Pre-trip Information and Route-Choice Decisions with Stochastic Travel Conditions: Theory

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**Key words:** Advanced Traveler Information Systems; incidents; congestion; road capacity; route choice; pre-trip information

**JEL codes:** D62, R41

May 5, 2014

Abstract

This paper studies the effects of pre-trip information on route-choice decisions when travel conditions are congested and stochastic. We adopt a model based on the classical two-route network in which free-flow travel times and/or capacities on each route vary unpredictably due to such shocks as bad weather, accidents, and special events. We show that the benefits of information depend on differences between routes in free-flow costs, the shape of the travel cost functions, the severity of congestion and capacity shocks, and the degree of correlation between routes in travel conditions. Information is more likely to be welfare-reducing when free-flow travel costs differ appreciably, travel cost functions are convex, shocks are similar in size on the routes, and route conditions are strongly and positively correlated.
1. Introduction

Traffic congestion imposes a high cost. In its 2012 Annual Urban Mobility Report, the Texas Transportation Institute estimates that in 2011 congestion in US urban areas imposed on motorists approximately 5.5 billion hours of travel delay and 2.9 billion gallons of extra fuel consumption with an estimated total cost of $121 billion (Schrank et al., 2012). Much of this congestion delay is due to unpredictable events such as bad weather, incidents, unannounced road work, malfunctioning traffic control devices, and special events. By one estimate, incidents (i.e., vehicle collisions and breakdowns) alone contribute some 52 - 58 percent of total delay in US urban areas (Schrank et al., 2011, Appendix B, p. B-27). Using Dutch data, Adler et al. (2013) show that the costs of incidents increase steeply with their duration, and they are particularly high at locations with high levels of recurrent congestion.

One way to mitigate the costs of nonrecurring congestion is to provide motorists with information about travel conditions. For decades, travel information has been available from newspapers, radio, television, and variable message signs. More recently, these media have been supplemented by Advanced Traveler Information Systems (ATIS) that compile information from various sources and convey it via traffic websites (e.g., waze.com), GPS devices, e-mail, mobile phones, 511 phone systems (in the US), and Personal Intelligent Travel Assistants. Motorists can use this information to adjust their trip destinations, departure times, route choices, parking locations, and other choices. The focus of this paper will be on the effects of information on route choice given a fixed number of vehicle trips.

The fact that substantial resources are devoted to collecting, processing, and communicating traffic information suggests that it is of private value to motorists. But because individual motorists do not internalize the costs of congestion delay and other external costs they impose on other motorists, the private cost of a trip that they face is less than the social cost. The world of second best applies and it is not a priori clear that more, or more precise, information is socially valuable. As a consequence, both the policy determination and management of traffic systems depend on better understanding of the relationship between the availability of travel information and congestion.

The effects of information on travel-related decisions have been examined in many analytical and simulation studies; see Chorus et al. (2006) and de Palma et al. (2012) for reviews. A
majority conclude that information is likely to be socially beneficial, although some studies identify conditions under which adverse responses occur. In theory, information can adversely affect route choice, departure time, and other decisions in three ways: concentration, overreaction, and oversaturation. *Concentration* occurs when many travelers make similar choices that exacerbate congestion. Concentration is an equilibrium phenomenon in the sense that individual travelers do not regret the choices they make. *Overreaction* occurs when travelers fail to anticipate how other travelers will react, and collectively respond too much to new information; see Mahmassani and Jayakrishnan (1991) and Emmerink et al. (1995). Overreaction is out-of-equilibrium behavior since those who respond end up worse off. *Oversaturation* occurs if drivers are faced with too much information, and either fail to use it effectively or become overwhelmed and resort to heuristic decision rules (Ben-Akiva et al., 1991).

A number of economic studies have explored the conditions under which more extensive, or more accurate, information can be detrimental. Arnott et al. (1996) show that when the number of trips is determined endogenously, and demand and capacity are stochastic, information can be welfare-reducing if the travel demand and cost functions have certain shapes. Arnott et al. (1991, 1999) show that with endogenous departure times, providing imperfect information can be welfare-reducing relative to no information. Other studies have looked at the effects of information on route choice as we do here. One recurrent finding is that welfare is maximized when only a fraction of drivers are informed. For example, Mahmassani and Jayakrishnan (1991) and Emmerink et al. (1995) conclude that beyond a penetration rate of about 20 percent, information has negative effects due to concentration and overreaction. The critical penetration rate can depend on the type of information provided, and the degree of inertia in driver response; see Emmerink et al. (1994) for a review.

In this paper, we conduct a detailed investigation of the effects of pre-trip information on route-choice decisions and system efficiency. We use deterministic user equilibrium as the solution concept and therefore focus on the possible adverse effects of concentration rather than overreaction or oversaturation. We study the classical "two-route network" whose origins trace back to Pigou (1920) and Knight (1924). We take as a starting point studies by de Palma and Lindsey (1994), Verhoef et al. (1996), and Emmerink et al. (1998), who analyze two-route networks with linear link travel cost functions and investigate the effects of providing information to motorists when link capacities are stochastic. Our work goes further by
demonstrating how the benefits of information depend on differences between routes in free-flow costs, the shape of the travel cost functions, the severity of capacity shocks, and the degree of correlation between routes in travel conditions.

There are many other studies of information and route choice, but most of them use simulation methods rather than equilibrium analysis, and many consider en-route rather than pre-trip information. One exception is Liu et al. (2009) who analyze a general road network with links that vary stochastically in capacity from day to day. Like us, they adopt static user equilibrium as the solution concept and consider pre-trip information. Their analysis is also more general in that they consider imperfect as well as perfect information about travel conditions. However, they focus on worst-case travel times (i.e., the price of anarchy) rather than the benefits of information. Their main finding is that with imperfect information the worst-case inefficiency of user equilibrium depends only on the shape of the link time functions, whereas with perfect information it also depends on the probabilities and magnitudes of capacity reductions.

In this paper we concentrate on the welfare gains or losses from information rather than on worst-case properties. We do not consider imperfect information but, rather, focus on the polar regimes of zero information and full (i.e., perfect) information. In the zero-information regime, drivers only know the unconditional probability distribution of states on the two routes, while in the full-information regime they learn the states before choosing a route. We begin by deriving conditions under which information is welfare-improving in the sense of decreasing expected travel costs. We then derive conditions under which information is welfare-reducing so that an "information paradox" occurs. We show that the paradox is more likely to arise when free-flow travel costs differ appreciably, travel cost functions are convex, shocks are similar in size on the two routes, and conditions are perfectly and positively correlated. These conditions generalize and extend those identified in the earlier studies. A companion paper (Rapoport et al., 2013) tests the predictions of the model in a laboratory experiment.

In addition to deriving the effects of information provision on route-choice decisions, we investigate how information affects travel costs in particular states (i.e., specific travel conditions on the routes). We show that under reasonable assumptions information is always welfare-improving in at least one state. However, information is often welfare-reducing in at least one state, and it can be detrimental a large fraction of the time.
Section 2 lays out the model. Section 3 derives general results on the welfare effects of information in particular states as well as expected travel costs overall. Section 4 presents numerical results for a parameterized version of the model. Section 5 concludes with a summary and ideas for future research.

2. The model

We adopt the classical "two-route network" model in which a single origin is connected to a single destination by two routes. Drivers or users are treated as a continuum and their measure is fixed at $N > 0$. Fixed demand is a reasonable assumption for a commuting corridor if workers cannot telecommute or switch to public transit on short notice. Travel conditions on each route vary from day to day because of bad weather, accidents, roadwork, or other shocks. The set of possible travel conditions for the two routes combined is denoted by $S$, where $S$ can be continuous or discrete. The number of users who choose route $i$ in state $s \in S$ is denoted by $N_{is}$. Individual private travel cost on route $i$ in state $s$ is given by a function $C_i(N_{is})$. Therefore, total travel costs on the network in state $s$ are

$$TC_s = C_{1s}(N_{1s})N_{1s} + C_{2s}(N_{2s})N_{2s}, \quad (1)$$

The travel cost functions on the routes are assumed to satisfy the following monotone properties:

**Assumption 1:** (Monotone travel cost functions)

For every state $s \in S$, $C_i(N_{is})$ is an increasing function of $N_{is}$, $i = 1, 2$, and it is strictly increasing on at least one route.

Assumption 1 guarantees that user equilibrium with full information is unique.

2.1 User equilibrium

User equilibrium (UE) follows Wardrop's first principle with modifications to allow for uncertainty about travel costs on each route. Users are assumed to be risk-neutral with respect to travel costs so that they prefer the route with lower expected travel cost. Users are also assumed to adopt pure strategies so that they choose a route deterministically on a given day whether or
not they know the state. Unless indicated otherwise, attention is restricted to interior equilibria in which both routes are used. Since each user must choose one of the two routes,

\[ N_{1s} + N_{2s} = N, \quad \forall s \in S. \quad (2) \]

Two information regimes are considered: full information (F) and zero information (Z). In the full-information regime, users learn the state \( s \) before choosing a route. Let \( N_{is}^F \) denote the number of users on route \( i \) in state \( s \). The UE division of traffic between routes in state \( s \) is then determined by eqn. (2) and the condition that travel costs on the two routes are equal:

\[ C_{1s} \left( N_{1s}^F \right) = C_{2s} \left( N_{2s}^F \right) = C_{2s} \left( N - N_{1s}^F \right), \quad \forall s \in S. \quad (3) \]

In the zero-information regime, users only know the unconditional probability distribution of states. Let \( N_{i}^Z \) denote the number of users who choose route \( i \). The UE division of traffic is determined by the analogue to equation (2), where \( N_{1}^Z + N_{2}^Z = N \), and the condition that expected travel costs on the two routes are equal:

\[ E \left[ C_{1s} \left( N_{1}^Z \right) \right] = E \left[ C_{2s} \left( N_{2}^Z \right) \right] = E \left[ C_{2s} \left( N - N_{1}^Z \right) \right], \quad (4) \]

where \( E[\ ] \) is the expectations operator.

