfare optimal passenger numbers. We turn to the relationship between the airfare, passenger numbers and the airport charge in the next section.
5 Airport Charges Under the Airport-Airline Vertical Structure

The airport does not directly control airfares but the airport charge. This is because airfares are determined by airlines, which take the airport charge as given. What is the welfare optimal airport charge in the two-stage game considered here, which includes a vertical structure with $n$ airlines in Cournot competition?

5.1 Airline behavior in the second stage

We determine the welfare optimal airport charge by backward induction and start by determining airline profits. For the total airline supply it holds

\[
\sum_x \sum_i q_{ix} = q. \tag{17}
\]

Substituting the sum on the left-hand side by the total equilibrium demand $D_B(p) + D_L(p)$ and solving for $p$ gives the equilibrium airfare, which depends on the total amount of passengers and is denoted by $P(q)$. After substituting $P(q)$ for $P_B(q_B, q_L)$ and $P_L(q_B, q_L)$, we can rewrite airline profits in (3) by

\[
\Pi_i = q_i [P(q) - \tau] \tag{18}
\]

with $q_i = q_{iB} + q_{iL}$. Expression (18) gives airline profits depending on airline quantities $q_1, q_2, \ldots, q_n$. 


In the second stage, each airline chooses quantity $q_i$ to maximize its profit. The corresponding best responses to the other airlines’ quantities $q_j$ with $j \neq i$, indicated by $r$, are

$$q^*_i = \arg \max_{q_i} \Pi_i.$$  \hspace{1cm} (19)

The associated first-order conditions are

$$q^*_i \frac{\partial P}{\partial q_i} + P - \tau = 0.$$  \hspace{1cm} (20)

The first two terms give the marginal revenue of passengers to airlines. The relationship between this marginal revenue and the number of passengers, $q_i$, is ambiguous; however, we concentrate on those cases where a negative relationship exists. The last term is determined by the airport charge, $\tau$.

Simultaneously solving the conditions in (20) for airline quantities, $q^*_i$, gives the airline supply in the Cournot-Nash equilibrium, which depends on the airport charge $\tau$ and is denoted by $q^N_i(\tau)$. Let $q^N(\tau)$ denote the total supply in equilibrium depending on $\tau$ with $q^N(\tau) = q^N_1(\tau) + \ldots + q^N_n(\tau)$. This in turn gives the airfare $P(q^N(\tau))$, the number of business passengers $D_B(P(q^N(\tau)))$, and the number of leisure passengers $D_L(P(q^N(\tau)))$ in the Cournot-Nash equilibrium all of which depend in the second stage on $\tau$. We find the following comparative static results between the airport charge, the airfare, passenger numbers and full prices.

**Lemma 2** With a given negative relationship between the marginal revenue of passengers to airlines and $q_i$ (i.e., $d[q_i(\partial P/\partial q_i) + P]/dq_i < 0$), the signs of

$$\frac{dq^N_i(\tau)}{d\tau} < 0, \frac{dP}{d\tau} > 0, \frac{dD_B}{d\tau} < 0, \frac{dD_L}{d\tau} > 0$$  \hspace{1cm} (21)
are clear-cut but the signs of

\[
\frac{dD_B}{d\tau} \quad \text{and} \quad \frac{d\rho_B}{d\tau}
\]

(22)

are ambiguous.

**Proof** See Appendix B.

Lemma 2 shows that a positive relationship between the airport charge \( \tau \) and the equilibrium airfare \( P(q^N(\tau)) \) exists. The other relationships mentioned directly follow from this result together with the results obtained in Lemma 1.

Substituting \( D_x(P(q^N(\tau))) \) for \( q_x \) in (4) gives the welfare depending on \( \tau \), \( W(D_B(P(q^N(\tau))), D_L(P(q^N(\tau)))) \). We need this expression to analyze the optimal airport charge.

