Online Product Reviews: Implications for Retailers and Competing Manufacturers

Young Kwark, Jianqing Chen, Srinivasan Raghunathan

The University of Texas at Dallas

youngk082@utdallas.edu, chenjq@utdallas.edu, sraghu@utdallas.edu

Abstract

This paper studies the effect of online product reviews on different players in a channel structure. We consider a retailer selling two substitutable products produced by different manufacturers, and the products differ in both their qualities and the fits to consumers’ needs. Online product reviews provide additional information for consumers to mitigate the uncertainty about the quality of a product and about its fit to consumers’ needs. We show that the effect of reviews on the upstream competition between the manufacturers is critical in understanding which firms gain and which firms lose. The upstream competition is affected in fundamentally different ways by quality information and fit information, and each information type has different implications for the retailer and manufacturers. The quality information homogenizes consumers’ perceived utility differences between the two products and increases the upstream competition, which benefits the retailer but hurts the manufacturers. The fit information heterogenizes consumers’ estimated fits to the products and softens the upstream competition, which hurts the retailer but benefits the manufacturers. Furthermore, reviews may also alter the the nature of upstream competition from one in which consumers’ own perception on the quality dimension plays a dominant role in consumers’ comparative evaluation of products to one in which fit dimension plays a dominant role. If manufacturers do not respond strategically to reviews and keep the same wholesale prices regardless of reviews (i.e., the upstream competition is assumed to be unaffected by reviews), then, we show that reviews never hurt the retailer and the manufacturer with favorable reviews, and never benefit the manufacturer with unfavorable reviews, a finding that demonstrates why reviews’ effect on upstream competition is critical for firms in online marketplaces.

Keywords: online product reviews; competition; game theory
1 Introduction

With the ubiquity of the internet and the prevalence of user-generated content, an increasing number of consumers read online product reviews before they make purchase decisions (Deloitte and Touche, 2008; Cone, 2010). In particular, online product reviews have become an important information source for consumers to mitigate the uncertainty about the quality of a product and about its fit to consumers’ needs (Chen and Xie, 2008). Consumer surveys report that online product reviews strongly influence consumers’ purchase decisions. According to Deloitte and Touche (2008), while 43% of surveyed consumers were reinforced of their original purchase intention by reviews, 43% of consumers changed their opinions about which product to buy and 9% of consumers even abandoned their purchase plan after reading the product reviews. These data suggest that consumers rely on their own assessments of products as well as others’ assessments embedded in the product reviews while evaluating competing products and making purchase decisions.

Third-party websites such as CNET.com and online retailer sites such as Amazon.com provide both expert and consumer reviews. An important feature of online product reviews is that they are public and common to all consumers as well as sellers. On the one hand, some information revealed by the reviews, such as the quality of a product, would have similar effect on purchase decisions of all consumers. Therefore, the public and common reviews would play an important role in shifting a product’s demand. On the other hand, the reviews also provide additional information about a product’s fit to consumers’ needs. Because different consumers may have different needs, the information in the fit dimension may have different effects on consumers’ purchase decisions. Many empirical studies have examined the effect of reviews on consumers’ purchase decisions and retailer’s sales (e.g., Chevalier and Mayzlin, 2006; Zhu and Zhang, 2010). Departing from these studies, we aim to analytically study the effect of online product reviews in a channel structure and investigate how reviews affect the upstream competition between manufacturers as well as the retailer in this paper.

The question of how online product reviews affect the price competition between substitutable products is important to both practitioners and academics. The question becomes especially important in a context of a dominant retailer selling competing products from different manufacturers, because such a two-level channel structure with one dominant retailer is commonly observed in practice (e.g., Amazon.com). In such contexts, while each manufacturer views the substitutable products from other manufacturers as competitors, the retailer may view the products as satisfying the different needs of different consumers. Therefore, reviews may have fundamentally different
effects on the manufacturers and retailer. Furthermore, from a retailer’s perspective, an analysis of reviews’ effects on both consumers (demand side) and manufacturers (supply side) is essential for a more complete understanding of reviews’ implications. However, the effect of online product reviews on the retailer and individual manufacturers remains unexplored and unclear despite the large volume of studies dedicated to the online review phenomenon.

To address this question, we develop a game theoretic model in which one retailer sells two substitutable products produced by different manufacturers. The products differ in both their qualities and the fits to consumers’ needs. While all consumers value high quality rather than low quality, different consumers have different needs, with some consumers perceiving one product more suitable than the other product while others perceiving the opposite way. Each consumer has her own assessment of the quality of each product and its fit to her need. The online product reviews provide additional information in both the quality and fit dimensions. We distinguish the case in which the consumers’ own perception about the quality dimension plays a dominant role in determining consumers’ perceived utility differences between the two products, and the case in which the fit dimension plays an important role such that the fit is critical for some consumers. We call the former the quality-dominates-fit case and the latter the fit-dominates-quality case. We use the scenario without product reviews as the benchmark and study the effect of online product reviews on the competition between the two manufacturers as well as on the retailer.

We find the information in the quality dimension and fit dimension embedded in the online product reviews has very different effects on the competition between the two products. We show that reviews reduce the heterogeneity of consumers’ perceived quality differences and thus increase the competition between the two manufacturers. We call this reduced heterogeneity resulting from the reviews variance-reducing effect, which generally hurts the manufacturers and benefits the retailer. Additionally, reviews shift the mean perceived quality difference in favor of the product with favorable reviews. We call this mean-shifting effect, which generally benefits the manufacturer with favorable reviews and the retailer. As a result, in the quality-dominates-fit case, the retailer benefits from the reviews. The manufacturer with unfavorable reviews suffers from the reviews not only because of unfavorable comments which shifts its demand away but also because of the increased competition. In equilibrium, we find that the manufacturer is induced to lower its wholesale price and earns less profit because of the reviews. Interestingly, even the manufacturer with favorable reviews would be worse off if the negative effect from the reduced heterogeneity dominates the positive effect from the favorable reviews.
In contrast, for the fit dimension, online product reviews enable consumers to better understand the products’ fits to their needs. We demonstrate that, because of the reviews, consumers are differentiated further from each other in their perceived fits. This result occurs because consumers have different underlying needs and they learn better about the products’ fits to their needs with the additional product information from the reviews. We call this increased heterogeneity resulting from the reviews variance-increasing effect. The variance-increasing effect softens the competition between the two manufacturers, which generally benefits the manufacturers and hurts the retailer. As a result, in the fit-dominates-quality case, reviews hurt the retailer if the negative impact of variance-increasing effect outweighs the positive impact of mean-shifting effect. The manufacturer with favorable reviews in the quality dimension benefits from the reviews in both the positive comments about its product quality and the reduced competition. Therefore, in equilibrium, the manufacturer charges a higher wholesale price and earns a higher profit. It is worth noting that the manufacturer with unfavorable reviews can also be better off if the positive effect of reduced competition offsets the negative effect from unfavorable comments about its product quality.

Interestingly, we find that reviews with sufficiently high precision may alter the nature of upstream competition from one in which quality dimension plays a dominant role in consumers’ utility assessment to one in which fit dimension plays a dominant role. Therefore, a retailer that benefits from reviews when the review precision is not high may be hurt by them if the review precision is high because, as explained in the previous two paragraphs, the reviews’ impact on the upstream competition is fundamentally different in the quality-dominates-fit and fit-dominates-quality cases.

All of the above results hold only when manufacturers respond strategically to reviews. If manufacturers keep the same wholesale prices regardless of reviews (i.e., reviews do not have any effect on upstream competition), then we show that reviews never hurt the retailer nor the manufacturer with favorable reviews, and never benefit the manufacturer with unfavorable reviews. Stated more generally, our main result is that when online product reviews mitigate consumers’ uncertainty about quality and fit dimensions of perceived utility difference between competing products, the effect of reviews on the upstream competition between manufacturers is critical in understanding which firms gain and which firms lose.

Several recent studies have analyzed the effect of product reviews on firms. Empirical studies have examined the impact of reviews on firm’s sales, and the findings have been mixed. For instance, while some studies found a significant positive association between rating valence and sales (Chevalier and Mayzlin, 2006; Clemons et al., 2006; Duan et al., 2008), others did not find a rela-
tionship between the two (Chen et al., 2004; Liu, 2006). The variance of product ratings (Clemons et al., 2006), the volume of ratings (Liu, 2006), text reviews (Archak et al., 2011), and the reviewer characteristics or product characteristics (Forman et al., 2008; Zhu and Zhang, 2010) have been found to have an impact on sales. Online product reviews have also been found to be subject to self-selection biases that impact consumer purchase behavior (Li and Hitt, 2008), and to reflect not only perceived quality but also the perceived value which is the difference between perceived quality and price (Li and Hitt, 2010). These empirical results suggest that sellers may have an incentive to manipulate reviews of their products to improve their competitive position. Dellarocas (2006) and Mayzlin (2006) analyze sellers’ incentives to manipulate reviews and show that reviews are informative even under seller manipulation. Jiang and Chen (2007) show that vendors have an incentive to induce higher product ratings by under-charging in early periods. Different from these studies, we view the reviews as information mitigating the uncertainty in a product quality and the fit to consumers’ needs, and investigate how this additional information affects upstream product competition.

Some existing analytical work has modeled product reviews as information that enables consumers to identify products matching their needs (Chen and Xie, 2008) or estimate their true utilities more accurately (Li et al., 2011; Sun, 2011). Many of these models focus on how the nature of a product in their market appeal (e.g., mass or niche products) affects review outcome (e.g., positive or negative), and in turn how the reviews affect consumers’ willingness-to-pay and therefore the product demand. In addition, these models typically consider the effect of reviews on sellers in a framework of direct selling from sellers to consumers. For example, Li et al. (2011) study how consumer reviews mediate the competition between two direct sellers when consumers face repeat purchases and a cost to switch products between periods, and consumer reviews convey product fits to them. In contrast, we consider a two-level channel structure with a retailer selling products from competing manufacturers and consumers facing two dimensional product uncertainty—both the product quality and the fit to their needs. Interestingly, we find that the quality information and fit information play very different roles in changing the upstream competition between the manufacturers, and the same reviews would have very different implications for the retailer and each manufacturer.

Another stream of research has modeled product reviews as free advertising and analyzed how sellers should adjust their own marketing mix strategies in the presence of the reviews (Chen and Xie, 2005; Jiang and Srinivasan, 2010). Most of the papers in this stream assume that sellers know
consumers’ ex ante expectation of product valuation and how reviews affect this expectation. One particularly related paper is Shaffer and Zettelmeyer (2002), which analyzes the effects of information provision on the profits of channel members when the information is supplied by third parties. In their model, all consumers have the same product information, additional information has the same qualitative impact (positive or negative) on every consumer, and sellers have perfect knowledge of all product information. In our model, we consider that consumers have private estimates of the products’ qualities and fits to their needs, and online product reviews provide public and common additional information about quality and private and idiosyncratic additional information about fit to consumers. Our results and insights lie in the changes in consumer heterogeneity resulting from the changes in the uncertainty facing consumers because of reviews, which differ from the existing work. Our paper is also related to the ones using horizontal differentiation models (Chen and Xie, 2008; Li et al., 2011; Gu and Xie, 2012; Shaffer and Zettelmeyer, 2002; Villias-Boas, 2004; Sun, 2011). All these papers consider consumer “fit” or “taste” as an important factor in consumers’ utility function. Different approaches have been used in the literature to model consumer fit/taste. Some papers use discrete taste models in which the fit dimension is modeled to be perfect match or mismatch and, in the case of mismatch, consumers incur some disutility (e.g., in Chen and Xie, 2008; Li et al., 2011; Gu and Xie, 2012). Others use continuous taste models in which the degree of fit is modeled as a continuous variable and different consumers may have different degrees of misfit and thus incur different disutilities (e.g., in Shaffer and Zettelmeyer, 2002; Villias-Boas, 2004; Sun, 2011). In particular, Hoteling model and its variations have been widely used for a duopoly setting: the misfit cost is modeled as the degree of misfit times a unit misfit cost, and a consumer’s degree of misfit to one product is negatively correlated to the misfit to the other product. Our model for consumer fit dimension belongs to the continuous taste models.

The rest of this paper is organized as follows. In the next section, we lay out the model. In Section 3, we derive the main results of the effect of the reviews on the upstream competition and the retailer. In Section 4, we show our results hold in more general settings than the one considered in the base model. Section 5 concludes the paper.

2 Model

We consider a retailer $R$ that sells two products, $A$ and $B$. The products are imperfect substitutes, and are produced by different manufacturers. We call the manufacturer that produces product $A$
(B) manufacturer A (B). The marginal production cost for each product is assumed to be zero. The manufacturers sell their products to the retailer and the retailer sells them to end consumers. Each consumer has a unit demand.

**Consumer Utility:** Each product is characterized by a quality attribute and a fit attribute. The quality attribute represents the vertical dimension in the sense that every consumer prefers high quality to low quality. The fit attribute represents the horizontal dimension in the sense that preferences vary across consumers. The quality of a product determines the maximum value that a consumer derives from the product, which is denoted as $x_i$, $i \in \{A, B\}$. The products may not have perfect fits to consumers and thus consumers incur misfit costs. As in typical “location” models of product differentiation, the misfit cost is modeled as the degree of misfit $\lambda$, $\lambda \in [0, 1]$, times a unit misfit cost $t$, and a consumer’s degree of misfit to product $A$ is negatively correlated to the misfit to product $B$. In particular, when the degree of misfit between a consumer and product $A$ is $\lambda$, the degree of misfit between the consumer and product $B$ is $(1 - \lambda)$. A consumer’s utility from a product, $U_i$, is the maximum value that the product offers net the misfit cost. A consumer’s net utility from a product is the utility net the retail price. Denoting the retail price as $p_i$, $i \in \{A, B\}$, we can formulate the net utilities derived from products $A$ and $B$ for the consumer with a degree of misfit $\lambda$ to product $A$ as follows.

\[
\begin{align*}
V_A &= U_A - p_A = x_A - \lambda t - p_A \\
V_B &= U_B - p_B = x_B - (1 - \lambda)t - p_B
\end{align*}
\]

Therefore, the net utility difference between product $A$ and $B$, $V_A - V_B$, for the consumer with the degree of misfit $\lambda$ to product $A$ is

\[
V_A - V_B = (U_A - U_B) - (p_A - p_B) = (x_A - x_B) + (1 - 2\lambda)t - (p_A - p_B)
\]

We also call $(x_A - x_B)$ the quality difference of the two products. Unless otherwise indicated, we call $\lambda$ the degree of misfit. We assume that the (true) quality difference between the products is zero.\(^1\) A continuum of consumers of measure 1 have different (true) degrees of misfit $\lambda$, which satisfies a uniform distribution.