Social welfare is given by expected users' surplus; given fixed \( N \), it can be measured by the negative of expected total costs. Let \( TC_{s}^r \) denote total costs in state \( s \) and regime \( r \), \( r = Z, F \). The welfare gained in state \( s \) by shifting from zero information to full information, \( G_{s}^{ZF} \), is therefore \( G_{s}^{ZF} = TC_{s}^{Z} - TC_{s}^{F} \). Let \( E \left[ TC_{s}^{r} \right] \) denote expected total costs in regime \( r \). The welfare gained by shifting from zero information to full information, \( G_{s}^{ZF} \), is then \( G_{s}^{ZF} = E \left[ TC_{s}^{Z} \right] - E \left[ TC_{s}^{F} \right] \) where expectations are taken over states. One of the main goals of the paper is to sign \( G_{s}^{ZF} \) as well as \( G_{i}^{ZF}, \ s \in S \).

### 2.2 System optimum

The system optimum (SO) is derived by applying Wardrop's second principle, again allowing for uncertainty about travel costs. The marginal social cost (MSC) of a trip on route \( i \) in state \( s \) is

\[ MSC_{is} \left( N_{is} \right) = \frac{\partial \left( C_{is} \left( N_{is} \right) N_{is} \right)}{\partial N_{is}} = C_{is} \left( N_{is} \right) + \frac{\partial C_{is} \left( N_{is} \right)}{\partial N_{is}} N_{is}. \quad (5) \]
The first term in eqn. (5) is the private cost of a trip, and the second term is the external cost which is positive as long as the cost function is upward-sloping and the route is used. Let $N_{is}^*$ denote SO usage of route $i$ in state $s$. With full information, the SO division of traffic between routes in state $s$ is realized when marginal social costs are equal:

$$MSC_{1s}(N_{1s}^*) = MSC_{2s}(N_{2s}^*).$$

(6)

The SO with full information serves as a benchmark for measuring the efficiency of UE outcomes.

3. Welfare effects of information

3.1 General results

The welfare effects of information depend — amongst other factors — on the functional forms of the travel cost functions on each route, on how the cost functions vary with the state, and on the correlation between travel conditions on the two routes. While general results are elusive, a few insights can be derived without making further assumptions. For example, if one of the routes is incident free and does not get congested, then total travel costs do not vary with travel conditions on the other route in either the zero-information or the full-information regimes. In that case, in equilibrium, information does not affect welfare.

A second observation is that if the UE with full information coincides with the SO in every state, then full information is necessarily welfare-improving. This result is formalized as:

**Proposition 1. (Value of information when user equilibrium is system optimal):**

Let Assumption 1 hold and assume that with full information the UE and SO are congruent in all states (i.e., $N_{1s}^{*F} = N_{1s}^{*}, \forall s \in S$). Then:

(a) full information is welfare-neutral or welfare-improving over zero information in each state (i.e., $G_{s}^{ZF} \geq 0, \forall s \in S$); and

(b) unless SO usage is the same in every state, full information is strictly welfare-improving over zero information (i.e., $G_{s}^{ZF} > 0$).
Part (a) of Proposition 1 is immediate because, with full information, the SO achieves a global minimum of total costs in each state. Part (b) follows from Assumption 1 and the fact that usage with zero information is independent of the state and therefore cannot be optimal in all states.

Proposition 1 provides a clear indication of when information is welfare-improving. However, it has limited empirical relevance because the UE and SO generally differ even on a two-route network with inelastic total demand. To see this, substitute eqn. (5) into eqn. (6) for the SO division of traffic to get $C_{1s}(N_{1s}^*) + \frac{\partial C_{1s}(N_{1s}^*)}{\partial N_{1s}} N_{1s}^* = C_{2s}(N_{2s}^*) + \frac{\partial C_{2s}(N_{2s}^*)}{\partial N_{2s}^*} N_{2s}^*$. This equation can be written:

$$C_{1s}(N_{1s}^*)\left(1 + \varepsilon_{ls}(N_{1s}^*)\right) = C_{2s}(N_{2s}^*)\left(1 + \varepsilon_{ls}(N_{2s}^*)\right),$$

(7)

where $\varepsilon_{ls}(N_{is}^*) \equiv \frac{\partial C_{is}(N_{is}^*)}{\partial N_{is}} \frac{N_{is}^*}{C_{is}(N_{is}^*)}$ is the elasticity of travel cost on route $i$ in state $s$, given usage of $N_{is}^*$. Comparing eqn. (7) with eqn. (3) it is evident that $N_{1s}^* = N_{ls}^*$ only if the cost elasticities on the two routes are the same at the UE traffic route split. This will not be the case in general. Usage of route 1 can be larger in the UE than SO in some states and smaller in other states, and there is no guarantee that information is welfare-improving. Indeed, it is possible for full information to be welfare-reducing in all states. To see this, consider the following example which is similar to Example 1 in Liu et al. (2009). Let $N=1$ and suppose that there are two equally likely states, $A$ and $B$. Travel costs on the two routes are:

In state $A$: $C_{1A}(N_{1A})=1$, $C_{2A}(N_{2A})=N_{2A}$

In state $B$: $C_{1B}(N_{1B})=N_{1B}$, $C_{2B}(N_{2B})=1$.

In the SO, usage in each state is divided equally between the two routes so that $N_{1s}^* = N_{1s}^* = 1/2$. Expected total costs are 3/4. Since expected travel costs are the same function of usage on each route, $N_{1s}^* = 1/2$. Therefore, user equilibrium with zero information coincides with the SO in both states. By contrast, in the UE with full information all users choose the more congestible route, which is the worst possible division of traffic. Expected total costs with full information are 1: a full one third higher than with zero information.\(^1\)

\(^1\)This example illustrates the price of anarchy with linear travel costs described by Roughgarden (2005).
In the example, one route is overused in one of the states and the other route is overused in the other state. Suppose now that one route is systematically overused relative to the other in the full-information UE. This is likely to be the case if one route is shorter or quicker than the other, and thus more prone to congestion because it is more heavily used. To be concrete, suppose route 1 is overused. It is then possible to establish:

**Proposition 2. (Route 1 systematically overused):**

Let Assumption 1 hold and assume that route 1 is always overused in the full-information UE (i.e., \( N_{1s}^F > N_{1s}^Z, \forall s \in S \)). Suppose further that full-information UE usage is not the same in all states. Then, full information is strictly welfare-improving relative to zero information in at least one state (i.e., \( G^Z_s > 0 \) for some \( s \)).

Proof: Let \( s' \) and \( s'' \) be two states such that \( N_{1s}^F \leq N_{1s}^F \leq N_{1s}^F, \forall s \in S \). It then follows that \( N_{1s}^F < N_1^Z < N_{1s'}^F \). By assumption, \( N_{1s'}^* < N_{1s'}^F \). Hence \( N_{1s'}^* < N_{1s'}^F < N_1^Z \). Full information is then welfare-improving relative to zero information in state \( s' \) because route 1 is overused less with full information than with zero information. ■

The proof of Proposition 2 exploits the fact that zero information induces higher usage of route 1 than full information in some states, thereby exacerbating overuse of route 1. This reasoning might suggest that full information must be welfare-reducing in state \( s'' \). Yet, this is not the case. It is true that \( N_1^Z < N_{1s}^F \), so that zero information induces less overuse of route 1 than full information. However, it is possible to have \( N_1^Z < N_{1s'}^* < N_{1s'}^F \), so that route 1 is underused with zero information. Moreover, the welfare loss from underuse can exceed the welfare loss from overuse with full information. This will be the case if the bias towards excessive usage of route 1 is relatively small, whereas travel conditions vary substantially so that usage with zero information deviates substantially from full information in some states.
3.2 Results for power functions

To achieve further progress with the analysis, the travel cost functions are now assumed to have the canonical U.S. Bureau of Public Roads (1964) form\(^2\):

\textbf{Assumption 2. (Power function travel cost functions):}

\begin{equation*}
C_{is}(N_i) = a_{is} + b_{is}N_i^d, \quad d \geq 0, \quad i = 1, 2, \forall s.
\end{equation*}

Parameter \(a_{is}\) will be called the \textit{free-flow travel cost} on route \(i\), parameter \(b_{is}\) the \textit{congestion coefficient}, and parameter \(d\) the \textit{power coefficient}. If the travel cost functions are homogeneous of degree zero in usage and capacity, then \(b_{is} = \beta_i k_{is}^{-d}\) where \(k_{is}\) denotes capacity in state \(s\) and \(\beta_i\) is a constant. If the \(k_{is}\) are stochastic, so are the \(b_{is}\).

The welfare effects of information are sensitive to whether travel conditions on the two routes are correlated. In the case of bad weather or special events that affect driving over extensive parts of the road network, the congestion coefficients \(b_{1s}\) and \(b_{2s}\) are likely to be positively correlated when the two routes are not far apart. For other shocks, such as accidents, \(b_{1s}\) and \(b_{2s}\) may be uncorrelated. Correlation between the free-flow travel cost parameters \(a_{is}\) and \(a_{2s}\), and between the \(a_{is}\) and \(b_{is}\) parameters on each route, will also depend on the nature of shocks.

As noted in the introduction, several earlier studies have explored the welfare effects of information on two-route networks when the travel cost functions are assumed to be linear (i.e., \(d = 1\)). de Palma and Lindsey (1994) allow for imperfect information about states.\(^3\) They identified two conditions under which better information is welfare-improving: (1) free-flow travel costs are equal on the two routes, and (2) free-flow travel costs differ by the same amount in each state and the congestion coefficients are statistically independent. They also showed that information can be welfare-reducing if the congestion coefficients are positively correlated.

\(^2\) The BPR function is normally used to specify travel time rather than travel cost. For travel cost, the value of the parameter \(d\) reflects the net sequential effects of usage on congestion delay and congestion delay on travel cost.