### 5.2 The optimal airport charge in the first stage

The welfare optimal airport charge is

\[
\tau^* = \arg \max_{\tau} W(D_B(P(q^N(\tau))), D_L(P(q^N(\tau)))).
\]

(23)

The associated first-order condition is

\[
\frac{dW}{d\tau} = \frac{dW}{dp} \cdot \frac{dP}{d\tau} = 0.
\]

(24)

This condition is composed of the first derivative of welfare with regard to the airfare, which is shown as the left-hand side of (14), i.e., the first-order condition for the optimal airfare \( p^* \), times the first derivative of the airfare \( P \) with respect to \( \tau \).
**Proposition 2** The optimal airport charge, $\tau^*$, leads to the social-optimal airfare, $p^*$, and the first-best optimal passenger numbers, $(q^*_B, q^*_L)$.

**Proof** Since $dP/d\tau > 0$, the first-order condition in (24) is satisfied if and only if the first-order condition for the optimal airfare in (14) is satisfied. Hence, the optimal airport charge, $\tau^*$, implies the social-optimal airfare, $p^*$, and social-optimal passenger numbers $(q^*_B, q^*_L)$.

Substituting $p^*$ for $P$ and $q^*_i$ for $q^*_i$ in (20) and rearranging leads to the optimal airport charge

$$\tau^* = p^* + q^*_i \frac{\partial P}{\partial q_i} = (q^*_B v_B + q^*_L v_L) C' + q^*_i \frac{\partial P}{\partial q}.$$  

(25)

The first part of the optimal airport charge is determined by the optimal airfare $p^*$, which is positive. Note that $p^*$ equals the optimal airport charge in a situation with an atomistic airline market where airline mark-ups are zero. The second part is negative and difficult to interpret. We rewrite the optimal airport charge in (25) to obtain a more transparent picture about the effect of passenger types, business and leisure, on the optimal airport charge.

Substituting $P_x(q_B, q_L)$ for $p$ in (5) and rearranging gives

$$P_x(q_B, q_L) = \rho_x(q_B, q_L) - v_x C(q_B, q_L)$$

(26)

with

$$\frac{dP_x}{dq_i} = \frac{\partial \rho_x}{\partial q} \frac{dq_x}{dq_i} - v_x C'.$$  

(27)
Note that $P_B(q_B, q_L) = P_L(q_B, q_L) = P(q)$ implies $\partial P/\partial q = dP_x/dq_i$. Then, substituting $dP_x/dq_i$ in (27) for $\partial P/\partial q$ in (25) yields

$$\tau^* = (q^*_B v_B + q^*_L v_L) C' + q^N_i \left( \frac{\partial \rho_x}{\partial q_x} \frac{dq_x}{dq_i} - v_x C' \right).$$  (28)

Letting $\varepsilon_x = -(dq_x/d\rho_x)(\rho_x/q_x)$ denote the (positive) demand elasticity with respect to the full price and noting that the market share of airline $i$ is $q_i^N/(q^*_B + q^*_L) = 1/n$, the optimal airport charge in (28) can be rewritten as

$$\tau^* = (q^*_B v_B + q^*_L v_L - q^N_i v_x) C' - \frac{q^N_i \rho_x}{q^*_x} \frac{dq_x}{dq_i} \varepsilon_x$$

$$= \left( 1 - \frac{1}{n} \frac{(q^*_B + q^*_L)v_x}{q^*_B v_B + q^*_L v_L} \right) (q^*_B v_B + q^*_L v_L) C' - \frac{1}{n} \frac{q^*_B + q^*_L}{q^*_x} \frac{dq_x}{dq_i} \frac{\rho_x}{\varepsilon_x}. \quad (29)$$