**Product Uncertainty and Online Product Reviews:** Different from the standard vertical

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\(^1\)We can show that our results generalize to the case when the true quality difference is non-zero if consumers' assessment (discussed in the next paragraph) of the true quality difference is unbiased.
differentiation and horizontal differentiation models, consumers are uncertain about both product quality and the misfit, where the online product reviews play a role as in Chen and Xie (2008). That is, consumers do not know the true quality difference or their true degrees of misfit. In the absence of online product reviews, based on the product description and other information sources, each consumer has her own assessment of the quality difference between the two products and of the misfit. We denote a consumer’s own assessment of the quality difference as $x_C$. Similar to the approach often used in the literature (e.g., Lewis and Sappington, 1994; Ruckes, 2004; Johnson and Myatt, 2006; McCracken, 2011; Petriconi, 2012), the uncertainty in the misfit is modeled as that a consumer observes a signal $s$, which reveals the consumer’s true degree of misfit with probability $\beta_C$, and with probability $(1 - \beta_C)$ is uninformative and follows the true distribution; that is, $\Pr(s = y|\lambda = y) = \beta_C$ and $\Pr(s \neq y|\lambda = y) = 1 - \beta_C$, where $y \in [0, 1]$. Based on Bayesian updating, we can derive $E(\lambda|s = y) = [\beta_C y + (1 - \beta_C)/2]$ (see the proof in the appendix). Substituting $x_C$ and the expected degree of misfit into Equation (1), the expected net utility difference between product $A$ and $B$ for the consumer with perceived quality difference $x_C$ and signal $s = y$ on the degree of misfit is then

$$E(V_A - V_B) = x_C + (1 - 2y)\beta_C t - (p_A - p_B) \tag{2}$$

Different consumers perceive different $x_C$ and receive different signals $y$. We assume that at the aggregate level consumers’ perceived quality differences satisfy a uniform distribution over $[-\epsilon, \epsilon]$. The uncertainty model for the signal about misfit implies that the signals satisfy a uniform distribution over $[0, 1]$. The retailer does not know an individual consumer’s perceived quality difference or signal about the misfit, but knows their distributions.

Online product reviews provide public information about the products, and consumers use this information in addition to their own assessments to evaluate the products. We denote as $x_R$ the perceived quality difference in the two products revealed by the online product reviews, which is common to all consumers. In the presence of online product reviews, consumers combine their own assessments $x_C$ and the public common assessment $x_R$ to form their judgment of the quality difference between the two products. As shown by Bates and Granger (1969) using the minimum variance estimation, the consumer’s expected quality difference becomes

$$rx_C + (1 - r)x_R \tag{3}$$

where $r, r \in (0, 1)$, depends on the relative precisions of the two information sources, and the weight
on the product reviews, \((1 - r)\), is high when the precision of the product review information is high. Intuitively, consumers adjust their quality assessments because of the additional information from the product reviews, and the extent to which the reviews affect consumers’ assessments depends on the relative precisions and confidence between their own assessments and the product reviews. For the fit dimension, with the additional information provided by online product reviews, consumers know better about their idiosyncratic fit. Similar to the scenario where reviews are unavailable, in the scenario where reviews are available the uncertainty in the misfit is modeled as that a consumer observes a signal. However, this signal, because of the additional information provided by online product reviews, is more informative than that in absence of online product reviews. In particular, this signal reveals a consumer’s true degree of misfit with probability \(\beta_R\) and with probability \((1 - \beta_R)\) is uninformative, where \(\beta_R > \beta_C\). Similarly, by Bayesian updating, we can derive \(\mathbb{E}(\lambda|s = y) = [\beta_R y + (1 - \beta_R)/2]\) in the presence of reviews. Notice that a consumer’s signal in the presence of reviews can be different from or the same as the one in the absence of reviews, but the signal is more accurate about the degree of misfit with reviews than without. Based on Equation (1), in the presence of online product reviews, the expected net utility difference between product \(A\) and \(B\) for the consumer with perceived quality difference \(x_C\) and signal \(s = y\) on the degree of misfit is then

\[
\mathbb{E}(V_A - V_B) = [rx_C + (1 - r)x_R] + (1 - 2y)\beta_R t - (p_A - p_B)
\]  

(4)

**Timing of the Game:** The sequence of events is as follows. In stage 1, manufacturers set wholesale prices \(w_i\) simultaneously. In stage 2, the retailer sets retail prices \(p_i\). In stage 3, consumers evaluate difference of product utility, and make their purchase decisions. We consider two scenarios: one without online product reviews and the other with reviews. We use the scenario without reviews as the benchmark to analyze the effect of reviews. In the scenario with reviews, reviews are observed by consumers, the manufacturers, and the retailer before they make their decisions. Therefore, consistent with many existing studies (e.g., Shaffer and Zettelmeyer, 2002), we do not model the review generating process and instead we examine the effect of reviews when product reviews are in a steady state.\(^2\) Consumers’ own estimates about product quality differences and their misfits are their private information. All other model parameters are common knowledge. All players are risk neutral.

\(^2\)The evidence is abundant that the reviews influence consumers’ purchase decisions and product demands (e.g., Deloitte and Touche, 2008) and firms adjust product prices in response to the reviews (e.g., Jiang and Wang, 2008; Shin et al., 2011).
Demand Functions: In stage 3 of the game, consumers learn the expected utility differences between two products. Based on Equations (2) and (4), we can uniformly formulate a consumer’s expected net utility difference for both the without-review and with-review scenarios as

\[ E(V_A - V_B) = E(U_A - U_B) - (p_A - p_B) = [\gamma x_C + (1 - \gamma)x_R] + (1 - 2y) \beta t - (p_A - p_B) \]  

in which \( \{\gamma, \beta\} \in \{(1, \beta_C),\{r, \beta_R\}\} \) with \( \gamma = 1 \) and \( \beta = \beta_C \) for the scenario without reviews and \( \gamma = r \) and \( \beta = \beta_R \) for the scenario with reviews. \( E(U_A - U_B) = [\gamma x_C + (1 - \gamma)x_R] + (1 - 2y) \beta t \) is the consumer’s expected utility difference. As in typical “location” models of horizontal product differentiation, we assume that the maximum value that each product delivers is high enough such that consumers derive positive net utility from each product.

Clearly, besides consumers’ own assessments, the product reviews affect consumers’ perceived net utility differences between the two products by changing \( \gamma \) and \( \beta \). We focus our analysis on the cases in which online product reviews play a mild or moderate role in changing the competition between the two manufacturers such that in equilibrium two manufacturers are comparably competitive. The extreme case in which the additional difference revealed by online product reviews is so dramatic such that one manufacturer has a dominant advantage in the market is not considered in this study.

We next distinguish two cases:

- Quality-dominates-fit case: in which consumers’ own perception on the quality dimension dominates the fit dimension such that there exist consumers who have the lowest fit with product A but derive higher net utility from it than from product B because their own assessment on the quality dimension is favorable toward product A, and there also exist consumers who have the lowest fit with product B but derive higher net utility from it. Figure 1a illustrates the quality-dominates-fit case.

- Fit-dominates-quality case: in which the fit dimension dominates consumers’ own assessment on quality dimension such that consumers who have perfect fit with a product always derive a higher net utility from that product, regardless of their own assessment on the quality dimension. Figure 1b illustrates the fit-dominates-quality case.

The essential features that distinguish the above two cases are similar to those that distinguish search goods and experience goods—two important concepts of product types discussed widely in the economics literature (e.g., Nelson, 1970; Garvin, 1984; Sutton, 1986). Search goods are likely to come under the quality-dominates-fit case, and experience goods come under the fit-dominates-quality
case. In general, products evaluated based on the objective indices such as product performance, reliability, and durability are likely to be quality-dominant (Garvin, 1984). Examples include digital camera, GPS, and hardware. Products evaluated based on subjective consumer-specific indices such as experience attributes, features, and aesthetics are more fit-dominant (Sutton, 1986). Examples include jewelry and video games. In the sequel, we derive the conditions under which each case occurs in equilibrium.

In quality-dominates-fit case, as illustrated in Figure 1a, for any consumer who receives signal $y$, $y \in [0,1]$, her perceived net utility difference would be positive or negative, depending on her perceived quality difference. By Equation (5), if her perceived quality difference is higher than $\frac{1}{\gamma} \left[ (p_A - p_B) - (1 - \gamma) x_R - (1 - 2y) \beta t \right]$, she derives higher net utility from product $A$; otherwise, she derives higher net utility from product $B$. Therefore, we can formulate the base demand for each product as

$$
\hat{D}_A = \int_0^1 \int_{p_A - p_B - (1 - \gamma) x_R - (1 - 2y) \beta t}^{p_A - p_B - (1 - \gamma) x_R - (1 - 2y) \beta t} \frac{1}{2\epsilon} dxdy = \frac{1}{2} - \frac{1}{2\epsilon} \gamma x_R \\
\hat{D}_B = \int_0^1 \int_{p_A - p_B - (1 - \gamma) x_R - (1 - 2y) \beta t}^{p_A - p_B - (1 - \gamma) x_R - (1 - 2y) \beta t} \frac{1}{2\epsilon} dxdy = \frac{1}{2} + \frac{1}{2\epsilon} \gamma x_R
$$

where the integral in product $i$'s base demand measures the consumers who derive higher net utility from product $i$ than from the other product, $i \in \{A,B\}$.

In fit-dominates-quality case, consumers who perceive a strong fit with product $A$ always derive higher net utility from product $A$, and consumers who perceive a strong fit with product $B$ always derive higher net utility from product $B$, regardless of the perceived quality difference. As illustrated in Figure 1b, we can denote the former consumer group as those who receive signal $y \in [0, y_A]$ and the latter as those who receive signal $y \in [y_B, 1]$ along the line, because of the monotonicity between the net utility difference and the fit dimension. The consumers who receive signals between $y_A$ and

![Figure 1: Competitive Cases](image)
may derive higher net utility from product $A$ or from product $B$, depending on their perceived quality differences. The marginal consumer $y_A$ ($y_B$) is the one who derives the same utility from the two products when perceiving the largest quality difference against product $A$ ($B$); that is, when $x_C = -\epsilon$ ($x_C = \epsilon$). By Equation (5), we have

$$
\begin{align*}
y_A &= \frac{1}{\beta t} \left[\gamma \epsilon + (1 - \gamma) x_R + \beta t - (p_A - p_B)\right] \\
y_B &= \frac{1}{\beta t} \left[\gamma \epsilon + (1 - \gamma) x_R + \beta t - (p_A - p_B)\right] 
\end{align*}
$$

We then can formulate the base demand for each product in fit-dominates-quality case as

$$
\tilde{D}_A = \int_{y_A}^{y_B} dy + \int_{y_A}^{y_B} \int_{-\epsilon}^{\frac{p_A - p_B - (1 - \gamma) x_R - (1 - 2\gamma) \beta t}{\beta t}} \frac{1}{2\epsilon} dx dy = \frac{1}{2} - \frac{1}{2\beta t} [p_A - p_B - (1 - \gamma) x_R] \\
\tilde{D}_B = \int_{y_A}^{y_B} \int_{-\epsilon}^{\frac{p_A - p_B - (1 - \gamma) x_R - (1 - 2\gamma) \beta t}{\beta t}} \frac{1}{2\epsilon} dx dy + \int_{y_A}^{y_B} dy = \frac{1}{2} + \frac{1}{2\beta t} [p_A - p_B - (1 - \gamma) x_R]
$$

Notice that the above (base) demand functions (6) and (8) are functions of $(p_A - p_B)$. Therefore, if consumers have no outside options, when the retailer increases the prices of both products by the same margin, the demand for each product stays the same. As a result, the retailer has incentive to increase the prices to the largest extent possible and the optimal retail prices are the corner solutions. To derive interior solutions, we assume that some consumers may find an outside option (e.g., buying from a different retailer) more attractive than buying their preferred product from the focal retailer, and that the number of such consumers is increasing in the prices charged by the retailer. This assumption ensures that the retailer suffers a penalty in the form of reduced demand for increasing prices. We model this outside option using parameter $\alpha$ ($\alpha \in \mathbb{R}^+$) which denotes the marginal decrease in the demand for a product from a marginal increase in its price. Essentially, when the price of a product is increased, the marginal decrease in demand for that product comes from two sources: one, some consumers switch from that product to the other product but still buy from the focal retailer (the marginal decrease because of switching between products within the focal retailer is $\frac{1}{2\gamma}$ in quality-dominates-fit versus $\frac{1}{2\beta t}$ in fit-dominates-quality), and two, some consumers switch from the focal retailer to an outside option (the marginal decrease because of switching to the outside option is $\alpha$). In our base setup, we model this outside option at an aggregate level in order to focus our attention on the effect of reviews and to ensure tractability. In the extension discussed in Section 4, we model the outside option more explicitly by considering retailer competition.\(^3\)

\(^3\)Furthermore, we also analyzed a model that does not incorporate any outside option. In that model, optimal retail prices are corner solutions which correspond to the maximum prices the retailer can charge. The results of this model regarding the effect of reviews are generally similar to those presented in this paper.
From Equations (6) and (8), we notice that each firm’s base demands in both cases take the same structure except the coefficients of the terms in the brackets (i.e., $\frac{1}{2\gamma}$ in quality-dominates-fit versus $\frac{1}{2\beta}$ in fit-dominates-quality). As a result, we can uniformly characterize the demand as follows, after incorporating demand loss because of outside option:

$$D_A = \tilde{D}_A - \alpha p_A = \left[ \frac{1}{2} + \frac{1}{2\tau} (1 - \gamma) x_R \right] - \left( \frac{1}{2\tau} + \alpha \right) p_A + \frac{1}{2\tau} p_B$$

$$D_B = \tilde{D}_B - \alpha p_B = \left[ \frac{1}{2} - \frac{1}{2\tau} (1 - \gamma) x_R \right] - \left( \frac{1}{2\tau} + \alpha \right) p_B + \frac{1}{2\tau} p_A$$

(9)

where $\tau \in \{\gamma, \beta\}$ with $\tau = \gamma$ for quality-dominates-fit case and $\tau = \beta$ for fit-dominates-quality case. This expression evidently demonstrates that the assumptions that we impose on consumers’ true and perceived preferences and distribution of consumers’ perceived quality difference are equivalent to the assumptions on linear demand functions, which have been commonly used in the literature (e.g., Choi, 1991). The online product reviews affect the competition between the two manufacturers via changing the parameters of the above demand functions. Notice that the terms $\left[ \frac{1}{2} \pm \frac{1}{2\tau} (1 - \gamma) x_R \right]$ in Equation (9) measure the market potential sizes for the products. To exclude trivial cases, we assume that the market potential sizes are positive; that is,

$$\tau > (1 - \gamma) |x_R|$$

(10)

3 Effect of Online Product Reviews

In this section, we first derive the subgame perfect equilibria for both the scenario without online product reviews and the one with reviews. We then analyze the effects of online product reviews on the retailer and manufacturers by comparing their equilibrium payoffs under the two scenarios.