\(^3\) In their model, imperfect information is conveyed by messages, and they assume that both states and messages are statistically independent between routes.
Verhoef et al. (1996) and Emmerink et al. (1998) assume that the congestion coefficient on each route is either low or high — corresponding respectively to good conditions and bad conditions. They also assume that the states of the two routes are statistically independent. Both studies allow for elastic (i.e., price-sensitive) demand by assuming that travel demand is a linear decreasing function of the price. Verhoef et al. (1996) show that full information is welfare-improving if the free-flow travel cost on each route is at least as high in bad conditions as good conditions. Emmerink et al. (1998) assume that there are two groups of agents: one informed and one uninformed. Result (1) of de Palma and Lindsey (1994) holds in their model. They also indicated that if free-flow costs differ between routes, then it may not be optimal for all the agents to be informed. They do not address whether informing all agents is either better or worse than informing none.

In this section, we extend the analysis of the earlier papers to the class of power functions specified in Assumption 2, delve further into the implications of stochastic free-flow travel costs, and examine in more detail the conditions under which information is, or is not, welfare-improving. Our analytical results are formalized in four propositions (proofs of all propositions are provided in the appendix):

**Proposition 3. (Excessive usage of the shorter route with full information):**

*Given Assumptions 1 and 2:*

(a) if one route has a lower free-flow travel cost than the other route (e.g., because it is shorter), then it is overused relative to the SO in the full-information UE;

(b) if the two routes have equal free-flow travel costs, then the full-information UE coincides with the SO.

Part (a) of Proposition 3 is explained as follows. In the UE, private travel costs on the two routes are equal. If route 1 has a lower free-flow travel cost than route 2, then it must be more heavily congested than route 2 and the congestion externality will be higher. Since individual users ignore the congestion externality, too many choose route 1. Part (b) of Proposition 3 was established for the case of deterministic costs by Barro and Romer (1987). When free-flow travel

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4 If informed agents use both routes in all states, then information is welfare-improving regardless of assumptions about free-flow travel costs.
costs are the same, so are congestion delays. Marginal social costs of usage are higher than
private costs on both routes, but given Assumption 2 the elasticities in eqn. (7) are the same, and
therefore the UE coincides with the SO. This is true even if congestion coefficients on the two
routes differ because the route that is less congestion-prone is used correspondingly more
heavily. For example, if \( d = 1 \) then the marginal congestion externality on route \( i \) is \( b_i N_i \). If
\( b_{2s} = 2b_{1s} \), then route 1 carries twice as much traffic as route 2 and \( b_{1s} N_{1s} = b_{2s} N_{2s} \).

**Proposition 4. (Equal free-flow travel costs):**

Assume that the free-flow travel costs on the two routes are always equal (i.e. \( a_{2s} = a_{1s}, \forall s \)). Then, given Assumptions 1 and 2:

(a) full information is welfare-improving over zero information (i.e., \( G^{\text{ZF}} \geq 0 \)); and

(b) full information is strictly welfare-improving (i.e., \( G^{\text{ZF}} > 0 \)) unless the congestion
coefficients are equiproportional in all states (i.e., \( b_{2s} / b_{1s} = \lambda \) for some \( \lambda > 0 \) and \( \forall s \)).

To see part (a), note that given equal free-flow travel costs and Assumption 2, the UE with full
information coincides with the SO in every state as per part (b) of Proposition 3. Full information
is therefore welfare-improving relative to zero information. Moreover, full information is strictly
welfare-improving unless SO usage is the same in every state — such as under the conditions
identified for part (b) of Proposition 4.

The zero-information and full-information regimes are not easy to compare when free-flow
travel costs differ because equations (3), (4), and (6) do not have closed-form solutions for
arbitrary \( d \). However, closed-form solutions exist for \( d=1 \) and \( d=2 \), and partial welfare rankings
can be derived for these two cases. They are formalized in Propositions 5 and 6.

**Proposition 5. (Linear travel costs):**

Let Assumptions 1 and 2 hold with \( d=1 \). Then:

(a) if the congestion coefficients and the difference in free-flow travel costs \( (a_{2s} - a_{1s}) \) on
the two routes are all statistically independent, then full information is welfare-improving
relative to zero information (i.e., \( G^{\text{ZF}} \geq 0 \));
(b) if the congestion coefficients are statistically independent of \((a_{2s} - a_{1s})\) and equiproportional in all states, then full information is welfare-neutral (i.e., \(G^{ZF} = 0\));

(c) if the free-flow travel cost varies with the congestion coefficient on each route according to some nondecreasing deterministic function \(a_i = a_i(b_i)\) and the states of the two routes are statistically independent, then full information is welfare-improving relative to zero information (i.e., \(G^{ZF} \geq 0\));

(d) even if the congestion coefficients are statistically independent of \(a_{2s} - a_{1s}\), if the congestion coefficients are interdependent and do not vary equiproportionally in all states, then full information can be welfare-reducing (i.e., \(G^{ZF} < 0\)).

Part (a) of Proposition 5 asserts that when the free-flow travel costs and congestion coefficients vary independently, information is welfare-improving. Independence assures that the bias towards excessive usage of the shorter route varies independently of the congestion characteristics of each route. Part (b) of Proposition 5 generalizes the result of Verhoef et al. (1996) for two states to an arbitrary set of states. It also shows that part (b) of Proposition 4 generalizes when routes differ in free-flow travel costs, and it also implies that information is welfare-neutral if the congestion coefficients are deterministic. Part (c) allows for correlation between the free-flow travel costs and congestion coefficients, but restricts the correlation to be positive. Therefore, it rules out the example presented above in which the parameters \(a_i\) and \(b_i\) are negatively correlated. Finally, part (d) asserts that information can be detrimental if the capacities are correlated, but do not vary by the same percentage amount in all states.

**Proposition 6.** *(Quadratic travel costs)*:

*Let Assumptions 1 and 2 hold with \(d=2\). Then:*

*(a) if the congestion coefficients and the difference in free-flow travel costs \((a_{2s} - a_{1s})\) on the two routes are all statistically independent, then full information can be welfare-improving or welfare-reducing relative to zero information; and*
(b) if the congestion coefficients are equiproportional in all states, and \( a_{2s} \neq a_{1s} \) in a set of states with positive probability measure, then full information is welfare-reducing (i.e., \( G^{ZF} < 0 \)).

Part (a) of Proposition 6 contrasts with part (a) of Proposition 5 which indicates that, under the same conditions, information is necessarily beneficial. Part (b) of Proposition 6 also contrasts with part (b) of Proposition 5 which establishes similar conditions under which full information is welfare-neutral rather than welfare-reducing. A heuristic explanation for these differences is that the elasticity of travel cost with respect to usage is twice as large with quadratic travel costs as with linear travel costs. Consequently, deviations in usage from the optimal division of traffic between routes create larger welfare losses in the quadratic case.

While Propositions 5 and 6 encompass only linear and quadratic cost functions, parts (a) and (b) of the two propositions suggest that information is more beneficial when conditions on the two routes vary independently rather than in tandem. To see why, note that when conditions are independent they can be good on one route and bad on the other. Bad conditions increase both the private cost and the marginal social cost of a trip so that it is both an equilibrium response and socially efficient for traffic to shift from the bad route to the good route. Consequently, it is usually beneficial to inform users about travel conditions. By contrast, when conditions are always similar on the two routes there is less to gain from reallocating traffic between them, and any UE shifts that do occur are less likely to improve system performance.

To pursue this reasoning further, consider the deadweight loss from non-optimal route split in the UE when the congestion coefficients are stochastic and free-flow travel costs are deterministic. With full information and linear travel costs, the deadweight loss in state \( s \) is \((a_2 - a_1)^2 / (4(b_1 + b_2))\). A deadweight loss occurs when free-flow travel costs differ because of the bias toward excessive usage of the shorter route described earlier. The deadweight loss is greatest when \( b_{1s} \) and \( b_{2s} \) are both small (i.e., both routes have good travel conditions), which happens more frequently when route conditions are positively correlated than when they are independent. With zero information, route usage is independent of correlation so that it does not affect efficiency of the zero-information regime.
3.3 Specific results

To proceed further, we now impose three additional assumptions that we maintain for the rest of the paper. First, the free-flow travel costs, \( a_i \) and \( a_z \), are deterministic. Second, \( a_z > a_i \). Therefore, Route 1 is the cheaper route and will sometimes be called the shorter route. Third, travel conditions on each route are either good (\( G \)) or bad (\( B \)), with \( b_{iB} > b_{iG} \), \( i=1,2 \). The probability of bad conditions on route \( i \) is \( \pi_i \). Unless conditions on the two routes are perfectly correlated, there are four possible states for the system: \( GG, GB, BG, \) and \( BB \), where the first subscript refers to the state of route 1, and the second to the state of route 2.

With full information there are four possible UE usage levels of route 1: \( N_{1GG}^F, N_{1GB}^F, N_{1BG}^F, \) and \( N_{1BB}^F \). Similarly, there are four possible SO usage levels: \( N_{1GG}^s, N_{1GB}^s, N_{1BG}^s, \) and \( N_{1BB}^s \). It is easy to deduce some general properties of the usage levels. Given Assumption 2, UE is systematically biased towards excessive usage of route 1. Therefore, part (a) of Proposition 3 applies and

\[
 N_{1s}^s < N_{1s}^F, \quad s = GG, GB, BG, BB. \tag{8}
\]

In the UE, usage of route 1 is lowest when conditions are bad on route 1 and good on route 2. Correspondingly, usage of route 1 is highest when it has good conditions and route 2 has bad conditions. Usage is intermediate when conditions on the two routes are the same. Usage levels are therefore ranked as follows:

\[
 N_{1BG}^F < \min \left[ N_{1GG}^F, N_{1BB}^F \right] \leq \max \left[ N_{1GG}^F, N_{1BB}^F \right] < N_{1GB}^F. \tag{9}
\]

An analogous ranking holds for the SO:

\[
 N_{1BG}^s < \min \left[ N_{1GG}^s, N_{1BB}^s \right] \leq \max \left[ N_{1GG}^s, N_{1BB}^s \right] < N_{1GB}^s. \tag{10}
\]

In both cases the ranking of usage in states \( GG \) and \( BB \) depends on parameter values. It is possible for the UE and SO rankings to differ so that \( \left( N_{1GG}^s - N_{1BB}^s \right) \left( N_{1GG}^F - N_{1BB}^F \right) < 0 \).