To identify the effect of passenger type on $\tau^*$, let us assume, for a moment, that only passengers of type $x$ exist (we use subscript $s$ to indicate the case of a single passenger group). In this situation, the optimal airport charge in (29) reduces to

$$\tau^*_s = \left( 1 - \frac{1}{n} \right) v_x q^*_x C' - \frac{1}{n} \frac{\rho_x}{\varepsilon_x}, \quad (30)$$

which reproduces the results obtained by Zhang and Zhang (2006). Hence, $\tau^*_s$ depends on market shares, $1/n$, and the elasticity of demand with respect to the full price $\rho_x, \varepsilon_x$. Thus, the airport-charge formula in (29) extends (30) to the case of two passenger types. Observe that there is an upper limit for $\tau^*_s$ in (30), which is determined by the first (positive) term on the right-hand side of (30). This upper limit is reached for $\varepsilon_x \to \infty$ and determines the part of the marginal congestion costs that, in a situation with $\varepsilon_x \to \infty$, would not be internalized by airlines and passengers. We call this part the **external part of the marginal congestion costs** in the remainder of this paper. Observe that this part depends on market shares as was pointed out earlier.
by, e.g., Daniel (1995), Brueckner (2002), Pels and Verhoef (2004), Zhang and Zhang (2006), and Basso and Zhang (2007) and Brueckner and van Dender (2008).\footnote{In particular, Brueckner and van Dender (2008), Brueckner (2009), and Basso and Zhang (2009) consider models with perfectly elastic passenger demands (i.e. $\epsilon_x \to \infty$); consequently, the second term on the right-hand side of (30) vanishes.} However, since elasticities are finite in reality, the optimal airport charge in (30) is supposed to be lower than the external part of the marginal congestion costs; this is to offset airline mark-ups and to reach the social-optimal passenger numbers as was shown by Pels and Verhoef (2004). In our context, this is shown by the second (negative) term on the right-hand side of (30). The picture changes substantially in a context where passenger types with different values of time exist:

**Proposition 3** In a context with business passengers who have a greater value of time than leisure passengers, $\tau^*$ can exceed the external part of the marginal congestion costs but will never reach the level of atomistic airport charges.

**Proof** To demonstrate that $\tau^*$ can exceed the external part of the marginal congestion costs, we first consider $x = L$. Note that $dq_L/dq_i > 0$ holds true because $D'_L < 0$ and $D'_L + D'_B < 0$ (see Lemma 1). This gives

\[
\frac{(q_B^* + q_L^*)v_L}{q_B^*v_B + q_L^*v_L} \leq 1 \tag{31}
\]

and

\[
\frac{1}{n} \frac{q_B^* + q_L^*}{q_x^*} \frac{dq_L}{dq_i} \frac{\rho_L}{\rho} \frac{\varepsilon_L}{\varepsilon} > 0. \tag{32}
\]

Equation (31) implies directly that the external part of the marginal congestion costs imposes no upper limit for $\tau^*$ when passenger types with different values of time exist. The same can be shown for $x = B$. Note that the sign of
\( dq_B/dq_i > 0 \) is ambiguous because the sign of \( D_B' \) is ambiguous (see Lemma 1). Therefore, the sign of the second term on the right-hand side of (29) is also ambiguous (in contrast to the sign of (32)), which shows again that the external part of the marginal congestion costs imposes no upper limit for the welfare optimal airport charge. Finally, \( p^* \), which is the optimal airport charge in a situation with an atomistic airline market, is an upper limit for \( \tau^* \) because the second term in (25) is negative.

Thus, the existence of passenger types with different values of time has significant consequences for the choice of airport charges. This is because with different passenger types it can be useful to charge oligopolistic airlines with a price that exceeds the level of the external part of the marginal congestion costs. This is in sharp contrast to the notion of Pels and Verhoef (2004) who argued that the welfare optimal airport charge is always lower than the external part of the marginal congestion costs when airline markets are oligopolistic. The intuition for our result is that greater airfares may be required to protect business passengers (with a great relative value of time) from congestion caused by leisure passengers who are less sensitive with regard to congestion than business passengers.