In stage 2 of the game, the retailer maximizing its profit by choosing the optimal price for each product; that is,

$$\max_{p_A, p_B} \pi_R = (p_A - w_A) D_A + (p_B - w_B) D_B$$

(11)

By the first-order conditions, we can derive the retailer’s optimal prices, which are functions of wholesale prices. In stage 1 of the game, anticipating the retailer’s reaction in response to the wholesale prices, the manufacturers maximize their profits by choosing their optimal prices; that is,

$$\max_{w_i} \pi_i = w_i D_i, \ i \in \{A, B\}$$

(12)
Based on the first-order conditions, we can obtain the optimal wholesale price for each manufacturer. Substituting the optimal wholesale prices back to retailer’s pricing, we can derive the equilibrium retail prices. Then, the equilibrium demand for each product and the profits for the retailer and manufacturers follow. We summarize the equilibrium outcome in the following lemma.

**Lemma 1.** The equilibrium wholesale prices, retail prices, and profits for each player in the scenarios with and without online product reviews are as follows.

(a) Wholesale prices:

\[
\begin{align*}
    w_A &= \frac{\tau}{1+4\alpha \tau} + \frac{(1-\gamma)x_R}{3+4\alpha \tau} \\
    w_B &= \frac{\tau}{1+4\alpha \tau} - \frac{(1-\gamma)x_R}{3+4\alpha \tau}
\end{align*}
\]

(b) Retail prices:

\[
\begin{align*}
    p_A &= \frac{1+6\alpha \tau}{4\alpha(1+4\alpha \tau)} + \frac{(5+6\alpha \tau)(1-\gamma)x_R}{4(1+\alpha \tau)(3+4\alpha \tau)} \\
    p_B &= \frac{1+6\alpha \tau}{4\alpha(1+4\alpha \tau)} - \frac{(5+6\alpha \tau)(1-\gamma)x_R}{4(1+\alpha \tau)(3+4\alpha \tau)}
\end{align*}
\]

(c) Profits:

\[
\begin{align*}
    \pi_A &= \frac{(1+2\alpha \tau)(3+4\alpha \tau) + (1+4\alpha \tau)(1-\gamma)x_R^2}{4\tau(1+4\alpha \tau)^2(3+4\alpha \tau)^2} \\
    \pi_B &= \frac{(1+2\alpha \tau)(3+4\alpha \tau) - (1+4\alpha \tau)(1-\gamma)x_R^2}{4\tau(1+4\alpha \tau)^2(3+4\alpha \tau)^2} \\
    \pi_R &= \frac{(1+2\alpha \tau)^2}{8\alpha(1+4\alpha \tau)^2} + \frac{(1+2\alpha \tau)^2(1-\gamma)^2x_R^2}{8\tau(1+\alpha \tau)(3+4\alpha \tau)^2}
\end{align*}
\]

where \( \tau \in \{\beta t, \gamma e\} \) and \( \{\gamma, \beta\} \in \{\{1, \beta_c\}, \{r, \beta_R\}\} \), with \( \tau = \gamma e \) for quality-dominates-fit case and \( \tau = \beta t \) for fit-dominates-quality case and in each case with \( \{\gamma, \beta\} = \{1, \beta_c\} \) for the scenario without reviews and \( \{\gamma, \beta\} = \{r, \beta_R\} \) for the scenario with reviews.

**Proof.** All proofs are in the appendix unless indicated otherwise.

We next examine the effect of online product reviews. We first discuss the effect in the quality-dominates-fit case and fit-dominates-quality case, and then we investigate the effect when reviews shift the nature of competition between the two cases. We can assert the effect by comparing equilibrium prices and profits between in the scenario without reviews and in the scenario with reviews. For the comparison purpose, we next use the regular notations (e.g., \( \pi_R \)) for the scenario without reviews and use the notations with hats (e.g., \( \hat{\pi}_R \)) for the scenario with reviews.
3.1 Quality-Dominates-Fit Case

Without loss of generality, we consider $x_R \geq 0$; that is, the online product reviews favor manufacturer $A$ on the quality dimension.

**Proposition 1.** In the quality-dominates-fit case, in the presence of online product reviews with $x_R \geq 0$:

(a) Product B’s wholesale price is lower and manufacturer B’s profit is lower; that is, $w_B > \hat{w}_B$ and $\pi_B > \hat{\pi}_B$;

(b) Product A’s wholesale price is lower (i.e., $w_A > \hat{w}_A$) if and only if

$$x_R < \frac{\epsilon (3+4\alpha r \epsilon)}{(1+4\alpha r)(1+4\alpha \epsilon)}$$

and manufacturer A’s profit is lower (i.e., $\pi_A > \hat{\pi}_A$) if and only if

$$x_R < \frac{\epsilon (3+4\alpha r \epsilon)}{(1-\epsilon)(1+4\alpha \epsilon)} - \frac{r \epsilon (3+4\alpha r \epsilon)}{(1-\epsilon)(1+4\alpha \epsilon)}$$

(c) Product B’s retail price is lower and the retailer’s profit is higher; that is, $p_B > \hat{p}_B$ and $\pi_R < \hat{\pi}_R$; Product A’s retail price is lower (i.e., $p_A > \hat{p}_A$) if and only if

$$x_R < \frac{2 \epsilon (1+\alpha \epsilon)(3+4\alpha r \epsilon)}{(1+4\alpha r)(1+4\alpha \epsilon)(5+6\alpha \epsilon)}$$

In the symmetric case with $x_R = 0$, we can verify that the conditions specified in the above proposition are all satisfied.

**Corollary 1.** In the presence of the symmetric product reviews (i.e., $x_R = 0$), (a) the wholesale prices and manufacturers’ profits are lower; that is, $w_i > \hat{w}_i$ and $\pi_i > \hat{\pi}_i$, $i \in \{A, B\}$; (b) the retail prices are lower and retailer’s profit is higher; that is, $p_i > \hat{p}_i$ and $\pi_R < \hat{\pi}_R$.

The intuition for the symmetric case is as follows. In general, the online product reviews homogenize consumers’ perceived quality differences and thus homogenize consumers’ perceived utility differences. In the scenario without product reviews, each consumer’s perceived quality difference is from her own private assessment. In the scenario with reviews, the consumer combines her own assessment with the quality difference assessment revealed by the online product reviews. Because the quality difference revealed by the reviews is public and common to all consumers, the presence of product reviews reduces the heterogeneity of consumers’ perceived quality difference and
thus reduces the heterogeneity of their perceived utility differences. Figure 2a illustrates the effect of the online product reviews on the perceived utility differences in this symmetric case. Because consumers put some weight on the common component—perceived quality difference revealed by the reviews—in evaluating the utility differences, the span of their evaluations is reduced and thus the variance of their evaluations is reduced because of the product reviews. We call this effect \textit{variance-reducing effect}.

The reduced heterogeneity in consumers’ perceived utility differences between the two products makes the two products more substitutable overall and makes consumers more price sensitive to a specific product, and thus it increases the competition between the two manufacturers. The increased substitutability and competition can also be seen from the demand functions. In this symmetric case, the demand functions in Equation (9) can be rewritten as

\begin{align*}
D_A &= \frac{1}{2} - \alpha p_A - \frac{1}{2\gamma \epsilon} (p_A - p_B) \\
D_B &= \frac{1}{2} - \alpha p_B - \frac{1}{2\gamma \epsilon} (p_B - p_A)
\end{align*}

Note that the coefficient of the price difference term (i.e., $\frac{1}{2\gamma \epsilon}$ in the case) measures the substitutability between the two products: the larger the coefficient is, the more substitutable the two products are. The effect of online product reviews on the demand function is that it reduces $\gamma$ from 1 to $r$ (where $r < 1$), and thus it increases the substitutability between the two products. The increased competition between the two manufacturers drives their wholesale prices down as well as their profits. On the other hand, the retailer benefits from the increased competition between the two manufacturers. With the lower wholesale prices, the retailer lowers its retail prices, which increases the demand for each product. In addition, the lower wholesale prices leave the retailer
with higher profit margin from each sale, which explains why the retailer’s profit is increased by online product reviews.

In the general case as prescribed in Proposition 1, in addition to the variance-reducing effect, the online product reviews have another asymmetric effect on each manufacturer. The favorable quality information toward product $A$ revealed by the reviews (i.e., $x_R > 0$) uniformly changes each consumer’s perceived quality difference between the two products favorably toward product $A$. As a result, in the presence of the favorable reviews toward product $A$, on average consumers’ perceived utility differences between the two products are favorable for product $A$. We call this effect *mean-shifting effect*. Figure 2b illustrates such an effect. With favorable reviews for product $A$, consumers are more likely to have a higher utility from product $A$. As a result, the mean of their perceived utility differences is shifted toward the right-hand side and is changed favorably for product $A$. In the demand functions outlined in Equation (9), the mean shifting is reflected in the shifting from product $B$’s potential market size to product $A$’s such that, compared to the symmetric case, manufacturer $A$’s potential market size increases (from $\frac{1}{2}$ to $\left[\frac{1}{2} + \frac{1}{2\epsilon}(1 - r)\right]$) and manufacturer $B$’s decreases (from $\frac{1}{2}$ to $\left[\frac{1}{2} - \frac{1}{2\epsilon}(1 - r)\right]$). Manufacturer $B$ suffers from the reduced potential market size resulting from unfavorable reviews, in addition to increased competition resulting from the variance-reducing effect as in the symmetric case. As a result, the wholesale price for product $B$ is reduced and manufacturer $B$’s profit is lower because of the reviews.

For manufacturer $A$, the favorable reviews have positive effect on its wholesale price and profit because of the boosted appeal in the market, whereas the increased competition resulting from the variance-reducing effect has a negative effect. Whether the manufacturer can benefit from the reviews depends on the relative strength of the two effects. In general, more favorable reviews make the positive effect more significant, which in turn makes manufacturer $A$ more likely to be better off from the reviews. When the reviews are highly favorable for $A$, compared to the scenario without reviews, the increased potential market size allows her to set a higher wholesale price without hurting the demand, which may compensate the loss from the increased competition and make manufacturer $A$ better off. Inequalities (20) and (21) pinpoint the conditions, which essentially show that only if the reviews are favorable enough, the wholesale price and profit for manufacturer $A$ become higher because of the reviews.

The retailer benefits from online product reviews both from the variance-reducing effect and from mean-shifting effect. First, as illustrated in the symmetric case, the variance-reducing effect intensifies the upstream competition, which, per se, reduces the wholesale prices and thus increases
the profit of retailer. Second, the mean-shifting effect makes the downstream demand asymmetric
in terms of their potential market sizes, which engenders more room for the retailer to exploit its
market and benefits the retailer. For example, shifting the potential demand from product B to
product A, per se, allows the retailer to charge a higher retail price for product A and receive a
higher realized demand for it, at the cost of a lower retail price with a lower realized demand for
product B. Notice the gain from the increased price and increased demand for product A outweighs
the loss from the decreased price and decreased demand for product B, because the changes in both
the price and demand are more significant for product A than product B due to A’s dominance
in the market potential. Therefore, any increase of the degree of the asymmetry in the market
potentials benefits the retailer. All together, the retailer obtains a higher profit in the presence of
the reviews because of the benefits from both effects. The retailer charges a lower price for product
B because of the lower wholesale price in the supply side and the lower demand in the demand side.
The retailer may charge a higher or lower price for product A, depending on the balance between
the variance-reducing effect and the mean-shifting effect, in which the variance-reducing effect tends
to lower the price whereas the mean-shifting effect tends to increase the price.

The condition for this quality-dominates-fit case to occur in equilibrium is that \( t < \frac{r \epsilon}{\beta R} - \frac{|x_R| (1-r)(1+8\alpha r(1+\alpha r))}{2\beta R(1+\alpha r)(3+4\alpha r)} \), which requires the weight \( t \) on the fit dimension is small such that the
quality dimension plays a relatively more important role in determining consumers’ perceived utility
differences between the two products.

3.2 Fit-Dominates-Quality Case

As in the quality-dominates-fit case, without loss of generality, we consider \( x_R \geq 0 \).