Usage in the UE with zero information is intermediate between the extremes of full information so that

\[
 N_{1BG}^F < N_{1s}^Z < N_{1GB}^F. \tag{11}
\]

Given inequalities (8) and (11) it follows that

\[
 N_{1BG}^s < N_{1BG}^F < N_{1s}^Z. \tag{12}
\]
Therefore, full information is welfare-improving in state BG: $G_{BG}^{ZF} > 0$. This is consistent with Proposition 2, which states that with systematic overuse of route 1 full information must be strictly welfare-improving in at least one state. Whether information is beneficial in the other three states depends on parameter values. Although it is possible with $d=1$ or with $d=2$ to derive closed-form expressions for $G_{GG}^{ZF}$, $G_{GB}^{ZF}$, and $G_{BB}^{ZF}$, the expressions are too unwieldy to be useful. To obtain further insights, we turn next to numerical examples.

4. Numerical examples

4.1 A preliminary illustrative example

To illustrate how information can be welfare-reducing we begin with a specific numerical example. Parameter values are set to: $N=1$, $d=2$, $a_{1G} = a_{1b} = 0$, $a_{2G} = a_{2b} = 3$, $b_{1G} = 4$, $b_{1b} = 16$, $b_{2G} = 2$, $b_{2b} = 8$, and $\pi_1 = \pi_2 = 0.25$. Because $d=2$, $a_2 > a_1$, and the congestion coefficients are equiproportional (i.e., $b_{2b}/b_{1b} = b_{2G}/b_{1G} = 1/2$), the assumptions of part (b) of Proposition 6 are satisfied. Consequently, the information paradox occurs when route conditions are perfectly correlated. The top panel of Figure 1 depicts the UE and SO when conditions on both routes are good (probability 0.75), and the bottom panel depicts UE and SO when conditions on both routes are bad (probability 0.25).

With zero information, the division of traffic is determined by eqn. (4). UE traffic on route 1 is $N_1^Z = 0.690$. When conditions are good, UE traffic with full information is $N_{1GG}^F = 0.871$ and SO usage is $N_{1GG}^* = 0.581$. Route 1 is overused by 0.871-0.581=0.29. The deadweight loss (DWL) from overuse is measured by the sum of the lightly-shaded and darkly-shaded areas in Figure 1 bounded by $MSC_{1G}$ and $MSC_{2G}$. With zero information, route 1 is overused only by 0.690-0.581=0.109, and the deadweight loss is limited to the darkly-shaded area. When conditions are bad, UE traffic on route 1 with full information is $N_{1BB}^F = 0.541$ and SO usage is $N_{1BB}^* = 0.458$. Route 1 is overused by 0.541-0.458=0.083. With zero information it is overused by 0.690-0.458=0.232.

Full information supports a more efficient outcome than zero information when conditions are bad, but a less efficient outcome when conditions are good. The net welfare effect of full
information depends on the probabilities of good and bad conditions, and on the relative sizes of the shaded areas which, in turn, depend on the curvature of the travel cost functions. In the present example, the welfare loss from information with good conditions is 0.729, and the welfare gain from information with bad conditions is 1.745. The probability-weighted gain is 0.75(-0.729) + 0.25(1.745) = -0.110. The expected gain is negative, and consequently an information paradox is realized.

--Insert Figure 1 about here--

If route conditions are independent, conditions can be good on one route and bad on the other. Diagrams similar to those in Figure 1 can be constructed for these states. (Such a diagram is presented in the next section.) With good conditions on route 1 and bad conditions on route 2, there is a welfare loss from information of $G_{ga}^{ZF}=0.629$. Conversely, with bad conditions on route 1 and good conditions on route 2, there is a welfare gain of $G_{BG}^{ZF}=2.693$. The expected gain from information is $G^{ZF}=(0.75)^2(-0.729) + (0.75)(0.25)(-0.629) + (0.25)(0.75)(2.693) +(0.25)^2(1.745) = -0.086$. As expected, the welfare gain is higher than with perfectly correlated route conditions, but since it is negative an information paradox still occurs.

4.2 Sensitivity analysis

We now explore an extended numerical example that spans a range of travel conditions and information welfare effects. Similar to the previous example, we set $d=2$ for the base case. As noted earlier, route conditions are likely to be uncorrelated for shocks such as accidents, and correlated for other shocks such as special events and bad weather. Most shocks reduce road capacity and thereby increase the corresponding congestion coefficients. Free-flow travel speeds are often unaffected unless speed limits are reduced (e.g., for major road repairs). The main

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5 Most traffic engineering studies have found that travel time is a strictly convex function of hourly flow, and most estimates of the power coefficient, $d$, in Assumption 1 are four or higher (Transportation Research Board, 2010). However, the model here is static, and the travel cost function should be interpreted as a reduced-form relationship between equilibrium travel cost and the number of trips taken over an endogenously-determined time period. Depending on the structure of trip-timing preferences the equilibrium travel cost function can be linear (Arnott et al., 1993), close to linear (de Palma and Marchal, 1999), quadratic (Fosgerau and Engelson, 2011) or some other function. The quadratic form assumed here comprises an intermediate case between linear and highly nonlinear functions. As reported later in this section, the results obtained for $d>2$ are very similar to those for $d=2$. 

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exception is adverse weather which usually causes some reduction in free-flow speeds.\(^6\) For weather-related shocks it is thus natural to assume not only that the parameters \(b_{1s}\) and \(b_{2s}\) are positively correlated, but also that the \(a_{is}\) parameters are stochastic and positively correlated with the \(b_{is}\). However, these assumptions are inappropriate for a scenario in which route conditions are uncorrelated. Moreover, shocks that increase free-flow costs on the two routes by the same amount do not affect usage. To focus on the effects of route correlation, it is desirable to use the same specifications of the travel cost functions for the uncorrelated and correlated cases. Therefore, free-flow costs are treated as deterministic.

The model is defined by ten parameters: \(N, d, \pi_1, \pi_2, a_1, a_2, b_{1G}, b_{1B}, b_{2G}, b_{2B}\). By adjusting the scale of the congestion cost coefficients we can normalize total usage to \(N=1\) without loss of generality. To allow perfect correlation between route conditions we require \(\pi_1 = \pi_2\), and we set the common value to 0.25. Since the absolute magnitude of the welfare effects of information with fixed demand depend only on the relative costs of travel on the two routes, the choice of free-flow travel time on the shorter route is arbitrary. We set \(a_1 = 6\) so that the lowest feasible travel cost on the network is $6: representative of a short commuting trip. We set \(a_2 = 9\) so that free-flow cost on route 2 is 50 percent longer than on route 1 due to some combination of a lower speed limit, greater length, or longer access time.

The two routes are assumed to have the same capacity in good conditions. The common value of the congestion coefficient is set to \(b_{1G} = b_{2G} = 15\). With full information this results in a UE cost of $11.40. Parameters \(b_{1B}\) and \(b_{2B}\) are varied continuously over the interval \([15,135]\) to cover a range of adverse conditions. As noted earlier, if the travel cost functions are homogeneous of degree zero in usage and capacity, \(b_{is} = \beta_i k_i^{-d}\) where \(k_i\) is the capacity of route \(i\) in state \(s\). An accident that blocks one of two lanes on a freeway reduces effective capacity to

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\(^6\) The size of the reduction depends on the type and severity of the weather disturbance (e.g., rain, snow, slush, ice, sleet, fog, and strong winds). Estimates vary widely. For example, according to FHWA Road Weather Management Program (2009), free-flow speeds drop by 2-13\% in light rain, 6-17\% in heavy rain, and 5-64\% in heavy snow. See US Federal Highway Administration (2010) for an extensive review of how inclement weather impacts road capacity, travel speeds, and travel demand.
about one third of its design value. With \( d = 2 \), the coefficient \( b_u \) will then increase by a factor of nine which corresponds to the upper end point of the range of \( b_u \) values considered.

To assess the size of the welfare gains or losses from information it is useful to report not only the absolute welfare gain, \( G^{ZF} \), but also a metric that is independent of scale and units of measurement. Such a metric is obtained by dividing \( G^{ZF} \) by the welfare gain from a first-best optimal intervention (i.e., in which usage is split between routes to maximize social surplus when the state is known). The first-best optimum can be implemented by providing full information, and then imposing state-dependent tolls to support the optimal division of traffic between routes. Let \( B^{F} \) denote the benefits from tolls given full information. Welfare is then measured with the index \( \omega \equiv G^{ZF} / \left( G^{ZF} + B^{F} \right) \).

The top panel of Figure 2 plots the welfare gain \( G^{ZF} \) as a function of the size of the shocks on each route, \( b_{1B} / b_{1G} \) and \( b_{2B} / b_{2G} \), when route conditions are uncorrelated. The welfare gain rises steadily in each dimension from zero at the origin up to \( $3.81 \) with \( b_{1B} / b_{1G} = b_{2B} / b_{2G} = 9 \). Thus, as might be expected, information is more valuable the larger are the shocks, and hence the more variable are travel conditions. The bottom panel of Figure 2 shows that the efficiency index \( \omega \) rises monotonically as well, reaching a maximum of \( \omega = 0.96 \) close to the theoretical limit of \( \omega = 1 \). With large shocks, providing information thus goes much further toward optimizing network performance than tolling.