5.3 Comparative static results

We are also interested in the relationship between various optimal variables, the optimal numbers of business and leisure passengers, optimal airfares, full prices, optimal airport charges and the business passengers’ benefits or their value of time. For this reason, we introduce parameter \( a(> 0) \) with

\[
\frac{\partial B_B'(q_B)}{\partial a} \geq 0 
\]

(33)
for all \( q_B > 0 \). Recall that \( B' \) is the business passengers’ inverse demand with respect to the full price \( \rho_B \). Hence, \( a \) establishes a positive relationship with these inverse demands. We obtain the following comparative static results:

**Proposition 4** In a context with business passengers who have a greater value of time than leisure passengers, the following clear-cut relationships exist:

\[
\frac{dq_B^*}{da} > 0, \quad \frac{dq_L^*}{da} > 0, \quad \frac{d\rho_L(q_B^*, q_L^*)}{da} < 0, \quad (34)
\]

\[
\frac{dq_B^*}{dv_B} < 0, \quad \frac{dq_L^*}{dv_B} > 0, \quad \frac{d\rho_B(q_B^*, q_L^*)}{dv_B} > 0 \quad \text{and} \quad \frac{d\rho_L(q_B^*, q_L^*)}{dv_B} < 0. \quad (35)
\]

In contrast, the relationship

\[
\frac{d\rho_B(q_B^*, q_L^*)}{da} \quad (36)
\]

is ambiguous, and the relationships

\[
\frac{d(q_B^* + q_L^*)}{dz}, \quad \frac{dp^*}{dz} \quad \text{and} \quad \frac{d\tau^*}{dz} \quad (37)
\]

are also all ambiguous in sign for all \( z \in \{a, v_B\} \).

**Proof** See Appendix C.

The comparative static results with regard to \( a \) in (34) show that a positive relationship between \( a \) and the optimal number of business passengers exists while this relationship is negative for leisure passengers. The latter negative relationship implies a positive relationship between \( a \) and the full price \( \rho_L(q_B^*, q_L^*) \). The relationship between \( a \) and \( \rho_B(q_B^*, q_L^*) \) is, in contrast, unclear because an increase in \( q_B^* \) is consistent with both an increase or a decrease in the business passengers’ full price when \( a \) increases.
On the other hand, the comparative static results with regard to the business passengers’ value of time, $v_B$, in (34) show that a negative relationship between $v_B$ and the number of business passengers, $q_B^*$, exists while this relationship is positive in the case of leisure passengers. This implies that the business passengers’ full price, $\rho_B$, increases in $v_B$ and the one for leisure passengers, $\rho_L$, decreases in $v_B$.

The comparative static results in (37) are ambiguous; hence, there is no clear-cut relationship between $a$ or $v_B$ and the total number of passengers, $q_B^* + q_L^*$, the optimal airfare, $p^*$, and the optimal airport charge, $\tau^*$. Take the relationship between $\tau^*$ and $v_B$ as an example. For low values of $v_B$ (but still greater than $v_L$) an increase of $\tau$ can be useful in protecting business passengers from excessive congestion. The picture changes for great values of $v_B$. In that situation, it may not be useful anymore to protect business passengers from congestion because their benefits from flying net of congestion costs are too small, which altogether provides an economic intuition for the ambiguous relationships in (37). In the next section, we present numerical simulations to illustrate the finding of the previous sections.

6 Numerical Simulations

In this section, we first present a specific model on which our numerical simulations will be based. Second, we present numerical results.