**Proposition 2.** In the fit-dominates-quality case, in the presence of online product reviews with \( x_R \geq 0 \):

(a) Product A’s wholesale price is higher and manufacturer A’s profit is higher; that is, \( w_A < \hat{w}_A \)
and \( \pi_A < \hat{\pi}_A \);

(b) Product B’s wholesale price is higher (i.e., \( w_B < \hat{w}_B \)) if and only if

\[
x_R < \frac{t(\beta R - \beta C)(3+4\alpha \beta R t)}{(1-r)(1+4\alpha \beta R t)(1+4\alpha \beta C t)}
\]

(24)
manufacturer B’s profit is higher (i.e., \( \pi_B < \hat{\pi}_B \)) if and only if

\[
x_R < \frac{\beta_R t(3+4\alpha\beta_R t)}{(1-r)(1+4\alpha\beta_R t)} - \frac{t(3+4\alpha\beta_R t)}{(1-r)(1+4\alpha\beta_C t)} \sqrt{\frac{(1+2\alpha\beta_C t)\beta_R \beta_C t}{1+2\alpha\beta_R t}}
\]

(c) Product A’s retail price is higher (i.e., \( p_A < \hat{p}_A \)); Product B’s retail price is higher (i.e., \( p_B < \hat{p}_B \)) if and only if

\[
x_R < \frac{2t(\beta_R - \beta_C)((1+\alpha\beta_R t)(3+4\alpha\beta_R t)}{(1-r)(1+4\alpha\beta_C t)(1+4\alpha\beta_R t)(5+6\alpha\beta_R t)}
\]

the retailer’s profit is lower (i.e., \( \pi_R > \hat{\pi}_R \)) if and only

\[
x_R^2 < \frac{4t^2(\beta_R - \beta_C)(1+\alpha\beta_R t)(3+4\alpha\beta_R t)^2(1+3\alpha\beta_R t + \alpha(3+8\alpha\beta_R t)\beta_C t)}{(1-r)^4(1+2\alpha\beta_R t)^2(1+4\alpha\beta_R t)^2(1+4\alpha\beta_C t)^2}
\]

In the symmetric case with \( x_R = 0 \), we can verify that the conditions derived in the above proposition are all satisfied.

**Corollary 2.** In the presence of the symmetric product reviews (i.e., \( x_R = 0 \)), (a) the wholesale prices and manufacturers’ profits are higher; that is, \( w_i < \hat{w}_i \) and \( \pi_i < \hat{\pi}_i \), \( i \in \{A, B\} \); (b) the retail prices are higher and retailer’s profit is lower; that is, \( p_i < \hat{p}_i \) and \( \pi_R > \hat{\pi}_R \).

The intuition for the symmetric case is as follows. Different from the quality dimension in which the true quality difference is the same for all consumers and the product reviews add a common component in evaluating the quality difference across all consumers, in the fit dimension consumers have different preferences and online product reviews provide more information for them to further calibrate their own fits. With the additional information from reviews, consumers’ signals are more accurate and they become less uncertain about their degrees of misfit than in the absence of reviews (from with probability \( \beta_C \) to with probability \( \beta_R \) revealing the true degrees of misfit). The reduced uncertainty thus makes consumers more heterogeneous in terms of their perceived fits, which tends to increase the heterogeneity in consumers’ perceived utility differences. Figure 3a illustrates the effect of the online product reviews on the perceived utility differences in this symmetric case. Contrary to the effect in the quality dimension, the information provided by the product reviews in the fit dimension tends to increase the variance of consumers’ perceived utility differences. We call this effect variance-increasing effect.

The increased heterogeneity in consumers’ perceived utility differences between the two products makes the two products less substitutable overall and makes consumers less price sensitive to a specific product, and thus it softens the competition between the two manufacturers. The decreased
substitutability and competition can also be seen from the demand functions. In this symmetric
case, the demand functions in Equation (9) can be rewritten as

\begin{align*}
D_A &= \frac{1}{2} - \alpha p_A - \frac{1}{2\beta_t} (p_A - p_B) \\
D_B &= \frac{1}{2} - \alpha p_B - \frac{1}{2\beta_t} (p_B - p_A)
\end{align*}  
(28)

As discussed previously, the larger the coefficient of the price difference term (i.e., $\frac{1}{2\beta_t}$ in the case)
is, the more substitutable the two products are. The effect of online product reviews on the demand
function is that it increases $\beta$ from $\beta_C$ to $\beta_R$ and thus it decreases the substitutability between
the two products. The softening of the competition between the two manufacturers increases their
wholesale prices as well as their profits. On the other hand, the retailer hurts from the decreased
competition between the two manufacturers. With the higher wholesale prices, the retailer increases
its retail prices, which decreases the demand for each product. In addition, the higher wholesale
prices leave the retailer with lower profit margin from each sale, which explains why the retailer’s
profit is decreased by online product reviews.

In the general case as prescribed in Proposition 2, in addition to the variance-increasing effect,
as in the quality-dominates-fit case and as illustrated in Figure 3b, the online product reviews also
have asymmetric mean-shifting effect on each manufacturer. In the demand functions outlined in
Equation (9), the mean shifting is reflected in an increase in manufacturer $A$’s potential market size
(from $\frac{1}{2}$ to $\left[\frac{1}{2} + \frac{1}{2\beta_R t} (1-r) x_R\right]$) and a decrease in manufacturer $B$’s (from $\frac{1}{2}$ to $\left[\frac{1}{2} - \frac{1}{2\beta_R t} (1-r) x_R\right]$).
Manufacturer $A$ benefits from the favorable reviews and the resulting increased potential market
size, in addition to softened competition resulting from the variance-increasing effect as in the
symmetric case. As a result, the wholesale price for product $A$ is increased and manufacturer $A$’s
profit is higher because of the reviews.
For manufacturer B, the unfavorable reviews have a negative effect on its wholesale price and profit because of the reduced appeal in the market, whereas the softened competition resulting from the variance-increasing effect, as in the symmetric case, has a positive effect. Whether the manufacturer can benefit from the reviews depends on the relative strength of the two effects. In general, less unfavorable reviews make the negative effect less significant, which in turn makes manufacturer B more likely to be better off from the reviews. When unfavorable reviews are mild, the softened competition effect dominates, which engenders the possibility for manufacturer B of charging a higher price and earning a higher profit compared to the case without reviews. Inequalities (24) and (25) pinpoint such conditions.

In the presence of favorable reviews for product A, the retailer charges a higher retail price for product A because of the increased wholesale price from the supply side and the enhanced demand from demand side. The retailer may charge a higher or lower price for product B, depending on the balance between the variance-increasing effect and the mean-shifting effect, in which the variance-increasing effect tends to increase the price whereas the mean-shifting effect tends to decrease the price. Inequality (26) is the condition from such a tradeoff. In terms of its profit, on the one hand, the retailer benefits from mean-shifting effect of the product reviews, as in the quality-dominates-fit case. On the other hand, the retailer hurts from the variance-increasing effect of the product reviews. Inequality (27) shows that when the mean-shifting effect reflected by the magnitude of $x_R$ is small, the product reviews are detrimental to the retailer’s profit.

The condition for this fit-dominates-quality case to occur in equilibrium is that $t > \max\{\frac{\epsilon}{\beta R} + \frac{|x_R|(1-r)(1+8\alpha\beta R(1+\alpha\beta R))}{2\beta R(1+\alpha\beta R)(3+4\alpha\beta R)}\}$, which requires the weight $t$ on the fit dimension is large such that the fit dimension plays a dominant role in determining some consumers’ perceived utility differences between the two products.

It is worth noting that the focus of this study is on the effect of product reviews on the upstream competition and the retailer. Although from this perspective our result suggests that in the fit-dominates-quality case retailer is hurt by online product reviews, our study does not preclude other possibilities that may offer retailers incentives to provide online product reviews even in the fit-dominates-quality case. For example, for some product categories, Amazon does not sell by itself and lets others sell directly to consumers for a commission on sales. In that case, Amazon simply provides a platform for other sellers and could benefit from reviews.
3.3 The Case when Reviews Change the Nature of Competition

So far, we have focused our discussion on the effect of online product reviews within the same quality-dominates-fit or fit-dominates-quality category. Under some conditions, the online product reviews would shift the competition from one category to the other. We next use the symmetric case with $x_R = 0$ to illustrate such possibility and the effect of online product reviews when this shift occurs.

First, we can verify that the shift in the nature of upstream competition from the fit-dominates-quality case to the quality-dominates-fit case cannot happen. We suppose that the competition without reviews is in the fit-dominates-quality category in which, compared to consumers’ own assessment on the quality dimension, the fit dimension plays a dominant role in evaluating products. With the reviews, consumers put some weight on the review’s assessment (indicating no quality difference) and put less weight on their own assessment in estimating the quality difference. The decreased weight on consumer’s own assessment of the quality makes the fit dimension more important and results in more determined consumers who derive higher net utility from product $A$ ($B$) regardless of her own perceived quality difference; that is, the marginal consumer $y_A$ ($y_B$) tends to be shifted toward the right (left) side because $\gamma$ is reduced from 1 to $r$ in Equation (7). Therefore, the competition in the presence of the reviews cannot be shifted to the quality-dominates-fit category in which consumers’ own assessment on the quality dimension plays a dominant role in evaluating products.

Next we derive the conditions under which online product reviews shift the competition from the quality-dominates-fit category to the fit-dominates-quality category, and then analyze the effect of online product reviews on the prices and each player’s profit.

**Proposition 3.** When $\beta_C < \frac{\zeta}{r} < \frac{\beta_R}{r}$, (a) in the presence of the symmetric product reviews (i.e., $x_R = 0$), the competition between the two products is in the fit-dominates-quality category, and in the absence of the reviews, the competition is in the quality-dominates-fit category; (b) if and only if $\beta_R t < \epsilon$, the wholesale prices, retail prices, and manufacturers’ profits are lower with the product reviews (i.e., $w_i > \hat{w}_i$, $p_i > \hat{p}_i$, and $\pi_i > \hat{\pi}_i$, $i \in \{A, B\}$), and retailer’s profit is higher with the product reviews (i.e., $\pi_R < \hat{\pi}_R$).

The condition $\beta_C < \frac{\zeta}{r}$ ensures that in the absence of the reviews, the competition between the two products is in the quality-dominates-fit category, and the condition $\frac{\zeta}{r} < \frac{\beta_R}{r}$ ensures that in the presence of the reviews, the competition between the two products is in the fit-dominates-quality category.
category. Intuitively, when the precision of consumers’ own assessments on the fit dimension is low and the variance of consumers’ own assessment on the quality dimension is high, the competition in the no-review scenario is likely to fall under the quality-dominates-fit case, and when the precisions of reviews on quality and fit dimensions are high (i.e., $\beta_R$ and $(1-r)$ are high) such that consumers’ own assessment on the quality dimension plays non-dominant role the competition in the review scenario is likely to fall under the fit-dominates-quality case. The reason for the shift of competition from the quality-dominates-fit case to fit-dominates-quality case is, after reading reviews, consumers put less weight on their own assessment on the quality dimension, and meanwhile consumers’ signals are more accurate and they become less uncertain about their degrees of misfit. Therefore, the relative importance between the fit dimension and consumers’ own assessment on the quality dimension can switch after and before reading reviews such that in the presence of reviews the fit dimension becomes to play a dominant role instead.

It is worth noting that what distinguishes the quality-dominates-fit case and fit-dominates-quality case is the relative importance between the fit dimension and consumers’ own assessment on the quality difference in their comparative evaluation of products. Each case bears different nature of competition, and a particular product category does not necessarily correspond to a specific case. For example, in evaluating a digital camera (experience good), consumers may put more weight on quality rather than fit. However, the competition in the digital market can fall under either the quality-dominates-fit case or fit-dominates-quality case, and as indicated in Proposition 3, reviews can even shift the nature of the competition. For instance, for two digital cameras, one from Canon and the other from Sony, in the absence of reviews, consumers’ own assessment on the quality difference may play a dominant role in their purchase decision. In the presence of symmetric reviews which accurately reveal the quality difference between the two cameras to be minimal, consumers’ own assessment on the quality difference is no longer that important and consumers’ perceived fit (e.g., consumers’ preference about features and design) may play a dominant role. In this case, reviews shift the nature of competition from quality-dominates-fit case to fit-dominates-quality case.

When the competition is shifted from the quality-dominates-fit category to the fit-dominates-quality category, the substitutability of the products is also changed across the two categories. As illustrated in Equations (23) and (28), the degrees of substitutability in the quality-dominates-fit category and in the fit-dominates-quality category are measured by $\frac{1}{2\gamma t}$ and $\frac{1}{2\beta t}$, respectively. Therefore, without reviews the substitutability is $\frac{1}{2\epsilon}$ (because $\gamma = 1$) and with reviews the substitutability is $\frac{1}{2\beta_R t}$. When the former is less than the latter (i.e., $\beta_R t < \epsilon$), the substitutability of the products
in the presence of the reviews is higher, and thus the wholesale prices and retail prices are lower, and manufacturers' profits are lower. The retailer's profit is higher because of the increased profit margin and increased demands, as discussed previously.

Corollary 1 and Proposition 3 offer surprising insights and implications regarding how review precision may affect the retailer. When precision of consumers' own assessments on the fit dimension is low such that quality dominates fit without reviews, Corollary 1 shows that if review precisions are not too large such that in the presence of reviews quality continues to dominate fit, reviews benefit the retailer (because reviews intensify the upstream competition). On the other hand, when review precisions are high such that fit dominates quality and $\beta_{Rt} > \epsilon$, Proposition 3 demonstrates that reviews hurt the retailer (because reviews soften the upstream competition). This result shows that improving reviews' precision can actually hurt the retailer by fundamentally altering the nature—from intensifying to softening—of upstream competition.\footnote{Li et al. (2011) showed a S-shaped relationship between review precision and competing direct sellers' profits. Our result pertains to the retailer in a channel structure and our context is different from theirs, as explained previously.}

The general asymmetric case can be similarly analyzed, but with more complexity. The additional complexity mainly comes from the mean-shifting effect discussed in the fit-dominates-quality case. For instance, if $\beta_{Rt} < \epsilon$, for the product with unfavorable reviews, its wholesale price, retail price, and the manufacturer's profit are lower with product reviews, because the manufacturer suffers from the unfavorable comments as discussed previously, in addition to the negative effect from the increased substitutability associated with the transition to fit-dominates-quality case caused by reviews (as shown in Proposition 3). On the other hand, the manufacturer with favorable reviews balances between the negative effect from the increased substitutability and the positive effect from favorable comments toward its product. Whether the manufacturer is better off depends on the magnitude of the favorable reviews.

3.4 The Case when Manufacturers Do Not React to Reviews

By Propositions (1), (2), and (3), it is worth highlighting that the manufacturer with favorable reviews on the quality dimension need not benefit from reviews and the manufacturer with unfavorable quality reviews is not necessarily harmed by reviews. In addition, the retailer can be harmed by product reviews, and in some cases, the harm occurs only when reviews have high precisions on the quality and fit dimensions. As explained in the discussions that follow the propositions, a reason for all our findings is the effect that online product reviews have on the upstream competition between manufacturers. The following result shows how reviews' effects change if the strategic effects of
reviews on manufacturers are ignored, that is, if manufacturers do not react to reviews and keep the same wholesale prices in both review and no-review scenarios. We continue to assume $x_R \geq 0$ for this result.