Figure 3 plots \( G^{ZF} \) and \( \omega \) over the same parameter space when route conditions are perfectly correlated. In contrast to Figure 2, the welfare gain is not always positive. When shocks are large on one route and small on the other, information is beneficial although the maximum gain of \( $1.28 \) is only about one-third as large as with no correlation. Information is beneficial with highly unequal shocks because conditions are effectively bad on one route but effectively still fairly good on the other. As explained with respect to Propositions 5 and 6, UE and SO responses to information then tend to be similar. In contrast, when shocks are comparable in size

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7 Transportation Research Board (2000, Chapter 22, Table 22-6).
8 This index has been used in a number of antecedent studies (e.g., Arnott et al., 1991; Verhoef et al., 1995). It has several virtues. It is bounded above by 1, and it is negative if information is welfare-reducing when implemented without tolls. It is unaffected if free-flow costs are changed by the same amount on each route. And it provides a sense of the gains or losses from information relative to the gains from congestion pricing (or, equivalently, the price of anarchy).
information is detrimental and the index $\omega$ reaches a minimum value of -1.15. At this point, conveying information induces a welfare loss that exceeds the welfare difference between the zero-information UE and the SO.\(^9\)

---Insert Figures 2 and 3 about here---

As explained in Section 3, the welfare gain from information, $G^{ZF}$, is a probability-weighted average of the welfare gain in each of the four states, $G_{GG}^{ZF}$, $G_{GB}^{ZF}$, $G_{BG}^{ZF}$, and $G_{BB}^{ZF}$. $G_{BG}^{ZF}$ is always positive, but the other three component gains may be negative. Figure 4 identifies which of the components are negative over the same parameter space considered in Figures 2 and 3. (Note that states $BG$ and $GB$ never occur if route conditions are perfectly correlated.) Consistent with Figure 2, in the two regions labeled "∅", where only one of the routes is susceptible to large shocks, information is beneficial in all four states. The reason for this is that the UE and SO tend to shift strongly, and in the same direction, in response to information about travel conditions on the highly variable route. But if both routes suffer only small shocks (the regions labeled "$GB, BB$" and "$GG, GB$" near the origin), information is detrimental in the $GB$ state as well as one of the $GG$ and $BB$ states. Information is less beneficial in these circumstances because it is more likely to exacerbate the UE bias toward overuse of route 1.

Finally, if both routes are susceptible to large shocks (the regions labeled "$BB$" and "$GG$") information is detrimental in one of the $GG$ and $BB$ states. It may seem surprising that information can be detrimental in state $GG$, which occurs with probability 0.5625 and includes a region of the parameter space where $\omega$ exceeds 0.8 (cf. Figure 3). The reason is that the welfare loss in state $GG$ is quite small compared to the welfare gains in the other states. For example, with $b_{1B} / b_{1G} = b_{2B} / b_{2G} = 9$, $G_{GG}^{ZF} = -0.2$, whereas $G_{GB}^{ZF} = 8.15$, $G_{BG}^{ZF} = 12.67$, and $G_{BB}^{ZF} = 0.33$.

---Insert Figure 4 about here---

A wide range of alternative parameter values was explored to assess the robustness of the results. The results of three parameter variants are worth reporting. The first variant entails setting different probabilities of bad conditions on the two routes while holding fixed the average probability at 0.25. With the base-case parameters, the maximum degree of correlation between

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\(^9\) As explained shortly, the pattern in which information is beneficial when shocks differ significantly in magnitude does not emerge with all parameter combinations.
route conditions is \( \rho = 1 \) and the state probabilities are as shown in panel (a) of Table 1. As the difference in probabilities increases, the maximum degree of correlation between route conditions decreases. Attention is focused here on the probability combinations \((\pi_1, \pi_2) = (0.375, 0.125)\) and \((\pi_1, \pi_2) = (0.125, 0.375)\). In both cases the maximum correlation is \( \rho = 0.488 \).

--Insert Table 1 about here--

The state probabilities with \((\pi_1, \pi_2) = (0.375, 0.125)\) and \(\rho = 0.488\) are shown in panel (b) of Table 1. Compared to the base case, the probability of state \(BG\) increases from 0 to 0.25, while the probability of \(GG\) drops by 0.125 to 0.625 and the probability of \(BB\) drops by 0.125 to 0.125. Information is welfare-improving for all sizes of shocks and the efficiency index exhibits a similar shape (not shown) to that in Figure 2 with the base-case parameter values and zero correlation. The main reason for this behavior is that with weaker correlation between route conditions, the \(GG\) and \(BB\) states in which information can be welfare-reducing occur less frequently, whereas state \(BG\) in which information is always beneficial (see proof of Proposition 2) now occurs one quarter of the time.

The sign pattern of the welfare gain with \((\pi_1, \pi_2) = (0.375, 0.125)\) does not, of course, depend on the correlation coefficient. Figure 5 reveals that the pattern differs in several ways from the base case shown in Figure 4. First, the regions labeled "\(\emptyset\)" in which information is beneficial in all states are larger in total area. Second, a central region labeled "\(GG, BB\)" now exists in which information is detrimental in both the \(GG\) and \(BB\) states. Third, there is a small region labeled "\(GG, GB, BB\)" near the origin in which information is detrimental in all three of states \(GG\), \(GB\) and \(BB\). With \(\rho = 0.488\), state \(GB\) never occurs but state \(GG\) or \(BB\) occurs with probability 0.75. Information is therefore counterproductive three-quarters of the time. Nevertheless, the gain from information in state \(BG\) is large enough that the expected value of information is still positive.

If the probabilities of bad conditions are flipped to \((\pi_1, \pi_2) = (0.125, 0.375)\), the state probabilities are as shown in panel (c) of Table 1. State \(GB\) now occurs with probability 0.25 while state \(BG\) never occurs. Figure 6 shows that the sign pattern differs from that in Figure 5 in several respects. First, most of the area labeled "\(\emptyset\)" in which information is always beneficial exists in the central portion of the figure where shocks are similar in size. Second, the relative
positions of the \( GG \) and \( BB \) regions are approximately reversed. Third, the "\( GG, BB \)" and "\( GG, GB, BB \)" regions are absent, but a new region labeled "\( GB \)" appears in which information is detrimental in state \( GB \). Information turns out to be welfare-improving for all sizes of shocks and the efficiency index exhibits a similar shape to that with \( (\pi_1, \pi_2) = (0.375, 0.125) \) and the base case.

The results with \( (\pi_1, \pi_2) = (0.125, 0.375) \) are surprising in two respects. First, information is more likely to be harmful when shocks are similar in magnitude rather than very different as is the case with \( (\pi_1, \pi_2) = (0.375, 0.125) \) and the base case. This illustrates that the basic intuition underlying Propositions 5 and 6 and the previous examples is not universally applicable. The second surprise is that the expected value of information is always positive even though state \( BG \) in which information is always beneficial never occurs.

--Insert Figures 5 and 6 about here--

The second parameter variation of interest entails reducing the congestion coefficients from their base case values to \( b_{1G} = b_{2G} = 3 \) so that the network is much less congested. As shown in Figure 7, even with uncorrelated route conditions information is now detrimental when shocks on route 1 are not too large. Figure 8 decomposes the gains and losses from information for the point \( (b_{1B} / b_{1G} = 2, b_{2B} / b_{2G} = 3) \) identified by a dot in Figure 7. Expected travel costs on the two routes are shown by dashed lines. With zero information, usage of route 1 is \( N_1^Z = 0.9 \) and expected costs are \( C^Z \approx 9.05 \). When conditions are good on route 1, all drivers take route 1 regardless of conditions on route 2, and UE occurs at the point labeled \( GG, GB \) and marked by a black dot. If conditions are bad on route 1 and good on route 2, equilibrium occurs at point \( BG \) with \( N_{1BG}^F \approx 0.73 \), and if conditions are bad on both routes equilibrium occurs at point \( BB \) with \( N_{1BB}^F \approx 0.76 \).

The corresponding SO usage levels, marked by open circles, are \( N_{1GG}^Z \approx 0.67 \), \( N_{1GB}^Z \approx 0.74 \), \( N_{1BG}^Z \approx 0.53 \), and \( N_{1BB}^Z \approx 0.62 \). User equilibrium usage exceeds SO usage by a substantial margin in all four states. Usage with zero information exceeds SO usage in every state as well. Therefore, net welfare effect of information depends on how it affects the degree of overuse of

...
route 1. In state \( GG \), the efficiency loss due to overuse with zero information is measured by area \( bca \) between curves \( MSC_{1G} \) and \( MSC_{2G} \) from \( N^*_{1GG} \) to \( N^Z_{1} \). With full information, the corresponding area is \( bed \) which is larger than \( bca \) by the shaded area \( acd \). In state \( GB \), the efficiency loss is measured between curves \( MSC_{1G} \) and \( MSC_{2B} \). With zero information it is area \( fcg \), and with full information by area \( fed \). The loss is greater with full information by the shaded area \( gced \). By similar reasoning one can deduce that in state \( BG \), full information is more efficient than zero information by the shaded area \( hija \) plus an additional area between curves \( MSC_{1B} \) and \( MSC_{2G} \) above the figure which is not shown to improve readability. In state \( BB \), full information is more efficient than zero information by the diagonally hatched area \( lmk \) plus an additional area between curves \( MSC_{1B} \) and \( MSC_{2B} \) above the figure. In the example, full information is welfare-reducing because the probability-weighted sum of the red shaded areas exceeds the probability-weighted sums of the green and blue shaded areas. It is clear that, even with only two states on each route, it is difficult to deduce the net effect of information from diagrams except in very simple cases.

--Insert Figures 7 and 8 about here--

The final parameter variation entails increasing the power coefficient in integer steps from \( d=2 \) to \( d=10 \) to examine how the shape of the travel cost function influences results. To maintain reasonable comparability with the base case, the parameters \( b_{1G} \) and \( b_{2G} \) were adjusted so that the full-information UE costs in state \( GG \) match the full-information UE costs in state \( GG \) with \( d=2 \). Similarly, the upper bound values for \( b_{1B} \) and \( b_{2B} \) were adjusted so that the full-information UE cost in state \( BB \) matches the full-information UE cost in state \( BB \) with \( d=2 \) and the upper bound values of \( b_{1B} = 9b_{1G} \) and \( b_{2B} = 9b_{2G} \).