Assume that passenger benefits are

$$B_B(q_B) = aq_B - \frac{q_B^2}{2} \quad \text{and} \quad B_L(q_L) = q_L - \frac{b q_L^2}{2} \quad (38)$$
with
\[ a \in \left[ \frac{1 + v_B}{2 + b}, \frac{1 + 2v_B}{1 + v_B} \right] \]
and \( b \geq 0 \). The lower limit for \( a \) ensures that the number of business passengers is non-negative while the upper limit ensures that the number of leisure passengers is non-negative. For the leisure passengers’ value of time it holds \( v_L = 1 \) and the business passengers’ value of time is
\[ v_B \in [a(2 + b) - 1, \min\{a(2 + b) - 1, 1 + b + \sqrt{2 + 3b + b^2}\}] \text{.} \]
The lower limit for \( v_B \) ensures that the number of leisure passengers is non-negative. The upper limit ensures that the number of business passengers is non-negative (this is the case if \( \min\{\cdot\} \) equals the first expression) or that the second-order condition for the welfare optimal passenger numbers are satisfied (this is the case if \( \min\{\cdot\} \) equals the second expression). Average delays are
\[ C(q_B, q_L) = q_B + q_L \text{.} \]
Furthermore, we denote the upper limit for the external part of the marginal congestion costs by
\[ \Gamma^* = (v_B q_B^* + q_L^*) \text{,} \]
which equals the optimal airport charge in a context with an atomistic airline market (see equation (29)).

Benefits in (38) and average delays in (41) imply first-best passenger numbers
\[ q_B^* = \frac{2a + ab - (v_B + 1)}{2bv_B + b - v_B^2 + 2v_B + 1} \quad \text{and} \quad q_L^* = \frac{1 + 2v_B - a(v_B + 1)}{2bv_B + b - v_B^2 + 2v_B + 1} \text{.} \]
and airfares

\[ p^* = \frac{abv_B + av_B - a - v_B^2 + v_B + 1}{2bv_B + b - v_B^2 + 2v_B + 1}. \]  

(44)

The optimal airport charge \( \tau^* \) is

\[ \tau^* = 1 + \frac{1}{n} \left( \frac{(n + 1)((ab + a - 2b - 1)v_B - a - b)}{2(b + 1)v_B + b - v_B^2 + 1} - \frac{b(a - 1)}{b + 1} \right) \]  

(45)

and depends on \( n \).

The results obtained in the recent literature indicate that the ratio of the optimal airport charge in an oligopolistic airline market with the optimal one in an atomistic airline market should be inversely related with an airline’s market share, i.e. a negative relationship between an airline’s market share and the optimal airport charge should exist (Daniel 1995; Brueckner 2002; Pels and Verhoef 2004; Zhang and Zhang 2006; Basso and Zhang 2007; Brueckner and van Dender 2008). Moreover, Pels and Verhoef (2004) pointed out that it can be useful to further decrease the degree of internalization so as to correct for market power and price mark-ups in an oligopolistic airline market. We use Figure 1 to illustrate that the existence of passengers with different values of time can change the picture and move the optimal airport charges in an oligopolistic airline market scenario considerably closer to the level that would hold under atomistic market conditions. Note that this result may provide some theoretical support for the empirical findings of Morrison and Whinston (2008), who conclude that, from a social viewpoint, atomistic airport charges may be better than airport charges that correct for market shares.\(^3\) Another explanation is the existence of a dominant airline that competes in a Stackelberg fashion with a large number of fringe carriers. The Stackelberg setting seems however

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\(^3\)This result may not be true in a Stackelberg setting as was shown by Brueckner and van Dender (2008).

\(^4\)Another explanation is the existence of a dominant airline that competes in a Stackelberg fashion with a large number of fringe carriers. The Stackelberg setting seems however
Figure 1: The ratios of the optimal airport charge and the one in a situation with an atomistic airline market, $\tau^*/\Gamma^*$. The solid horizontal line equals $2/3$, which equals the total market share of 2 firms. Parameters: $a = 15/10$ (solid line) or $a = 16/10$ (dashed line) and $n = 3$. Functions are shown for the relevant ranges of $v_B$.