**Proposition 4.** If manufacturers do not react to reviews and keep the same wholesale prices in both review and no-review scenarios, in the presence of reviews: (a) Retailer’s profit and manufacturer A’s profit are never lower; (b) Manufacturer B’s profit is never higher.

This result highlights the critical role played by reviews’ effects on upstream competition in the overall impact of reviews on various players in the channel. Under the assumption that only the retailer adjusts its pricing strategies in response to reviews and manufacturers do not, we obtain straightforward and apparently intuitive results regarding the effect of reviews on different players. Proposition 4 and the previous propositions clearly demonstrate that ignoring reviews’ effects on upstream players’ strategic responses to reviews can lead to incorrect conclusions about the impact of reviews on various players.

4 Extension: Competing Retailers

In the baseline model, we use $\alpha$ to model that some consumers may have outside options and leave the focal retailer depending on the retail prices offered by the retailer. In this section, we explicitly model the consumers’ outside option by considering retail competition. In particular, we consider, in addition to the focal retailer, another retailer sells products $A$ and $B$ as well. Each retailer has its own “loyal” consumers in the sense that each retailer has its frequent shoppers who prefer to visit the retailer’s platform as the first stop. We assume that the two retailers are symmetric and each has “loyal” consumers of measure 1. The loyal consumers visit their preferred retailers and compare the net utility difference between the two products provided by the preferred retailer. As in the baseline model, on retailer $k$’s platform, $k \in \{1, 2\}$, some of its loyal consumers derive higher net utility from product $A$ and others derive higher net utility from product $B$. We similarly denote as $\tilde{D}_A$ (\tilde{D}_B) the base demand for product $A$ ($B$) on retailer $k$’s platform, which is the measure of the “loyal” consumers on retailer $k$’s platform who derive higher net utility from product $A$ ($B$).

In the presence of retailer competition, the demand for product $i$ on retailer $k$ $D_{ik}$ realized from its base demand $\tilde{D}_{ik}$ not only depends on the price that retailer $k$ charges for product $i$, but also depends on the prices that the competing retailer offers. We use the commonly used channel
competition framework (e.g., Ingene and Parry 2004) to model the retail competition, which can properly capture the interaction between the demands across the two retailers with tractability. In particular, we denote as $p_{ik}$, $i \in \{A, B\}$ and $k \in \{1, 2\}$, the price that retailer $k$ charges for product $i$. Based on the channel competition framework, we can extend the demand functions in Equation (9) as follows:

$$D_{Ak} = \tilde{D}_{Ak} - (a_1 p_{Ak} - b p_{Ak}) - (a_2 p_{Ak} - c p_{Bk})$$
$$D_{Bk} = \tilde{D}_{Bk} - (a_1 p_{Bk} - b p_{Bk}) - (a_2 p_{Bk} - c p_{Ak})$$

where $\{k, \bar{k}\} = \{1, 2\}$, and $a_1$, $a_2$, $b$, and $c$ are price sensitivity parameters. Intuitively, if retailer $k$ increases the price for product $i$, less of the base demand for the product converts to the sales. On the other hand, if its rival raises the price for product $A$ or $B$, some of its “loyal” consumers may switch to retailer $k$. By simple algebra, we can reorganize the above demand functions as

$$D_{Ak} = \tilde{D}_{Ak} - a p_{Ak} + b p_{Ak} + c p_{Bk}$$
$$D_{Bk} = \tilde{D}_{Bk} - a p_{Bk} + b p_{Bk} + c p_{Ak}$$

where $a \equiv a_1 + a_2$. The parameter $a$ measures “loyal” consumers’ price sensitivity to their preferred products on their preferred retailer platform. The cross-retailer parameters $b$ and $c$ measure the sensitivity of retailer $k$’s demands in its rival’s prices. Taking the standard assumption in the literature (Choi, 1996; Mcguire and Staelin, 1983), we let $a > b + c$, which ensures that an increase in prices decreases the total demand and the own price elasticity is larger than the cross-retailer ones.

Similar to the baseline model, in stage 2 of the game, the retailers maximize their profits by choosing the optimal price for each product; that is,

$$\max_{p_{Ak}, p_{Bk}} \pi_{Rk} = (p_{Ak} - w_{Ak})D_{Ak} + (p_{Bk} - w_{Bk})D_{Bk}$$

Different from the baseline case, each retailer’s demands for products $A$ and $B$ depends not only on its own prices but also on its rival’s prices. Given the wholesale prices, by the first-order conditions, we can derive each retailer’s optimal prices as the best response to its rival prices. Based on their best response prices, we can derive retailers’ optimal prices, which are functions of wholesale prices.

In stage 1 of the game, anticipating the competition between the two retailers and their reactions in response to the wholesale prices, the manufacturers maximize their profits by choosing their
optimal prices; that is,

$$\max_{w_{i1}, w_{i2}} \pi_i = w_{i1}D_{i1} + w_{i2}D_{i2}, \ i \in \{A, B\}$$  \hspace{1cm} (30)$$

As in the baseline model, based on the first-order conditions, we can obtain the optimal wholesale price for each manufacturer. Substituting the optimal wholesale prices back to retailer’s pricing, we can derive the equilibrium retail prices. We can then derive the equilibrium demand for each product for each retailer and the profits for the retailers and manufacturers. We summarize the equilibrium outcome in the following proposition.

**Proposition 5.** The equilibrium wholesale prices, retail prices, and profits for each player in the scenarios with and without online product reviews are as follows.

(a) Wholesale prices:

$$w_{Ak} = \frac{a\tau[2+(2a-b+c)\tau]}{2(2a-b-c)J} + \frac{(2a-b-c)(1-\gamma)(1+\alpha\tau)x_R}{2[2+(2a-b-c)\tau]T^{2}}$$  \hspace{1cm} (31)$$

$$w_{Bk} = \frac{a\tau[2+(2a-b+c)\tau]}{2(2a-b-c)J} - \frac{(2a-b-c)(1-\gamma)(1+\alpha\tau)x_R}{2[2+(2a-b-c)\tau]T^{2}}$$  \hspace{1cm} (32)$$

(b) Retail prices:

$$p_{Ak} = \frac{J+2a^{2}\tau[2+(2a-b+c)\tau]}{2(2a-b-c)J} + \frac{(1-\gamma)(T+2(2a-b-c)(1+\alpha\tau)^{2})x_R}{2[2+(2a-b-c)\tau]T^{2}}$$  \hspace{1cm} (33)$$

$$p_{Bk} = \frac{J+2a^{2}\tau[2+(2a-b+c)\tau]}{2(2a-b-c)J} - \frac{(1-\gamma)(T+2(2a-b-c)(1+\alpha\tau)^{2})x_R}{2[2+(2a-b-c)\tau]T^{2}}$$  \hspace{1cm} (34)$$

(c) Profits:

$$\pi_A = \left[\frac{[a\tau(2+(2a-b+c)\tau)T+(2a-b-c)(1+\alpha\tau)(1-\gamma)x_R]^{2}L}{(2a-b-c)\tau[2+(2a-b-c)\tau]T^{2}J^{2}}\right]$$  \hspace{1cm} (35)$$

$$\pi_B = \left[\frac{[a\tau(2+(2a-b+c)\tau)T-(2a-b-c)(1+\alpha\tau)(1-\gamma)x_R]^{2}L}{(2a-b-c)\tau[2+(2a-b-c)\tau]T^{2}J^{2}}\right]$$  \hspace{1cm} (36)$$

$$\pi_{Rk} = \frac{aL^{2}}{2[2+(2a-b-c)\tau]T^{2}J^{2}} + \frac{(1-\gamma)^{2}(1+\alpha\tau)L^{2}x_{R}^{2}}{2[2+(2a-b-c)\tau]T^{2}J^{2}}$$  \hspace{1cm} (37)$$

where, \(i \in \{A, B\}, \ k \in \{1, 2\}, \ \tau \in \{\beta_t, \gamma\epsilon\} \ \text{and} \ \{\gamma, \beta\} \in \{\{1, \beta_c\}, \{r, \beta_R\}\}, \ \text{with} \ \tau = \gamma\epsilon \ \text{for quality-dominates-fit case and} \ \tau = \beta_t \ \text{for pit-dominates-quality case} \ \text{and in each case with} \ \{(\gamma, \beta) = \{1, \beta_c\}\} \ \text{for the scenario without reviews and} \ \{(\gamma, \beta) = \{r, \beta_R\}\} \ \text{for the scenario with reviews.} \ T, \ L, \ \text{and} \ J
are defined as

\[
T = 6a - 3(b + c) + [14a^2 + 3(b^2 - c^2) - 2a(7b + c)]\tau + 2a[4a^2 + 2(b^2 - c^2) - a(6b - c)]\tau^2
\]

\[
L = 2a - b - c + [6a^2 + b^2 - c^2 - 2a(3b + c)]\tau + 2a[2a^2 + b^2 - c^2 - 3ab]\tau^2
\]

\[
J = L + 2a(a - b - c)\tau [2 + (2a - b + c)\tau]
\]

Based on the above equilibrium outcome, we can conduct the same analysis for both the quality-dominates-fit case and the fit-dominates-quality case, as in the baseline model. We can show that, considering the retailer competition, the main results from the baseline model stay the same qualitatively and the main insights continue to be valid. For instance, in the quality-dominates-fit case, as in Proposition 1, in the presence of online product review with \(x_R \geq 0\), product B’s wholesale price and retail price are lower, product A’s wholesale price is lower if and only if \(x_R\) is smaller than some threshold, and similarly product A’s retail price is lower if and only if \(x_R\) is small; The retailers’ profits are higher; Manufacturer B’s profit is lower, and manufacturer A’s profit is lower if and only \(x_R\) is smaller than some threshold. Therefore, the insight of how the upstream competition is affected by the information revealed by online product reviews regarding the product quality difference is the same. The difference from the baseline case lies in the threshold values—the threshold values now are also functions of the price sensitivity parameters, in addition to the parameters in the baseline model.

5 Conclusion

We examine the effect of online product reviews in a channel structure with a retailer selling substitutable products from competing manufacturers. We consider that consumers face uncertainty in both the product quality and fits to their needs, and product reviews provide additional information and reduce their uncertainties. Consumers agree on the preference order of the attributes in the quality dimension and have idiosyncratic preferences for the same attribute in the fit dimension. We identify the quality-dominates-fit case in which consumers’ own assessment on the quality dimension plays a dominant role in determining the perceived utility differences of the competing products and the fit-dominates-quality case in which the fit dimension plays a more important role. We demonstrated how the impacts of reviews on the retailer and manufacturers can be widely different and how these impacts can be different in the quality-dominates-fit and fit-dominates-quality cases.

Our research generates several managerial implications for online retailing. Online retailers have
been deploying a variety of technologies and platforms to mitigate consumer uncertainty and match consumers with their preferred products. Online review platforms and recommendation systems are a few examples of these. Our research highlights that the effect of reviews on upstream competition is critical in determining the effects of reviews on various players in a channel. Ignoring reviews’ effects on upstream players’ strategic responses to reviews can lead to incorrect conclusions about the effect of reviews on various players. Our study thus illustrates that the effects of reviews cannot be determined in isolation, and calls for in-depth investigation of the effect of reviews on upstream competition in order to properly evaluate the overall effects of reviews on different players involved.

Our results also suggest the information revealed by reviews on the quality or fit dimension has different effects on the players in a channel. Retailers generally benefit from the information in the quality dimension revealed by reviews and therefore should welcome, encourage, and even induce consumers and/or third parties to generate relevant reviews. For instance, retailers should make the review platform easy to use to facilitate the review generating process, and, in particular, they may provide some review templates to direct users toward generating information about product qualities.

The incentive of manufacturers’ fostering reviews generation is not necessarily aligned with that of retailers. While retailers are incentivized to promote product reviews to reveal product qualities, manufacturers should encourage and induce information in the fit dimension in product reviews. For large manufacturers with bargaining power over retailers, they should influence or force retailers to direct consumers for generating information in the fit dimension on retailers’ review platforms. In addition, all manufacturers should encourage providing product specifics from consumers or third parties on their own review platforms (if any) or other third-party platforms.

Another implication of our findings is that the appropriate design of review platforms depends critically on the product type, because the nature of the upstream competition—whether consumers’ own assessment on the quality dimension or the fit dimension dominates the other dimension—demands for different design of review platforms, and each product type is likely to fall under a particular competition category. Therefore, our study underscores the importance for retailers and manufacturers to know the type each product belongs to before choosing features of the review platform. However, identifying the product type may not be straightforward. Recent research by Hong et al. (2012) illustrates a mechanism based on product reviews to distinguish between search and experience goods, and their findings complement ours in achieving a more comprehensive understanding of impact of reviews on retailers and manufacturers.
Our results also have implications for further research on the effect of online reviews. Current empirical research mainly focuses on the effect of reviews using the sales/revenue data of retailers that sell to end consumers. Analytical research has also examined either monopoly sellers and competing firms that sell directly to end consumers. Results that show how reviews affect sales or the demand side of marketplaces provide a partial characterization of the effect of reviews. We provide a first set of results that show the effect of reviews on both demand and supply sides in online marketplaces.