The results for the higher values of \( d \) are very similar to those for \( d=2 \). In all cases, information is welfare-improving with uncorrelated route conditions and welfare-reducing with correlated route conditions when the two routes suffer shocks of similar size. Figure 9 plots the relative welfare gain for perfectly correlated conditions with \( d=10 \). The pattern is nearly

\[10 \text{ For all values of } d \text{ between } 3 \text{ and } 10, \text{ the recalibrated values of } b_{1B} \text{ and } b_{2B} \text{ are close to nine times the recalibrated values of } b_{1G} \text{ and } b_{2G}.\]
identical to that in Figure 3 for $d=2$. Index $\omega$ ranges from -1.24 to 0.79 over the parameter space. The corresponding range in Figure 3 is -1.15 to 0.89. Information becomes marginally less effective as the cost function become more convex, but the change is slight.

---Insert Figure 9 about here---

In summary, the propositions developed in Section 3 and the numerical examples presented in Section 4 show that full information can be welfare-reducing in a range of conditions. Table 2 lists the instances reported here. The information paradox occurs when routes differ in free-flow costs, and it appears to be more prevalent when travel costs are convex and route conditions are perfectly correlated. However, the paradox can also occur with linear costs and no correlation.

---Insert Table 2 about here---

5. Concluding Remarks

In this paper, we have studied the effects of pre-trip information on route-choice decisions when travel conditions are congested and vary unpredictably. We adopt a variant of the classical two-route network in which travel conditions on each route fluctuate randomly due to weather, accidents, and other disturbances. We show that the welfare effects of pre-trip information depend on a number of factors: free-flow travel costs on the routes, the shape of the trip cost functions, the severity of congestion, the absolute and relative sizes of shocks on the two routes, and the correlation in conditions between routes. Information has adverse effects in more cases when the free-flow costs differ appreciably, cost functions are convex, shocks are similar in size on the two routes, and conditions on the two routes are positively correlated. These conditions are all practically relevant; to our knowledge they were not systematically examined before.

An important lesson is that the effects of information are sensitive to similarities and differences between the routes. If travel costs are described by the widely used Bureau of Public Roads formula (Assumption 2), information is always beneficial if the two routes have the same free-flow travel costs (cf. Proposition 4). When focusing on variability in route capacities due to accidents and some other types of shocks, it may seem natural to assume that free-flow costs are the same. But this creates a bias toward overestimating the benefits from information as well as overlooking the possibility of an information paradox. Another pitfall is to assume that routes are
susceptible to the same magnitude of shocks. We have shown that this can create a bias in the opposite direction toward underestimating the benefits from information (cf. Figure 3).

5.1 Extensions

Our analysis could be extended in several directions. We have treated the probabilities of shocks as exogenous. This is realistic for weather-related disturbances, but not for shocks due to crashes which depend on traffic volumes. This limitation might be overcome by specifying the probability of a shock on each route as an increasing function of usage.

A second extension considers more complex networks with multiple origin-destination pairs and links. Examples in the literature include Emmerink et al. (1997), who consider two O-D pairs and seven links with linear cost functions, and Liu et al. (2009), who study a general network with polynomial link travel times. Larger network are often relevant in practice, but analytical results and basic insights are more difficult to derive with them.

A third extension combines information provision with congestion tolls as a way to internalize congestion externalities and avoid adverse information effects. Several theoretical studies have examined the joint implementation of Advanced Traveler Information Systems and congestion pricing (e.g., de Palma and Lindsey, 1994; 1998; Verhoef et al., 1996; Yang, 1999; Fernández et al., 2009). It may also be possible to reduce congestion by providing travelers with rewards for re-timing their trips from peak to off-peak periods (Ben-Elia and Ettema, 2011). While this approach encourages too much travel overall if total demand is elastic, it has the advantage of greater acceptability to motorists.

5.2 Imperfect or partial information

Our model features two polar information regimes. Perfect information is an idealization that is practically unattainable for several reasons (Bonsall, 2008). Information on travel conditions may be collected with a delay, or not at all. Information about conditions downstream may be obsolete by the time a user gets there. Users may ignore or misinterpret information updates. Information systems can malfunction. And there is a problem of consistency: a message about conditions that induces changes in user behavior can lead to changes in the conditions themselves that invalidate the message. These complications can be accommodated by considering imperfect information that is conveyed in the form of messages that may be
imprecise or wrong. Some theoretical studies have taken this approach (e.g., Kobayashi, 1994; Arnott et al., 1996, 1999; Emmerink et al., 1998; Lam et al., 2008; Liu et al., 2009), but more work would be useful. Ben-Elia et al. (2013) have recently studied the effects of ex-ante information on route-choice decisions in a laboratory experiment. Interestingly, they find that subjects may be willing to follow advice even when it has low accuracy.

Related to the question of imperfect information is whether a policy of conveying all, rather than only some, information to motorists is desirable. We have shown that, even if information reduces expected total costs, it can be welfare-reducing in one or more states. This suggests that system efficiency might be improved by disseminating information only in certain states. Strategic or selective information provision has recently been explored by Lee and Shin (2011) and Lindsey (2012) using models similar to the one here that employ a variant of Crawford and Sobel's (1982) model of cheap talk. These studies assume that there is only one information source. Yet, as noted in the introduction of our paper, information about traffic conditions is available from various sources. It may be difficult for any one source to effectively withhold information from travelers. Moreover, subscribers to paid private information services may demand prompt and full dissemination of any information that may be useful to them. In instances where full information is likely to be detrimental, a more direct means of intervention could be warranted such as ramp metering or police-directed traffic control.

Our model adopts deterministic user equilibrium as the solution concept. This implies that users are fully rational, and that adverse information effects can operate only through concentration, not overreaction or oversaturation. The theory also makes predictions only about the fraction of users who take each route, not the choices of individual users. To attain an equilibrium route split, it is necessary for users to independently make choices in the right proportions. This is a difficult coordination problem, and it is far from clear that users will solve it even if they encounter similar circumstances repeatedly.

Laboratory experiments are well-suited for investigating how drivers actually make decisions in stochastic environments, with or without information. In a companion paper (Rapoport et al., 2013), we test the predictions of the model in a laboratory experiment using a numerical instance of the theoretical framework developed here. We find that the predictions of the model are

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Allon et al. (2011) also adopt this approach to study how much information about queueing delays a firm will find it profitable to reveal to its customers.
confirmed on the aggregate level: full information is welfare-improving when route conditions are uncorrelated, but welfare-reducing when they are perfectly correlated. One difference from the theoretical framework is that the number of subjects involved in the laboratory experiments is constrained to be an integer. To generate accurate predictions, it is necessary to treat users as discrete entities in the sense that, when a route is congestible, each user adds a non-negligible amount to travel time on it. The companion paper incorporates this modification, and discusses the existence and uniqueness of equilibrium when users are discrete.

6. Acknowledgments

We are grateful to the Editor and three anonymous referees for very helpful comments.

7. Role of the funding source

We gratefully acknowledge financial support from the National Science Foundation grant SES-0752662 awarded to the University of Arizona. The NSF was not involved in any specific aspects of this paper or in the decision to submit the paper for publication.

8. List of tables and figures

Table 1. Probabilities of states with maximum correlation between routes.
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9. **Notational Glossary**

- $a_{is}$: free-flow travel cost on route $i$ in state $s$
- $b_{is}$: congestion coefficient on route $i$ in state $s$
- $\beta_i$: constant in travel cost function
- $B$: bad state
- $C_{is}(N_{is})$: private travel cost on route $i$ in state $s$
- $d$: power coefficient
- $E[\ ]$: expectations operator
- $\epsilon_{is}$: elasticity of travel cost with respect to usage on route $i$ in state $s$
- $F$: full-information regime
- $G$: good state
- $G_{ZF}^{ZF}$: expected welfare gain from shifting from zero information to full information
- $G_{zs}^{ZF}$: welfare gain from shifting from zero information to full information in state $s$
- $i$: index of route
- $k_{is}$: capacity of route $i$ in state $s$
- $MSC$: marginal social cost of trip
- $N$: total number of users
- $N_i$: number of users on route $i$
- $N_{is}$: number of users on route $i$ in state $s$
- $N_{is}^*$: system-optimal number of users on route $i$ in state $s$
- $r$: index of information regime
$\rho$  index of correlation between route conditions

$s$  state

$S$  set of states

$Z$  zero-information regime
10. References


Allon, G., Bassamboo, A., Gurvich, I., 2011. We will be right with you: Managing customer expectations with vague promises and cheap talk. Operations Research 59 (6), 1382-1394.


Appendix

Proof of Proposition 3

The UE condition given by equation (3) in the text is $a_{1s} + b_{1s} \left( N_{1s}^{F} \right)^{d} = a_{2s} + b_{2s} \left( N - N_{1s}^{F} \right)^{d}$ which can be written as

$$b_{2s} \left( N - N_{1s}^{F} \right)^{d} - b_{1s} \left( N_{1s}^{F} \right)^{d} = a_{1s} - a_{2s}. \quad (A1)$$

The SO condition given by equation (6) in the text is $a_{1s} + (d+1)b_{1s} \left( N_{1s}^{*} \right)^{d} = a_{2s} + (d+1)b_{2s} \left( N - N_{1s}^{*} \right)^{d}$ which can be written as

$$a_{2s} - a_{1s} + (d+1) \left( b_{2s} \left( N - N_{1s}^{*} \right)^{d} - b_{1s} \left( N_{1s}^{*} \right)^{d} \right) = 0. \quad (A2)$$

The left-hand side (LHS) of equation (A2) is a strictly decreasing function of $N_{1s}^{*}$. Evaluating the LHS of (A2) with $N_{1s}^{*}$ set at $N_{1s}^{F}$ as given by (A1) one obtains $a_{2s} - a_{1s} + (d+1)(a_{1s} - a_{2s}) = d(a_{1s} - a_{2s}) < 0$.