Figure 2: The optimal passengers numbers, $q_B^*$ and $q_L^*$, and the optimal total passenger number, $q_B^* + q_L^*$. Parameters: $a = 15/10$ (solid line) or $a = 16/10$ (dashed line) and $n = 3$. Functions are shown for the relevant ranges of $v_B$. 
Figure 1 depicts the ratios of the optimal airport charge and the one in a situation with an atomistic airline market, $\tau^*/\Gamma^*$. The solid horizontal line equals $2/3$, which equals the total market share of 2 firms. Parameters are $a = 15/10$ (the solid line) or $a = 16/10$ (the dashed line) and $n = 3$. Functions are shown for the relevant ranges of $v_B$, which are determined by (40). We know from the previous literature that, with a single group of passengers, the ratio of the optimal airport charge, $\tau^*$, and the one in an atomistic setting, $\Gamma^*$, should be less than the total market share of $n - 1$ airlines (see equation (30)), which is equal to $2/3$ for the current instance. Figure 1 reproduces this result because $\tau^*/\Gamma^* < 2/3$ for $v_B = 1$ (recall that $v_L = 1$). However, the figure also shows that when $v_B$ increases, the value of $\tau^*/\Gamma^*$ may also increase and exceed the level of $2/3$ substantially. This effect is even stronger for a larger value of $a$ (compare the solid line with $a = 15/10$ with the dashed line with $a = 16/10$).

Figure 2 depicts the optimal passengers numbers, $q_B^*$ and $q_L^*$, and the optimal total passenger number, $q_B^* + q_L^*$, for the same parameter constellations as the ones that we used to plot Figure 1. It shows that the optimal number of business passengers is decreasing in $v_B$ while the optimal number of leisure passengers is increasing in $v_B$. Observe that the relationship between the total number of passengers and $v_B$ is ambiguous. Altogether, these graphs illustrate some of our findings in Proposition 4.

7 Conclusions

We have considered a model with a monopolistic and congested public airport together with an oligopolistic airline market. Airlines served two types of rather restrictive compared to the assumption of passenger types with different values of time.
passengers, which included business passengers with a great relative value of
time and leisure passengers with a low relative value of time. All passengers
are charged with a single ticket price. We demonstrated that the existence of
passenger types with different values of time can have a substantial impact
on the choice of airport charges. In this situation, it can be useful to increase
airport charges and thus airfares to protect business passengers from excessive
congestion caused by leisure passengers who are less sensitive with regard to
congestion. As a consequence, the welfare optimal airport charge can exceed
the level of the external part of the marginal congestion costs.

There are two critical elements in our model. One is that airlines charge
all passengers with a single ticket price. In practice, airlines may try to
price discriminate between business and leisure passengers. One way is to
increase airfares when departure times get close. This can be an effective way
to charge business passengers with a higher airfare than leisure passengers
because leisure passengers often book their flights far in advance, which is not
necessarily true for business passengers.\textsuperscript{5} In this paper, we have abstracted
away from airline price discrimination to concentrate on the differences in
passenger types with regard to their value of time and the consequences
for airport congestion pricing. Moreover, airline pricing strategies may not
always be successful in practice, and it is therefore useful to understand the
effects of passenger types on airport congestion pricing in a context without
airline price discrimination. It would however be interesting to consider the
effect of passenger types together with airline price discrimination on optimal
airport charges in a future research project.

Another critical element is the consideration of only two values of time.
In practice, many more passenger types with different values of time are

\textsuperscript{5}Hazledine (2006) argued that airline price discrimination would be based on an inverse
relationship between passengers’ benefits, and the time left before departure time.
supposed to exist. It would therefore be useful to analyze a model that is more general with regard to the distribution of values of time. Overall, the research provided in this paper captures the effect of passenger types on airport congestion pricing in a simple model with two types, which has not yet been explored by other economists. However, future research is required to obtain a more complete picture of the role of passenger types.
A Proof of Lemma 1