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A Appendix

A.1 Proof of conditional expectation of misfit

*Proof.* The cumulative density function of \( s \), conditional on the consumer’s true degree of misfit \( \lambda \) being \( z \), can be formulated as

\[
P(s \leq y|\lambda = z) = (1 - \beta)y + \beta H(y - z)
\]  

(38)
where $H(\cdot)$ is the Heaviside step function that evaluates to zero if the argument is negative, and to one otherwise. The corresponding probability density function is

$$P(s = y|\lambda = z) = (1 - \beta) + \beta \delta(y - z)$$  \hspace{1cm} (39)

where $\delta(x)$ is the Dirac delta distribution that satisfies

$$\int_{-\infty}^{\infty} \delta(x)dx = 1 \text{ and } \delta(x) = \begin{cases} 0 & \text{for } x \neq 0 \\ \infty & \text{for } x = 0 \end{cases}$$

Using the Bayes’ Law,

$$P(\lambda = z|s = y) = \frac{P(s = y|\lambda = z) P(\lambda = z)}{P(s = y)} = (1 - \beta) + \beta \delta(y - z)$$  \hspace{1cm} (40)

and the conditional expectation is

$$E(\lambda|s = y) = \int_0^1 \lambda [(1 - \beta) + \beta \delta(y - \lambda)] d\lambda = \int_0^1 (1 - \beta) \lambda d\lambda + \beta y = \frac{1 - \beta}{2} + \beta y$$

A.2 Proof of Lemma 1

Proof. We denote $a_A \equiv \frac{1}{2} + \frac{1}{2\tau} (1 - \gamma)x_R$, $a_B \equiv \frac{1}{2} - \frac{1}{2\tau} (1 - \gamma)x_R$, $b \equiv \frac{1}{2\tau} + \alpha$, and $c \equiv \frac{1}{2\tau}$. The demand functions in Equation (9) then can be rewritten as

$$D_A = a_A - bp_A + cp_B$$
$$D_B = a_B - bp_B + cp_A$$  \hspace{1cm} (41)

The retailer’s optimization problem in stage 2 is characterized by the first-order conditions of Equation (11):

$$\frac{\partial \pi_R}{\partial p_A} = a_A - 2bp_A + 2cp_B + bw_A - cw_B = 0$$
$$\frac{\partial \pi_R}{\partial p_B} = a_B - 2bp_B + 2cp_A + bw_B - cw_A = 0$$

from which we can derive the retailer’s optimal retail prices as functions of the wholesale prices:

$$p_A = \frac{w_A}{2} + \frac{a_Ab + a_Bc}{2(b^2 - c^2)}$$
$$p_B = \frac{w_B}{2} + \frac{a_Bb + a_Ac}{2(b^2 - c^2)}$$  \hspace{1cm} (42)
The manufacturers’ optimization problems in stage 1 are characterized by the first-order conditions of Equation (12) (noticing that $p_i$ in $D_i$ is a function of $w_i$ as in Equation (42)):

\[
\frac{\partial \pi_A}{\partial w_A} = a_A - bp_A + cp_B - \frac{b}{2} w_A = \frac{a_A}{2} - bw_A + \frac{c}{2} w_B = 0 \\
\frac{\partial \pi_B}{\partial w_B} = a_B - bp_B + cp_A - \frac{b}{2} w_B = \frac{a_B}{2} - bw_B + \frac{c}{2} w_A = 0
\]

from which we can derive the optimal wholesale prices:

\[
w_A = \frac{2a_A b + a_B c}{4b^2 - c^2} \\
w_B = \frac{2a_B b + a_A c}{4b^2 - c^2}
\] (43)

Substituting the above optimal wholesale prices into Equation (42), we derive the optimal retail prices:

\[
p_A = \frac{2a_A b + a_B c}{2(4b^2 - c^2)} + \frac{a_A b + a_B c}{2(2b^2 - c^2)} \\
p_B = \frac{2a_B b + a_A c}{2(4b^2 - c^2)} + \frac{a_B b + a_A c}{2(2b^2 - c^2)}
\] (44)

Substituting the above optimal retail prices into the demand functions in Equation (41), we have the equilibrium demands:

\[
D_A = a_A - \frac{a_A (2b^2 - c^2 - a_B bc)}{2(4b^2 - c^2)} - \frac{a_A}{2} = \frac{(2a_A b + a_B c)b}{2(4b^2 - c^2)} \\
D_B = a_B - \frac{a_B (2b^2 - c^2 - a_A bc)}{2(4b^2 - c^2)} - \frac{a_B}{2} = \frac{(2a_B b + a_A c)b}{2(4b^2 - c^2)}
\]

With the above equilibrium demands, the optimal wholesale prices in Equation (43), and the optimal retail prices in Equation (44), we have equilibrium profits:

\[
\pi_A = w_A D_A = \frac{(2a_A b + a_B c)^2 b}{2(4b^2 - c^2)^2} \\
\pi_B = w_B D_B = \frac{(2a_B b + a_A c)^2 b}{2(4b^2 - c^2)^2}
\]

\[
\pi_R = (p_A - w_A)D_A + (p_B - w_B)D_B = \frac{(a_A - a_B)^2 b^3}{4(b^2 - c^2)(2b^2 - c^2)^2} + \frac{(2a_A b + a_B c)(2a_B b + a_A c)b^2}{2(b^2 - c^2)(4b^2 - c^2)^2}
\]

Lemma 1 follows by substituting $a_A, a_B, b$, and $c$ into the above optimal wholesale prices, retail prices, and equilibrium profits.

A.3 Proof of Proposition 1

**Proof.** By Lemma 1, with $\tau = \epsilon$ we have the equilibrium outcome for the scenario without reviews and with $\tau = r\epsilon$ we have the equilibrium outcome for the scenario with reviews.
(a) Part (a) follows from that

\[ w_B = \frac{\epsilon}{1+4\alpha\epsilon} > \frac{\epsilon}{1/r + 4\alpha\epsilon} \geq \frac{r\epsilon}{1+4\alpha\epsilon} - \frac{(1-\gamma)x_R}{3+4\alpha\epsilon} = \hat{w}_B \]

\[ \pi_B = \frac{(1+2\alpha\epsilon)e}{4(1+4\alpha\epsilon)^2} > \frac{(1+2\alpha\epsilon)e}{4\epsilon(1+4\alpha\epsilon)^2} \]

where the last inequality is because \( re(3+4\alpha\epsilon) > (1+4\alpha\epsilon)(1-\gamma)x_R \) by Equation (10).

(b) \( w_A > \hat{w}_A \) if and only if

\[ w_A - \hat{w}_A = \frac{\epsilon}{1+4\alpha\epsilon} - \frac{r\epsilon}{1+4\alpha\epsilon} - \frac{(1-\gamma)x_R}{3+4\alpha\epsilon} > 0 \]

which leads to the condition in Inequality (20).

\[ \pi_A > \hat{\pi}_A \] if and only if

\[ \pi_A - \hat{\pi}_A = \frac{(1+2\alpha\epsilon)e}{4(1+4\alpha\epsilon)^2} - \frac{(1+2\alpha\epsilon)(3+4\alpha\epsilon) + (1+4\alpha\epsilon)(1-\gamma)x_R}{4\epsilon(1+4\alpha\epsilon)^2} > 0 \]

which leads to the condition in Inequality (21).

(c) We have

\[ \hat{p}_B = \frac{1+6\alpha e}{4\alpha(1+4\alpha\epsilon)} > \frac{1+6\alpha e}{4\alpha(1+4\alpha\epsilon)} \geq \frac{1+6\alpha e}{4\alpha(1+4\alpha\epsilon)} - \frac{(5+6\alpha e)(1-\gamma)x_R}{4(1+4\alpha\epsilon)(3+4\alpha\epsilon)} = \hat{\pi}_B \]

\[ \hat{p}_R = \frac{(1+2\alpha\epsilon)^2}{8\alpha(1+4\alpha\epsilon)^2} < \frac{(1+2\alpha\epsilon)^2}{8\alpha(1+4\alpha\epsilon)^2} \leq \frac{(1+2\alpha\epsilon)^2}{8\alpha(1+4\alpha\epsilon)^2} + \frac{(5+6\alpha e)(1-\gamma)x_R}{4(1+4\alpha\epsilon)(3+4\alpha\epsilon)} = \hat{\pi}_R \]

\[ p_A > \hat{p}_A \] if and only if

\[ p_A - \hat{p}_A = \frac{1+6\alpha e}{4\alpha(1+4\alpha\epsilon)} - \frac{1+6\alpha e}{4\alpha(1+4\alpha\epsilon)} - \frac{(5+6\alpha e)(1-\gamma)x_R}{4(1+4\alpha\epsilon)(3+4\alpha\epsilon)} > 0 \]

which leads to the condition in Inequality (22).

\[ \square \]

A.4 Proof of Proposition 2

Proof. By Lemma 1, with \( \tau = \beta_C t \) we have the equilibrium outcome for the scenario without reviews and with \( \tau = \beta_R t \) we have the equilibrium outcome for the scenario with reviews.

(a) Part (a) follows from that

\[ w_A = \frac{\beta_C t}{1+4\alpha\beta_C t} < \frac{\beta_R t}{1+4\alpha\beta_R t} \leq \frac{\beta_R t}{1+4\alpha\beta_R t} + \frac{(1-\gamma)x_R}{3+4\alpha\beta_R t} = \hat{w}_A \]
\[
\pi_A = \frac{(1+2\alpha \delta_C t)\delta_C t}{4(1+4\alpha \delta_C t)^2} < \frac{(1+2\alpha \beta_R t)\beta_R t}{4(1+4\alpha \beta_R t)^2} \leq \frac{(1+2\alpha \beta_R t)[\beta_R t(3+4\alpha \beta_R t)+(1+4\alpha \beta_R t)(1-\gamma)x_R]^2}{4\beta_R t(1+4\alpha \beta_R t)^2(3+4\alpha \beta_R t)^2} = \hat{\pi}_A
\]

which leads to the condition in Inequality (24).

\[\pi_B < \hat{\pi}_B\] if and only if

\[
\pi_B - \hat{\pi}_B = \frac{(1+2\alpha \beta_C t)\delta_C t}{4(1+4\alpha \delta_C t)^2} - \frac{(1+2\alpha \beta_R t)[\beta_R t(3+4\alpha \beta_R t)-(1+4\alpha \beta_R t)(1-\gamma)x_R]^2}{4\beta_R t(1+4\alpha \beta_R t)^2(3+4\alpha \beta_R t)^2} < 0
\]

which leads to the condition in Inequality (25).

\[p_A < \hat{p}_A\] because

\[
p_A = \frac{1+6\alpha \beta_C t}{4\alpha(1+4\alpha \beta_C t)} < \frac{1+6\alpha \beta_R t}{4\alpha(1+4\alpha \beta_R t)} \leq \frac{1+6\alpha \beta_R t}{4\alpha(1+4\alpha \beta_R t)} + \frac{(5+6\alpha \beta_R t)(1-\gamma)x_R}{4(1+4\alpha \beta_R t)(3+4\alpha \beta_R t)} = \hat{p}_A
\]

which leads to the condition in Inequality (26).

\[\pi_R > \hat{\pi}_R\] if and only if

\[
\pi_R - \hat{\pi}_R = \frac{(1+2\alpha \beta_C t)^2}{8\alpha(1+4\alpha \beta_C t)^2} - \frac{(1+2\alpha \beta_R t)^2}{8\alpha(1+4\alpha \beta_R t)^2} - \frac{(1+2\alpha \beta_R t)(1-\gamma)^2x_R^2}{8\alpha(1+4\alpha \beta_R t)(3+4\alpha \beta_R t)^2} > 0
\]

which leads to the condition in Inequality (27).

A.5 Proof of Proposition 3

Proof. (a) We here consider the symmetric equilibrium (i.e., \(p_A = p_B\) in equilibrium). By the definition in Equation (7), \(y_A + y_B = 1\). So when \(y_A = \frac{1}{2\gamma t}(-\gamma \epsilon + \beta t) > 0\), the case falls under the \(t\)-dominates-\(\text{quality}\) case and when \(y_A < 0\), the case falls under the \(\text{quality}\)-dominates-\(t\) case. Therefore, the scenario with reviews is in the \(t\)-dominates-\(\text{quality}\) case if \(\tau \epsilon < \beta_R t\), and the scenario without reviews is in the \(t\)-dominates-\(\text{quality}\) case if \(\beta_C t < \epsilon\).

(b) By Lemma 1, the equilibrium outcomes for both scenarios can be formulated in a uniform manner: \(w_i = \frac{\tau}{1+4\alpha \tau}, p_i = \frac{1+6\alpha \tau}{4\alpha(1+4\alpha \tau)}, \pi_i = \frac{(1+2\alpha \tau)^2}{4(1+4\alpha \tau)^2}\), and \(\pi_R = \frac{(1+2\alpha \tau)^2}{8\alpha(1+4\alpha \tau)^2}\), where \(i \in \{A, B\}\), and
\( \tau = \epsilon \) for the scenario without reviews and \( \tau = \beta_R t \) for the scenario with (symmetric) reviews. By checking the first-order derivatives, we can verify that \( w_i, p_i \), and \( \pi_i \) are increasing in \( \tau \). For instance,

\[
\frac{\partial w_i}{\partial \tau} = \frac{1}{1+4\alpha \tau} - \frac{4\alpha \tau}{(1+4\alpha \tau)^2} = \frac{1}{(1+4\alpha \tau)^2} > 0
\]

\( \pi_R \) is decreasing because

\[
\frac{\partial \pi_R}{\partial \tau} = \frac{4\alpha (1+2\alpha \tau)}{8\alpha (1+4\alpha \tau)^2} - \frac{8\alpha (1+2\alpha \tau)^2}{8\alpha (1+4\alpha \tau)^3} = -\frac{(1+2\alpha \tau)}{2(1+4\alpha \tau)^2} < 0
\]

The conclusions in Part (b) then follow.

A.6 Proof of Proposition 4

Proof. We denote as \( w \) the equilibrium wholesale price in no-review scenario (noticing \( w_A = w_B \)). The optimal retail prices in Equation (42), by substituting \( a_A, a_B, b, \) and \( c \) in, can be formulated as

\[
p_A = \frac{w}{2} + \frac{1}{4\alpha} + \frac{(1-\gamma) x_R}{4\tau}
\]
\[
p_B = \frac{w}{2} + \frac{1}{4\alpha} - \frac{(1-\gamma) x_R}{4\tau}
\]

By substituting \( p_A \) and \( p_B \) into Equation (41), the demand functions can be formulated as

\[
D_A = \frac{1}{4} - \frac{\alpha w}{2} + \frac{(1-\gamma) x_R}{4\tau}
\]
\[
D_B = \frac{1}{4} - \frac{\alpha w}{2} - \frac{(1-\gamma) x_R}{4\tau}
\]

Notice that \( \gamma = 1 \) represents no-review scenario and \( \gamma = r \) represent review scenario in the above formula. Therefore, the demand for manufacturer \( A \) is increased by \( \frac{(1-r) x_R}{4\tau} \) because of reviews and thus its profit is also increased. Similarly, the conclusion on manufacturer \( B \) follows. The retailer’s profit can be formulated as

\[
(p_A - w)D_A + (p_B - w)D_B = \frac{(1-2\alpha w)^2}{8\alpha} + \frac{(1-\gamma)^2 x_R^2}{8\tau (1+\alpha \tau)}
\]

Therefore, the retailer’s profit is increased by \( \frac{(1-r)^2 x_R^2}{8\tau (1+\alpha \tau)} \) because of reviews.