Assume without loss of generality that $a_{1s} \leq a_{2s}$. If $a_{1s} < a_{2s}$, then $N_{1s}^{F} > N_{1s}^{*}$ and route 1 is overused in the UE with full information. This proves part (a). If $a_{1s} = a_{2s}$, $N_{1s}^{F} = N_{1s}^{*}$. This proves part (b).

Proof of Proposition 4

Given $a_{2s} = a_{1s} \forall s$, the UE and SO coincide in every state. Equations (A1) and (A2) yield:

$$N_{2s}^{F} / N_{1s}^{F} = N_{2s}^{*} / N_{1s}^{*} = \left( b_{1s} / b_{2s} \right)^{1/d}. \quad (A3)$$

With zero information, the UE is defined by condition (3) in the text:

$$E[a_{1s}] + E[b_{1s}] \left( N_{1}^{Z} \right)^{d} = E[a_{2s}] + E[b_{2s}] \left( N_{2}^{Z} \right)^{d} = E[a_{2s}] + E[b_{2s}] \left( N - N_{2}^{Z} \right)^{d},$$

which simplifies to

$$N_{2}^{Z} / N_{1}^{Z} = \left( E[b_{1s}] / E[b_{2s}] \right)^{1/d}. \quad (A4)$$

Since the right-hand-side (RHS) of equation (A3) generally differs from the RHS of (A4), full information is generally superior to zero information and $G^{ZF} > 0$. However, if (and only if) $b_{2s} = \lambda b_{1s}$ for some $\lambda > 0$ and all $s$, so that the probability distributions of the congestion coefficients are perfectly correlated and the same except for scale, then SO usage is the same in all states for any value of parameter $d$, and $G^{ZF} = 0$. Information has no welfare effect in this case because it does not affect usage.

\[ \blacksquare \]
Proof of Proposition 5

Let \( \hat{x} \) denote the expected value of \( x \). With full information, \( \hat{x} \) is the realized value of \( x \), and with zero information \( \hat{x} \) is the unconditional expectation of \( x \), designated \( \overline{x} \). Let \( E[ \ ] \) denote the unconditional expectations operator and \( E_i[ \ ] \) the marginal expectations operator with respect to \( b_i, i=1,2 \). With \( d=1 \), the user equilibrium equation (3) yields

\[
N_1 = \frac{\hat{a}_2 - \hat{a}_1}{b_1 + b_2} + \frac{\hat{b}_2}{b_1 + b_2} N.
\]

Expected total costs are

\[
\hat{C} = \hat{a}_i + \frac{\hat{b}_1}{b_1 + b_2} (\hat{a}_2 - \hat{a}_1) + \frac{\hat{b}_2}{b_1 + b_2} N.
\]

The expected benefit in shifting from zero information to full information is

\[
G^{2E} = a_i + \frac{\bar{b}_1}{b_1 + b_2} (a_2 - a_1) + \frac{\bar{b}_2}{b_1 + b_2} N - E \left[ a_i + \frac{b_1}{b_1 + b_2} (a_2 - a_1) + \frac{b_1 b_2}{b_1 + b_2} N \right]
\]

\[
= \bar{b}_i \frac{\bar{a}_2 - \bar{a}_1}{b_1 + b_2} - E \left[ \frac{b_1}{b_1 + b_2} (a_2 - a_1) \right] + \left( \frac{\bar{b}_1 \bar{b}_2}{b_1 + b_2} - E \left[ \frac{b_1 b_2}{b_1 + b_2} \right] \right) N
\]

\[
= (\bar{a}_2 - \bar{a}_1) \left( \frac{\bar{b}_1}{b_1 + b_2} - E \left[ \frac{b_1}{b_1 + b_2} \right] \right) + \left( \frac{\bar{b}_1 \bar{b}_2}{b_1 + b_2} - E \left[ \frac{b_1 b_2}{b_1 + b_2} \right] \right) N.
\]

where the last line follows from the assumption that \( a_{2s} - a_{1s} \) is stochastically independent of \( b_{1s} \) and \( b_{2s} \). Assume \( \phi \geq 0 \) (similar logic applies if \( \phi < 0 \)). Since route 1 is always used, \( a_i \leq a_{2s} + b_{2s} N \) for all \( s \).

Hence, \( a_{2s} - a_{1s} \geq -b_1^w N \) and also \( \bar{a}_2 - \bar{a}_1 \geq -b_2^w N \) where \( b_2^w \equiv \min b_{2s} \). Substituting this inequality into (A5):

\[
\frac{G^{2E}}{N} \geq \frac{b_2^w}{N} \left( E \left[ \frac{b_1}{b_1 + b_2} \right] - \frac{\bar{b}_1}{b_1 + b_2} \right) + \frac{\bar{b}_1 \bar{b}_2}{b_1 + b_2} - E \left[ \frac{b_1 b_2}{b_1 + b_2} \right] = \bar{b}_1 \frac{\bar{b}_2 - b_2^w}{b_1 + b_2} + E \left[ \frac{b_1 (b_2^w - b_{2s})}{b_1 + b_2} \right].
\]

Part (a): independent congestion coefficients

If \( b_{1s} \) and \( b_{2s} \) are statistically independent, inequality (A6) can be written as
\[ \frac{G^{ZF}}{N} \geq \frac{\bar{b}_2 - b_2^m}{\bar{b}_1 + \bar{b}_2} - E_i \left[ \frac{b_i, E_2 \left[ b_{2s} - b_2^m \right]}{b_{i_1} + b_{2s}} \right]. \]

By Jensen's inequality, \( E_i \left[ \frac{b_i, E_2 \left[ b_{2s} - b_2^m \right]}{b_{i_1} + b_{2s}} \right] \leq \frac{\bar{b}_1 - b_2^m}{\bar{b}_1 + \bar{b}_2}. \) Hence \( G^{ZF} \geq 0. \)

Part (b): Equiproportional congestion coefficients

If \( b_{2s} = \lambda b_s \) for all \( s \), equation (A5) simplifies to

\[ G^{ZF} = (\bar{a}_2 - \bar{a}_1) \left( \frac{\bar{b}_1}{(1 + \lambda)\bar{b}_1} - E_i \left[ \frac{b_{i_1}}{b_{i_1} + b_{2s}} \right] \right) + \left( \frac{\lambda}{(1 + \lambda)\bar{b}_1} \right)^2 E_i \left[ \frac{\lambda b_{i_1}^2}{(1 + \lambda)\bar{b}_1} \right] N = 0. \]

Part (c): Correlated free-flow costs and congestion coefficients

To prove part (c) we first establish the following two lemmas:

Lemma 1: Let \( x \) be a random variable. Let \( g(x) \) be an increasing function of \( x \) and \( h(x) \) a decreasing function of \( x \). Then, \( J = E\left[ g(x) h(x) \right] \leq E[g(x)]E[h(x)] = K. \)

Proof of Lemma 1: By the intermediate value theorem for integrals there exists \( \bar{x} \) within the range of \( x \) such that \( E[g(x)] = g(\bar{x}) \). Expectation \( J \) can then be written

\[ J = E\left[ (g(\bar{x}) + g(x) - g(\bar{x})) h(x) \right] = g(\bar{x}) E\left[ h(x) \right] + E\left[ (g(x) - g(\bar{x}))(h(x) + h(x) - h(\bar{x})) \right] \]

\[ = K + h(\bar{x}) E\left[ g(x) - g(\bar{x}) \right] + E\left[ (g(x) - g(\bar{x}))(h(x) - h(\bar{x})) \right] \]

\[ = K + E\left[ (g(x) - g(\bar{x}))(h(x) - h(\bar{x})) \right] \leq K, \]

where the last inequality follows from the monotonicity assumptions on \( g(x) \) and \( h(x) \).

Lemma 2: Let \( x \) be a random variable. Let \( g(x) \) and \( h(x) \) be increasing functions of \( x \). Then,

\[ E\left[ g(x) h(x) \right] \geq E[g(x)]E[h(x)]. \]

Lemma 2 is proved using similar steps as for Lemma 1.
Proof of part (c) continued: The proof of part (a) is applicable as far as the middle line of eqn. (A5):
\[
G^{ZF} = \frac{\bar{b}_1}{\bar{b}_1 + \bar{b}_2} \bar{a}_2 - \bar{a}_1 - E \left[ \frac{b_{i_1}}{b_{i_1} + b_{2s}} (a_{2_1} - a_{1_1}) \right] + \left( \frac{\bar{b}_1 \bar{b}_2}{\bar{b}_1 + \bar{b}_2} - E \left[ \frac{b_{i_1} b_{2s}}{b_{i_1} + b_{2s}} \right] \right) N. \tag{A7}
\]

It is necessary to establish an upper bound on the middle term of (A7). Now
\[
E \left[ \frac{b_{i_1}}{b_{i_1} + b_{2s}} (a_{2_1} - a_{1_1}) \right] = E_1 \left[ E_2 \left[ \frac{b_{i_1}}{b_{i_1} + b_{2s}} a_{2_1} \right] \right] - E_2 \left[ E_1 \left[ \frac{b_{i_1}}{b_{i_1} + b_{2s}} a_{1_1} \right] \right].
\]

Taking \( b_{2s} \) as the random variable, we have by Lemma 1,
\[
E_1 \left[ E_2 \left[ \frac{b_{i_1}}{b_{i_1} + b_{2s}} a_{2_1} \right] \right] \leq E_2 \left[ E_1 \left[ \frac{b_{i_1}}{b_{i_1} + b_{2s}} a_{2_1} \right] - \frac{b_{i_1}}{b_{i_1} + b_{2s}} \right] \bar{a}_2. \tag{A8}
\]

Taking \( b_{1_1} \) as the random variable, we have by Lemma 2,
\[
E_2 \left[ E_1 \left[ \frac{b_{i_1}}{b_{i_1} + b_{2s}} a_{1_1} \right] \right] \geq E_1 \left[ \frac{b_{i_1}}{b_{i_1} + b_{2s}} a_{1_1} \right] = E \left[ \frac{b_{i_1}}{b_{i_1} + b_{2s}} \right] \bar{a}_1. \tag{A9}
\]

Combining (A8) and (A9), it follows that
\[
E \left[ \frac{b_{i_1}}{b_{i_1} + b_{2s}} (a_{2_1} - a_{1_1}) \right] \leq E \left[ \frac{b_{i_1}}{b_{i_1} + b_{2s}} \right] (\bar{a}_2 - \bar{a}_1). \tag{A10}
\]

Substituting (A10) into (A7)
\[
G^{ZF} \geq (\bar{a}_2 - \bar{a}_1) \left( \frac{\bar{b}_1}{\bar{b}_1 + \bar{b}_2} - E \left[ \frac{b_{i_1}}{b_{i_1} + b_{2s}} \right] \right) + \left( \frac{\bar{b}_1 \bar{b}_2}{\bar{b}_1 + \bar{b}_2} - E \left[ \frac{b_{i_1} b_{2s}}{b_{i_1} + b_{2s}} \right] \right) N. \tag{A11}
\]

The proof is concluded by using the proof for part (a).