Substituting $p$ for $P_x$ in (1) and rearranging gives the two equilibrium conditions

$$p - B'_x + v_x C = 0.$$  \hfill (46)

The partial derivatives of these conditions with regard to $q_B$ and $q_L$ give the determinant

$$\Psi = \det \begin{pmatrix}
\frac{\partial (p - B'_B + v_B C)}{\partial q_B} & \frac{\partial (p - B'_B + v_B C)}{\partial q_L} \\
\frac{\partial (p - B'_L + v_L C)}{\partial q_B} & \frac{\partial (p - B'_L + v_L C)}{\partial q_L}
\end{pmatrix}
= B''_B B''_L - C'(v_L B''_B + v_B B''_L) > 0. \hfill (47)

Using Cramer’s rule to solve for $dq_B/dp$ gives

$$\frac{dq_B}{dp} = \frac{1}{\Psi} \cdot \det \begin{pmatrix}
\frac{\partial (p - B'_B + v_B C)}{dp} & \frac{\partial (p - B'_B + v_B C)}{\partial q_L} \\
\frac{\partial (p - B'_L + v_L C)}{dp} & \frac{\partial (p - B'_L + v_L C)}{\partial q_L}
\end{pmatrix}
= \frac{1}{\Psi} \cdot \left( B''_L + (v_B - v_L) C' \right) < 0. \hfill (48)

The sign of the numerator in (48) is ambiguous and hence the relationship between the business passenger demand and airfare is also ambiguous. In contrast, using Cramer’s rule to solve for $dq_L/dp$ gives a clear-cut result:

$$\frac{dq_L}{dp} = \frac{1}{\Psi} \cdot (B''_B - (v_B - v_L) C') < 0. \hfill (49)$$
Summing (48) and (49) also yields a clear-cut result:

\[ \frac{d(q_B + q_L)}{dp} = \frac{1}{\Psi} \cdot (B_L'' + B_B'') < 0. \tag{50} \]

The rest of comparative-static results can be shown in similar fashion.

B Proof of Lemma 2

To satisfy the first-order condition in (20), \( q_i \) must be reduced in response to an increase of \( \tau \) if a negative relationship between the marginal revenue of passengers, \( q_i (\partial P/\partial q) + P \), and \( q_i \) exists. In this situation, \( dq^N/d\tau \) directly follows. Furthermore, this implies a positive relationship between the airfare and the airport charge, \( dP/d\tau > 0 \), due to Lemma 1, which shows that a negative relationship between \( p \) and the total number of passengers in equilibrium, \( D_B + D_L \), exists. From Lemma 1, it also follows that there is a negative relationship between \( \tau \) and \( q_L \), which implies a positive relationship between \( \tau \) and \( \rho_L(q_B^N, q_L^N) \), because the relationship between \( p \) and \( D_L \) is negative. On the other hand, the relationship between \( \tau \) and \( q_B^N \) or \( \rho_B(q_B^N, q_L^N) \) is ambiguous because the relationship between \( p \) and \( D_B \) is ambiguous.

C Proof of Proposition 4

To determine the signs of \( dq^*_x/dz \), \( dp^*/dz \), and \( d\tau^*/dz \) with \( z \in \{a, v_B\} \), the first step is to determine the second partial derivatives of welfare in function (4), which give

\[ \frac{\partial^2 W}{\partial q^2_x} = B_x'' - 2v_x C' - (q_B v_B + q_L v_L) C' < 0 \tag{51} \]
and
\[ \frac{\partial^2 W}{\partial q_B \partial q_L} = -v_B C' - v_L C' - (q_B v_B + q_L v_L) C'' < 0. \tag{52} \]

We also need the second partial derivatives of welfare in (4) with respect to \( q_x \) and \( z \), which give
\[ \frac{\partial^2 W}{\partial q_B \partial a} > 0, \quad \frac{\partial^2 W}{\partial q_L \partial a} = 0, \quad \frac{\partial^2 W}{\partial q_B \partial v_B} = -C - q_B v_B C' < 0, \tag{53} \]
and
\[ \frac{\partial^2 W}{\partial q_L \partial v_B} = 0. \tag{54} \]