A.10 Proof of Proposition 5

Proof. We denote \( l_A \equiv \frac{1}{2} + \frac{1}{2 \tau} (1-\gamma) x_R \), \( l_B \equiv \frac{1}{2} - \frac{1}{2 \tau} (1-\gamma) x_R \), and \( h \equiv \frac{1}{2r} \). Retailer \( k \)'s optimization
problem in stage 2 is characterized by the first-order conditions of Equation (29):

\[
\frac{\partial \pi_{ik}}{\partial p_{Ak}} = l_A - 2(h + a)p_{Ak} + 2hp_{Bk} + (h + a)w_{Ak} - hw_{Bk} + cp_{Bk} + bp_{Ak} = 0
\]

\[
\frac{\partial \pi_{ik}}{\partial p_{Bk}} = l_B - 2(h + a)p_{Bk} + 2hp_{Ak} + (h + a)w_{Bk} - hw_{Ak} + cp_{Ak} + bp_{Bk} = 0
\]

Based on the retailers’ best responses to each other, we can derive their optimal retail prices as functions of the wholesale prices:

\[
p_{i1} = \frac{(\phi_{in} + \phi_1 w_{i1} + \phi_2 w_{i2} + \phi_3 w_{i1}^2 + \phi_4 w_{i2}^2)}{\phi_d}
\]

\[
p_{i2} = \frac{(\phi_{in} + \phi_1 w_{i2} + \phi_2 w_{i1} + \phi_3 w_{i2}^2 + \phi_4 w_{i1}^2)}{\phi_d}
\]

(45)

where \(i \in \{A, B\}\), \(\phi_d = [(2a)^2 - (b + c)^2][(2a + 4h)^2 - (b - c)^2]\), and

\[
\phi_{in} = (2a + b + c)(2a - b + 2h)(2a + b - c + 4h)l_i + (2a + b + c)(c + 2h)(2a + b - c + 4h)l_i
\]

\[
\phi_1 = 2 \left[4a^4 + 16a^3h - 2a(b + c)^2h - 2(b + c)^2h^2 - a^2(b^2 + c^2 - 16h^2)\right]
\]

\[
\phi_2 = [ab + (b + c)h] [-b^2 + c^2 + 4a(a + 2h)]
\]

\[
\phi_3 = 4 [ab + (b + c)h] [ac + (b + c)h]
\]

\[
\phi_4 = (4a^2 + b^2 - c^2 + 8ah) [ac + (b + c)h]
\]

The manufacturers’ optimization problems in stage 1 are characterized by the first-order conditions of Equation (30) (noticing that \(p_{ik}\) in \(D_{ik}\) is a function of \(w_{ik}\) as in Equation (45)):

\[
\frac{\partial \pi_i}{\partial w_{ik}} = [w_{ik}m_1 + w_{ik}m_2 + 2w_{ik}m_3 + 2w_{ik}m_4 - (a - b + v)\phi_{in} + (c + v)\phi_{in} + l_i\phi_d] / \phi_d = 0
\]

where \(i \in \{A, B\}\), \(k \in \{1, 2\}\), \(m_1 = [\phi_1 + h\phi_2 + b\phi_3 - (a + h)\phi_4]\), \(m_2 = [v\phi_1 + h\phi_2 - (a + h)\phi_3 + b\phi_4]\), \(m_3 = [-a + h)\phi_1 + b\phi_2 + h\phi_3 + c\phi_4]\), and \(m_4 = [b\phi_1 - (a + h)\phi_2 + c\phi_3 + h\phi_4]\). Based on the manufacturers’ best responses to each other, we can derive the optimal wholesale prices:

\[
w_{Ak} = \frac{2m_A(m_3 + m_4) - m_B(m_1 + m_2)}{(m_1 + m_2)^2 - 4(m_3 + m_4)^2}
\]

\[
w_{Bk} = \frac{2m_B(m_3 + m_4) - m_A(m_1 + m_2)}{(m_1 + m_2)^2 - 4(m_3 + m_4)^2}
\]

(46)

where \(k \in \{1, 2\}\), \(m_A = [l_A\phi_d - (a - b + h)\phi_{An} + (c + h)\phi_{Bn}]\), and \(m_B = [l_B\phi_d - (a - b + h)\phi_{Bn} + (c + h)\phi_{An}]\). Substituting the above optimal wholesale prices into Equation (45), we derive the optimal retail prices. Substituting the optimal retail prices and wholesale prices into Equation (30), we can derive the equilibrium profits as shown in Proposition 5.
A Effect of Review Precisions

In this section, we examine the effects of review precisions in both the quality dimension and fit dimension (measured by $r$ and $\beta_R$, respectively) on the wholesale prices, retail prices, and each player’s profit in the scenario with online product reviews. We also study the effect of the degree of misfit disutility (measured by $t$) on the prices and profits. Without loss of generality, we continue to assume $x_R \geq 0$.

A.1 Precision in Quality Dimension

As discussed around Equation (3), in the presence of the reviews, consumers combine their own quality assessment information with the review information, and the relative weights depend on the relative precision of the two information sources. When reviews have a better precision in quality, consumers put more weight $(1 - r)$ to the reviews in forming their new assessments. Therefore, $(1 - r)$ reflects the review precision in the quality dimension, and a better precision means a higher $(1 - r)$.

With online product reviews, in the demand functions Equation (9), $\tau = r\epsilon$ for the quality-dominates-fit case and $\tau = \beta_R t$ for the fit-dominates-quality case. Evidently, the review precision in the quality dimension $(1 - r)$ affects the market potentials, or the mean-shifting effect, for both cases. The precision also affects the product substitutability between the two products (reflected by $\frac{1}{\tau^2}$ as discussed previously), or the variance-reducing effect, in the quality-dominates-fit case. We next show the effect of the review precision in the quality dimension by showing how the change in $(1 - r)$ affects the equilibrium prices and profits in the scenario with reviews.

Proposition 6. The effects of a precision improvement in the quality dimension are as follows.

(a) The improvement lowers product B’s wholesale price and manufacturer B’s profit (i.e., $\frac{\partial \hat{w}_B}{\partial (1 - r)} \leq 0$ and $\frac{\partial \hat{\pi}_B}{\partial (1 - r)} \leq 0$) in both the quality-dominates-fit and fit-dominates-quality cases.

(b) The improvement generically increases product A’s wholesale price and manufacturer A’s profit (i.e., $\frac{\partial \hat{w}_A}{\partial (1 - r)} \geq 0$ and $\frac{\partial \hat{\pi}_A}{\partial (1 - r)} \geq 0$) except in the quality-dominates-fit case with a small $x_R$. In particular, in the quality-dominates-fit case, $\frac{\partial \hat{w}_A}{\partial (1 - r)} > 0$ if and only if

$$x_R > \frac{\epsilon(3+4\alpha r\epsilon)^2}{(3+4\alpha\epsilon)(1+4\alpha r\epsilon)^2}$$

(47)
and \( \frac{\partial \widetilde{\pi}_A}{\partial r_{t}} > 0 \) if and only if
\[
x_R > \frac{r \epsilon (3+4 \alpha \epsilon r)}{(1+4 \alpha \epsilon r)^2 (3+r(3+4 \alpha r(3+r(2+4 \alpha r))))}
\]

(c) The improvement increases the retailer’s profit and lowers product B’s retail price (i.e., \( \frac{\partial \widetilde{\pi}_R}{\partial r_{t}} \geq 0 \) and \( \frac{\partial \widetilde{\pi}_B}{\partial (1-r)} \leq 0 \)). The improvement generically increases product A’s retail price (i.e., \( \frac{\partial \widetilde{\pi}_A}{\partial (1-r)} \geq 0 \)) except in the quality-dominates-fit case with a small \( x_R \). In particular, in the quality-dominates-fit case, \( \frac{\partial \widetilde{\pi}_A}{\partial (1-r)} > 0 \) if and only if
\[
x_R > \frac{2 \epsilon (1+4 \alpha \epsilon r)^2 (3+4 \alpha \epsilon r)^2}{(1+4 \alpha \epsilon r)^2 (15+\alpha(17+2r(18+\alpha(20+r(11+12 \alpha))))}.
\]

**Proof.** Substituting \( \tau = r \epsilon \) into the expressions in Lemma 1, we have the equilibrium outcome for the scenario with reviews in the quality-dominates-fit case, and substituting \( \tau = \beta_R t \) into the expressions, we have the equilibrium outcome for the scenario with reviews in the fit-dominates-quality case.

In the fit-dominates-quality case,
\[
\frac{\partial \hat{\psi}_A}{\partial r} = - \frac{\partial \hat{\psi}_B}{\partial r} = - \frac{x_R}{3+4 \alpha \beta_R t} \leq 0
\]
\[
\frac{\partial \hat{\psi}_B}{\partial r} = - \frac{\partial \hat{\psi}_B}{\partial r} = - \frac{(5+6 \alpha \beta_R t)x_R}{4(1+\alpha \beta_R t)(3+4 \alpha \beta_R t)} \leq 0
\]
\[
\frac{\partial \hat{\psi}_A}{\partial r} = \frac{(1+2 \alpha \beta_R t)^2 (1-r) x_R}{4 \beta_R t (1+\alpha \beta_R t)(3+4 \alpha \beta_R t)^2} \geq 0
\]
where the last inequality is because \( \beta_R t (3+4 \alpha \beta_R t) > (1+4 \alpha \beta_R t)(1-\gamma) x_R \) by Equation (10).

In the quality-dominates-fit case,
\[
\frac{\partial \hat{\psi}_B}{\partial r} = \frac{\epsilon}{(1+4 \alpha \epsilon r)^2} + \frac{(3+4 \alpha r)x_R}{(3+4 \alpha \epsilon r)^2} > 0
\]
by noticing \( r \epsilon (3+4 \alpha r) > (1+4 \alpha \epsilon r)(1-\gamma) x_R \) by Equation (10). The above two inequalities together with Inequality (50) and Inequality (54), we conclude Part (a) (by noticing that \( \frac{\partial \hat{\psi}_X}{\partial (1-r)} = - \frac{\partial \hat{\psi}_X}{\partial r} \)).
\[
\frac{\partial \hat{w}_A}{\partial r} < 0 \text{ if and only if } \frac{\partial \hat{w}_A}{\partial r} = \frac{\epsilon}{(1+4\alpha r \epsilon)^2} - \frac{(3+4\alpha r) x_R}{(3+4\alpha r)^2} < 0
\]

which leads to the condition in Inequality (47). Together with Inequality (50), the conclusion on \( \frac{\partial \hat{w}_A}{\partial (1-r)} \) in Part (b) follows.

\[
\frac{\partial \hat{w}_A}{\partial r} < 0 \text{ if and only if } \frac{\partial \hat{w}_A}{\partial r} = \frac{\epsilon}{(1+4\alpha r \epsilon)^2} - \frac{(3+4\alpha r) x_R}{(3+4\alpha r)^2} < 0
\]

which leads to the condition in Inequality (48). Together with Inequality (53), the conclusion on \( \frac{\partial \hat{w}_A}{\partial (1-r)} \) in Part (b) follows.

We have

\[
\frac{\partial \hat{\pi}_B}{\partial r} = \frac{\epsilon}{2(1+4\alpha r \epsilon)^2} + \frac{x_B[15+\alpha(17+2r(18+\alpha(20+r(11+12\alpha)))]}{4(1+\alpha r \epsilon)^2(3+4\alpha r)^2} > 0
\]

Together with Inequality (51), the conclusion on \( \frac{\partial \hat{\pi}_B}{\partial (1-r)} \) in Part (c) follows.

\[
\frac{\partial \hat{p}_A}{\partial r} < 0 \text{ if and only if } \frac{\partial \hat{p}_A}{\partial r} = \frac{\epsilon}{2(1+4\alpha r \epsilon)^2} - \frac{x_B[15+\alpha(17+2r(18+\alpha(20+r(11+12\alpha)))]}{4(1+\alpha r \epsilon)^2(3+4\alpha r)^2} < 0
\]

which leads to the condition in Inequality (49). Together with Inequality (51), the conclusion on \( \frac{\partial \hat{p}_A}{\partial (1-r)} \) in Part (c) follows.

We have

\[
\frac{\partial \hat{p}_A}{\partial r} = \frac{\epsilon}{2(1+4\alpha r \epsilon)^2} - \frac{x_B[15+\alpha(17+2r(18+\alpha(20+r(11+12\alpha)))]}{4(1+\alpha r \epsilon)^2(3+4\alpha r)^2} < 0
\]

Together with Inequality (52), the conclusion on \( \frac{\partial \hat{p}_A}{\partial (1-r)} \) in Part (c) follows.

As discussed previously, manufacturer B may suffer from unfavorable reviews \( x_R \) \((x_R > 0)\) in two aspects: the mean-shifting effect and the variance-reducing effect. For any given \( x_R \) unfavorable to manufacturer B, an increase in the precision shifts more demand away from product B, because with an increased precision consumers become more confident in the product reviews, take the public unfavorable information \( x_R \) even more seriously, and therefore are more inclined to buy product A. The reduced potential market for product B drives product B’s wholesale price further down as well as manufacturer B’s profit. In addition, in the quality-dominates-fit case, with an increased precision, the variance-reducing effect becomes more salient because consumers put more
weight on the public information source $x_R$, which further reduces the heterogeneity in consumers’ perceived utility differences of the two products. In this case, the increased variance-reducing effect also contributes to driving product $B$’s wholesale price further down and lowering manufacturer $B$’s profit.

Manufacturer $A$ benefits from the mean-shifting effect but suffers from the variance-reducing effect. With an increased precision, the mean-shifting effect is even more favorable to manufacturer $A$. The enhanced market potential allows manufacturer $A$ to charge a higher wholesale price and leads to a higher profit. Therefore, in the fit-dominates-quality case, the improvement in the precision increases product $A$’s wholesale price and manufacturer $A$’s profit. In the quality-dominates-fit case, similar to manufacturer $B$, manufacturer $A$ is also negatively affected by an increase in the review precision in the variance-reducing effect. As a result, whether manufacturer $A$ will benefit from an increase in the precision depends on the balance between the gain from the mean-shifting effect and the loss from the variance-reducing effect. When the reviews are strongly favorable to manufacturer $A$ (i.e., when $x_R$ is large), the gain from the mean-shifting effect outweighs the loss from the variance-reducing effect, and as a result product $A$’s wholesale price is higher and so is manufacturer $A$’s profit.