Part (d): Congestion coefficients perfectly correlated, but not equiproportional

To show that full information can be welfare-reducing it suffices to provide an example. Consider the following variant of the example in Section 4, which is similar to Example 4 in de Palma and Lindsey (1994). Set \( N=1 \). Assume free-flow travel costs are deterministic, with \( a_1 = 6 \) and \( a_2 = 9 \). Travel conditions are either good on both routes or bad on both routes. Bad conditions occur with
probability 1/4. In good conditions, \( b_1 = b_2 = 7.8 \). In bad conditions, \( b_1 = 25 \) and \( b_2 = 35 \). It is straightforward to show that \( E[TC^Z] = 13.98 \), \( E[TC^F] = 14.01 \), and \( G^{ZF} = -0.032 \).

Proof of Proposition 6

Part (a): independent congestion coefficients

Suppose \( a_{2s} = a_{1s} \) \( \forall s \). Then, by Proposition 1, \( G^{ZF} \geq 0 \) and \( G^{ZF} > 0 \) unless the congestion coefficients are equiproportional in all states. When \( a_{2s} \neq a_{1s} \), a case in which \( G^{ZF} < 0 \) obtains with parameter values: \( d = 2 \), \( N = 1 \), \( \pi_1 = \pi_2 = 0.25 \), \( a_1 = 0 \), \( a_2 = 3 \), \( b_{1u} = 4 \), \( b_{1l} = 6 \), \( b_{2u} = 2 \), \( b_{2l} = 3 \). For this example, \( G^{ZF} = -0.01396 \).

Part (b): equiproportional congestion coefficients

With \( d = 2 \) and \( \hat{b}_s \neq \hat{a}_s \), the user equilibrium equation (3) yields

\[
N_1 = \frac{-\hat{b}_1 N + \left( \hat{b}_2 N^2 - (\hat{b}_1 - \hat{b}_2)(a_1 - a_2) \right)^{1/2}}{\hat{b}_1 - \hat{b}_2}.
\]

If \( \hat{b}_2 = \hat{b}_1 = \hat{b} \), the solution is \( N_1 = \left( \hat{a}_2 - \hat{a}_1 + \hat{b} N^2 \right) / (2\hat{b} N) \). Attention is restricted here to the case \( \hat{b}_2 \neq \hat{a}_1 \); the case \( \hat{b}_2 = \hat{a}_1 \) is treated similarly. Expected total costs are

\[
\hat{C} = \hat{a}_1 + \hat{b}_1 \left( \frac{\hat{b}_1 \hat{b}_2 N^2 + \left( \hat{b}_1 - \hat{b}_2 \right) \hat{A}^{1/2} - \hat{b}_2 N}{\hat{b}_1 - \hat{b}_2} \right)^2,
\]

where \( \hat{A} = \hat{a}_2 - \hat{a}_1 \). If \( b_{2s} = \lambda b_{1s} \), \( \forall s \),

\[
G^{ZF} = \frac{2\lambda N}{(\lambda - 1)^2} \left[ E_i \left[ \left( \lambda b_{1s}^2 N^2 - (\lambda - 1)b_{1s} A \right)^{1/2} \right] - \left( \lambda \bar{b}_{1s}^2 N^2 - (\lambda - 1)\bar{b}_1 \bar{A} \right)^{1/2} \right]
\]

\[
= E \left[ \left( \lambda b_{1s}^2 N^2 - (\lambda - 1)b_{1s} A \right)^{1/2} \right] - \left( \lambda \bar{b}_{1s}^2 N^2 - (\lambda - 1)\bar{b}_1 \bar{A} \right)^{1/2}
\]
where \( s \) means has the same sign as. Define \( Y_s = \lambda b_i^2 N^2 - (\lambda - 1) b_i A_i \). Differentiating the term \( Y_s^{1/2} \) in (A12) twice with respect to \( b_i \) one gets

\[
\frac{\partial^2 G^\text{ZF}}{\partial b_i^2} = -\frac{1}{4} (\lambda - 1)^2 A_i Y_s^{-3/2} < 0 \text{ if } \lambda \neq 1,
\]

\[
\frac{\partial^2 G^\text{ZF}}{\partial A_i^2} = -\frac{1}{4} (\lambda - 1)^2 b_i^2 Y_s^{-3/2} < 0 \text{ if } \lambda \neq 1,
\]

\[
\frac{\partial^2 G^\text{ZF}}{\partial b_i \partial A_i} = \frac{1}{4} (\lambda - 1)^2 b_i A_i Y_s^{-3/2},
\]

\[
\frac{\partial^2 G^\text{ZF}}{\partial b_i^2} \frac{\partial^2 G^\text{ZF}}{\partial A_i^2} - \left( \frac{\partial^2 G^\text{ZF}}{\partial b_i \partial A_i} \right)^2 = 0.
\]

By Jensen's inequality for multivariate functions, it follows that \( G^\text{ZF} \leq 0 \).
Table 1. Probabilities of states with maximum correlation between routes

(a) Base-case parameters. Correlation coefficient 1.00

<table>
<thead>
<tr>
<th>Route 1 conditions</th>
<th>Route 2 conditions</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Good</td>
<td>Bad</td>
</tr>
<tr>
<td>Good</td>
<td>0.75</td>
<td>0</td>
</tr>
<tr>
<td>Bad</td>
<td>0</td>
<td>0.25</td>
</tr>
</tbody>
</table>

(b) \( \pi_1 = 0.375, \pi_2 = 0.125 \). Correlation coefficient 0.488

<table>
<thead>
<tr>
<th>Route 1 conditions</th>
<th>Route 2 conditions</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Good</td>
<td>Bad</td>
</tr>
<tr>
<td>Good</td>
<td>0.625</td>
<td>0</td>
</tr>
<tr>
<td>Bad</td>
<td>0.25</td>
<td>0.125</td>
</tr>
</tbody>
</table>

(c) \( \pi_1 = 0.125, \pi_2 = 0.375 \). Correlation coefficient 0.488

<table>
<thead>
<tr>
<th>Route 1 conditions</th>
<th>Route 2 conditions</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Good</td>
<td>Bad</td>
</tr>
<tr>
<td>Good</td>
<td>0.625</td>
<td>0.25</td>
</tr>
<tr>
<td>Bad</td>
<td>0</td>
<td>0.125</td>
</tr>
</tbody>
</table>
Table 2. Instances in which full information is welfare-reducing

<table>
<thead>
<tr>
<th>Route correlation</th>
<th>Assumptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposition 5 (linear costs), part (d).</td>
<td>Perfect, positive</td>
</tr>
<tr>
<td>Proposition 6 (quadratic costs) and example in Figure 1.</td>
<td>Perfect, positive</td>
</tr>
<tr>
<td>Example with quadratic costs in Figures 3 &amp; 4.</td>
<td>Perfect, positive</td>
</tr>
<tr>
<td>Example with quadratic costs in Figure 7.</td>
<td>None</td>
</tr>
<tr>
<td>Example with highly nonlinear costs in Figure 9.</td>
<td>Perfect, positive</td>
</tr>
</tbody>
</table>
Figure 1. The information paradox with perfectly correlated route conditions. Top panel: good conditions on both routes; Bottom panel: bad conditions on both routes.

Source: Authors’ calculation

Note: DWL = Deadweight loss
Figure 2. Benefits from full information with no correlation between route conditions, base-case parameter values. Top panel: Welfare gain $G^{ZF}$; Bottom panel: efficiency index $\omega$.

Source: Authors’ calculation
Figure 3. Benefits from full information with perfect correlation between route conditions, base-case parameter values. Top panel: Welfare gain \( G^{ZF} \); Bottom panel: efficiency index \( \omega \).

Source: Authors’ calculation
Figure 4. States in which full information is welfare-reducing, base-case parameter values

Source: Authors’ calculation
Figure 5. States in which full information is welfare-reducing, \( \pi_1 = 0.375, \pi_2 = 0.125 \).

Source: Authors’ calculation
Figure 6. States in which full information is welfare-reducing, \( \pi_1 = 0.125, \pi_2 = 0.375 \).

Source: Authors’ calculation
Figure 7. Efficiency index $\omega$ for full information and no correlation between route conditions, $b_{1G} = 3, b_{2G} = 3$.

Source: Authors’ calculation
Figure 8. Effects of full information with no correlation between route conditions, $b_{1G} = 3$, $b_{2G} = 3$, $b_{1B} = 6$, $b_{2B} = 9$.

Source: Authors’ calculation
Figure 9. Efficiency index $\omega$ for full information with perfect correlation between route conditions, $d=10$.

Source: Authors’ calculation