Using Cramer’s rule to solve for \( dq_B^*/da \) and recalling that \( \Omega > 0 \) (by assumption) yields

\[
\frac{dq_B^*}{da} = \frac{1}{\Omega} \det \begin{pmatrix}
\frac{\partial^2 W}{\partial q_B \partial a} & \frac{\partial^2 W}{\partial q_B \partial q_L} \\
\frac{\partial^2 W}{\partial q_L \partial a} & \frac{\partial^2 W}{\partial q_L \partial v_B}
\end{pmatrix} = \frac{1}{\Omega} \left( -\frac{\partial B'_B}{\partial a} [B'_L - 2v_L C' - (q_B v_B + q_L v_L) C''] \right) > 0, \tag{55} \]

and solving for \( dq_L^*/da \) yields
\[
\frac{dq_L^*}{da} = \frac{1}{\Omega} \left( -\frac{\partial B'_L}{\partial a} [(v_B + v_L) C' + (q_B v_B + q_L v_L) C''] \right) < 0. \tag{56} \]

Since \( a \) increases the business passengers’ inverse demand with regard to the full price, the relationship between \( a \) and \( \rho_B(q_B^*, q_L^*) \) remains unclear although a positive relationship between \( a \) and \( q_B^* \) exists. On the other hand, from (56), \( d\rho_L(q_B^*, q_L^*)/da > 0 \) directly follows. Summing (55) and (56) yields
\[
\frac{d(q_B^* + q_L^*)}{da} = \frac{1}{\Omega} \left( -\frac{\partial B'_B}{\partial a} \right) (B'_L + (v_B - v_L) C'). \tag{57} \]
Since $v_B \geq v_L$, (57) is ambiguous in sign.

Solving for $dq_B^*/dv_B$ yields

$$\frac{dq_B^*}{dv_B} = (C + q_B C') \left[ B''_L - 2v_L C' - (q_B v_B + q_L v_L) C'' \right] < 0, \tag{58}$$

and solving for $dq_L^*/v_B$ yields

$$\frac{dq_L^*}{dv_B} = - \left[ -(v_B + v_L) C' - (q_B v_B + q_L v_L) C'' \right] (C + q_B C') > 0. \tag{59}$$

From (58) and (59), it follows $d\rho_B(q_B^*, q_L^*)/dv_B > 0$ and $d\rho_L(q_B^*, q_L^*)/dv_B < 0$.

Furthermore, summing (58) and (59) yields

$$d(q_B^* + q_L^*)/dv_B = \frac{1}{\Omega} \left( B''_L + (v_B - v_L) C' \right) (C + q_B C'), \tag{60}$$

which is ambiguous in sign.

We turn to airfares and airport charges. For the optimal airfare $p^*$ in (16), it holds

$$\frac{dp^*}{da} = \left( \frac{dq_B^*}{da} v_B + \frac{dq_L^*}{da} v_L \right) C'' \cdot \frac{d(q_B^* + q_L^*)}{da}, \tag{61}$$

and

$$\frac{dp^*}{dv_B} = \left( \frac{dq_B^*}{dv_B} v_B + q_B^* + \frac{dq_L^*}{dv_B} v_L \right) C'' \cdot \frac{d(q_B^* + q_L^*)}{dv_B}. \tag{62}$$

Both relationships in (61) and (62) are ambiguous because the signs of $d(q_B^* + q_L^*)/dz$ are ambiguous for all $z \in \{a, v_B\}$. Since the optimal airport charge $\tau^*$ in (25) depends on $p^*$, the sign of $d\tau^*/dz$ is also ambiguous for all $z \in \{a, v_B\}$.
References