With the decreased wholesale price for product $B$ resulting from the increased review precision, the retailer is induced to charge a lower retail price for product $B$ in balancing the price and demand. For product $A$, as for manufacturer $A$, the retail price depends on the tradeoff between the mean-shifting effect and the variance-reducing effect. As a result, if the reviews are strongly positive for product $A$, the market potential can increase so significantly such that the mean-shifting effect dominates the variance-reducing effect and the retailer charges a higher retail price for product $A$.

As discussed, the retailer benefits from online product reviews from both the variance-reducing effect and the mean-shifting effect. With an increased precision, the variance-reducing effect becomes more salient which further homogenizes consumers’ perceived utility differences between the two products and thus increases the retailer’s profit. Also, given any $x_R > 0$, with an increased precision, the market potentials for the two products become even more diverse and asymmetric. As discussed in Section 3.1, the asymmetry in the market potentials benefits the retailer, and therefore the increased asymmetry resulting from the increased review precision further boosts the retailer’s profit.
A.2 Precision in Fit Dimension

We next show the effect of the review precision in the fit dimension on the equilibrium prices and profits. As discussed around Equation (3), with the product review information, each consumer re-assesses her fit to the product and perceives her location misfit incurred by product \( A \) at as \( y_C \) with probability \( \beta_R \). When the precision of the fit dimension increases, consumers are more certain about their fits to each product; that is, consumers perceive their location misfit with a higher probability \( \beta_R \). Therefore, \( \beta_R \) reflects the review precision in the fit dimension, and a better precision means a higher \( \beta_R \).

In the quality-dominates-fit case, as illustrated by the demand functions Equation (9) (where \( \tau = r \epsilon \)), \( \beta_R \) does not play any role in the aggregate demands for products \( A \) and \( B \). Therefore, the change in \( \beta_R \) should bear no effect on the equilibrium prices and profits in that case.

In the fit-dominates-quality case, \( \tau = \beta_R t \) in the demand functions Equation (9) for the scenario with reviews. The change in \( \beta_R \) affects both the market potential (the mean-shifting effect) and the substitutability between the two products measured by \( \frac{1}{2 \beta_R^2} \) (which is the variance-increasing effect in this case). We next show the effect of the review precision in the fit dimension by showing how the change in \( \beta_R \) affects the equilibrium prices and profits in the scenario with reviews.

**Proposition 7.** The effects of a precision improvement in the fit dimension are as follows. In the quality-dominates-fit dimension, the improvement has no effect on the equilibrium outcome. In the fit-dominates-quality case,

(a) the improvement increases product \( B \)'s wholesale price and manufacturer \( B \)'s profit (i.e., \( \frac{\partial \hat{w}_B}{\partial \beta_R} > 0 \) and \( \frac{\partial \hat{\pi}_B}{\partial \beta_R} > 0 \));

(b) the improvement increases product \( A \)'s wholesale price and manufacturer \( A \)'s profit if and only if \( x_R \) is small; in particular, \( \frac{\partial \hat{w}_A}{\partial \beta_R} > 0 \) if and only if

\[
x_R < \frac{(3+4a\beta_R t)^2}{4a(1+4a\beta_R t)^2(1-r)}
\]

and \( \frac{\partial \hat{\pi}_A}{\partial \beta_R} > 0 \) if and only if

\[
x_R < \frac{\beta_R t(3+4a\beta_R t)^2}{(1-r)(1+4a\beta_R t)^2(3+4a\beta_R t(3+4a\beta_R t))}
\]

(c) the improvement lowers the retailer's profit and increases product \( B \)'s retail price (i.e., \( \frac{\partial \hat{\pi}_R}{\partial \beta_R} < 0 \)),

(55)

and

(56)
0 and \( \frac{\partial p_B}{\partial \beta_R} > 0 \); the improvement increases product A’s retail price (i.e., \( \frac{\partial \hat{p}_A}{\partial \beta_R} > 0 \)) if and only if

\[
x_R < \frac{2(1+\alpha\beta_{Rt})^2(3+4\alpha\beta_{Rt})^2}{\alpha(1+4\alpha\beta_{Rt})^2(17+8\alpha\beta_{Rt}(5+3\alpha\beta_{Rt}))(1-\tau)}
\]  

(57)

**Proof.** Substituting \( \tau = \beta_{Rt} \) into the expressions in Lemma 1, we have the equilibrium outcome for the scenario with reviews in the fit-dominates-quality case.

(a) We have

\[
\frac{\partial \hat{w}_B}{\partial \beta_R} = \frac{t}{(1+4\alpha\beta_{Rt})^2} + \frac{4\alpha t(1-\tau)x_R}{(3+4\alpha\beta_{Rt})^2} > 0
\]

which leads to the condition in Inequality (55).

(b) \( \frac{\partial \hat{w}_A}{\partial \beta_R} > 0 \) if and only if

\[
\frac{\partial \hat{w}_A}{\partial \beta_R} = \frac{t}{(1+4\alpha\beta_{Rt})^2} - \frac{4\alpha t(1-\tau)x_R}{(3+4\alpha\beta_{Rt})^2} > 0
\]

which leads to the condition in Inequality (56).

(c) We have

\[
\frac{\partial \hat{p}_B}{\partial \beta_R} = \frac{t}{2(1+4\alpha\beta_{Rt})^2} + \frac{\alpha t(17+8\alpha\beta_{Rt}(5+3\alpha\beta_{Rt}))(1-\tau)x_R}{4(1+4\alpha\beta_{Rt})^2(3+4\alpha\beta_{Rt})^2} > 0
\]

which leads to the condition in Inequality (57).

The increase in the precision in the fit dimension benefits manufacturer B in two ways. First, with the increased precision, consumers are more certain about their fits to the products and they are more differentiated from each other. The increased differentiation further softens the competition.
between the two manufacturers and the variance-increasing effect discussed in Section 3.2 becomes more salient, which induces manufacturer $B$ to charge a higher wholesale price and leads to a higher profit. Second, as illustrated in Equation (9), the increase in the precision in the fit dimension mitigates the negative effect of the negative reviews on product $B$'s market potential (i.e., its market potential $\frac{1}{2} - \frac{1}{2\beta_{R_t}}(1 - \gamma) x_{R_{t}}$ increases with the increase in the precision). Consumers face two dimensional uncertainty, quality and fit, in assessing the utility differences between the two products. In the case with a higher precision in the fit dimension, the estimate in the fit dimension plays a more important role than in the case with a low precision and the estimate in the quality plays a less important role. As a result, the increased precision in the fit dimension makes consumers less sensitive to the negative reviews against product $B$ which benefits manufacturer $B$ in both its wholesale price and profit.

Similarly, the changes in the variance-increasing effect and the market potential also influence product $A$’s wholesale price and manufacturer $A$’s profit. As that for product $B$, the increased precision in the fit dimension makes the variance-increasing effect more salient, which tends to drive up product $A$’s wholesale price and manufacturer $A$’s profit. Also, the increased precision makes consumers less sensitive to the product reviews for the quality difference. Notice that the review comments in the quality dimension are in favor of product $A$. Therefore, the increased fit precision makes manufacturer $A$ benefit less from the positive comments for its product in the quality dimension of the reviews. The degree of the decrease in the benefit from the positive comments (i.e., the benefit from the mean-shifting effect) depends on the magnitude of the positive comments. If the reviews are strongly positive, the decrease in the benefit would be greater than the increase in the benefit from the increased variance-increasing effect. As a result, manufacturer $A$ is induced to charge a lower price and earns less profit in equilibrium; in other words, manufacturer $A$ suffers from the increase in the fit precision. If the reviews are mild, manufacturer $A$ benefits from the precision improvement.

With the increased wholesale price for product $B$ resulting from the increased review precision in the fit dimension, the retailer is induced to charge a higher retail price for product $B$ in balancing the price and demand. For product $A$, as for manufacturer $A$, the retail price depends on the tradeoff between the mean-shifting effect and the variance-increasing effect. As a result, if the reviews are strongly positive for product $A$, the market potential can decrease so significantly such that the variance-increasing effect outweighs the mean-shifting effect and the retailer charges a lower retail price for product $A$. 

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As discussed, the retailer suffers from online product reviews in the variance-increasing effect and benefits in the mean-shifting effect. With an increased precision, the variance-increasing effect becomes more salient and thus decreases the retailer's profit. Also, with a higher fit precision in the reviews, the quality difference $x_R$ revealed by the product reviews plays a less important role while the fit information plays a more important role. As a result, the increased fit precision makes the market potentials for the two products less asymmetric. Again, as discussed previously in Section 3.1, the asymmetry in the market potentials benefits the retailer, and therefore the decreased asymmetry resulting from the increased review precision tends to lower the retailer's profit as well.

A.3 Effect of Relative Weight between Quality and Fit Dimensions

In this section, we examine the effect of consumers’ relative weight between the quality and fit dimensions in assessing utility differences between the two products. As formulated in Equation (1), we have $t$ as the misfit unit cost. When $t$ is large, consumers experience high misfit disutility, and vice versa. Since we do not impose a weight on the quality dimension or the weight for the quality dimension (i.e., the coefficient for $\theta$ in Equation (1)) can be viewed as being normalized to 1, the weight $t$ on the fit dimension can also be interpreted as the relative weight between the fit and quality dimensions: the larger the $t$, the more important role the fit dimension plays in determining consumers’ perceived utility differences.

We are interested in the effect of relative weight $t$ because the fit dimension may play a more important role in some cases than in others, and the relative weight seems to have different effects on the outcome in the scenario without reviews and in the scenario with reviews in some cases. First, in the quality-dominates-fit case, similar to the explanation for the absence of $\beta_R$ in the demand function, $t$ does not play a role in the aggregate demand for each product, as illustrated in Equation (9) (where $\tau = \gamma \epsilon$). Therefore, the relative weight $t$ has no effect on the equilibrium prices and profits.

In the fit-dominates-quality case, because now the change in the relative weight affects both the scenario without reviews and the one with reviews, the analysis of the effect of relative weight on the difference is more complicated than the previous analysis on the effect of review precisions (although it can be similarly formulated as before). Because of the page limit, we omit the analysis of the general case. Instead, we next use the symmetric case for the scenario with reviews to illustrate how the change in relative weight $t$ affects price and profit differences between the scenario without reviews and the scenario with reviews.
Proposition 8. The effects of the relative weight \( t \) in the presence of symmetric reviews are as follows. In the quality-dominates-fit case, the relative weight \( t \) has no effect on the equilibrium prices and profits. In the fit-dominates-quality case,

(a) \( \frac{\partial (\hat{\psi}_i - w_i)}{\partial t} > 0 \) and \( \frac{\partial (\hat{\psi}_i - p_i)}{\partial t} > 0 \), \( i \in \{A, B\} \), if \( \beta_R < \frac{1}{16a^2t^2c^2} \); otherwise, both are negative.

(b) \( \frac{\partial (\hat{\psi}_i - \pi_i)}{\partial t} > 0 \), \( i \in \{A, B\} \), if and only if

\[
\beta_R < \frac{1}{8(\alpha t(1+4\alpha t\beta_C)\sqrt{\alpha t\beta_C(1+\alpha t\beta_C)})+\alpha^2t^2\beta_C(3+4\alpha t\beta_C)}
\]

(c) \( \frac{\partial (\hat{p}_R - \pi_R)}{\partial t} > 0 \) if and only if

\[
\beta_R > \frac{t\alpha(1+4\alpha t\beta_C)\sqrt{t(1+32\alpha t\beta_C(1+2\alpha t\beta_C))+6t-4t^2\alpha^2\beta_C(3+8\alpha t\beta_C)}}{64t^3\alpha^3\beta_C(1+2\alpha t\beta_C)}
\]

Proof. For the quality-dominates-fit case, substituting \( \tau = \beta_C t \) into the expressions in Lemma 1 leads to the equilibrium outcome for the scenario without reviews, and substituting \( \tau = \beta_R t \) into the expressions leads to the equilibrium outcome for the scenario with reviews.

(a) For \( i \in \{A, B\} \), we have

\[
\frac{\partial \hat{\psi}_i}{\partial t} - \frac{\partial w_i}{\partial t} = \frac{\beta_R}{(1+4\alpha t\beta_R)^2} - \frac{\beta_C}{(1+4\alpha t\beta_C)^2} = \frac{(\beta_R-\beta_C)(1-16\alpha^2\beta_R\beta_Ct^2)}{(1+4\alpha t\beta_R)^2(1+4\alpha t\beta_C)^2}
\]

Therefore, \( \frac{\partial (\hat{\psi}_i - w_i)}{\partial t} > 0 \) if and only if \( 16\alpha^2\beta_R\beta_Ct^2 < 1 \). Similarly,

\[
\frac{\partial \hat{p}_i}{\partial t} - \frac{\partial p_i}{\partial t} = \frac{\beta_R}{2(1+4\alpha t\beta_R)^2} - \frac{\beta_C}{2(1+4\alpha t\beta_C)^2} = \frac{(\beta_R-\beta_C)(1-16\alpha^2\beta_R\beta_Ct^2)}{2(1+4\alpha t\beta_R)^2(1+4\alpha t\beta_C)^2}
\]

which leads to the condition in Inequality (58).

(b) For \( i \in \{A, B\} \), \( \frac{\partial (\hat{\psi}_i - \pi_i)}{\partial t} > 0 \) if and only if

\[
\frac{\partial \hat{\psi}_i}{\partial t} - \frac{\partial \pi_i}{\partial t} = \frac{\beta_R}{4(1+4\alpha t\beta_R)^2} - \frac{\beta_C}{4(1+4\alpha t\beta_C)^2} = \frac{(\beta_R-\beta_C)[1-48\alpha^2\beta_R\beta_Ct^2-64\alpha^3\beta_R\beta_Ct^4(\beta_C+\beta_R)]}{4(1+4\alpha t\beta_R)^2(1+4\alpha t\beta_C)^2}
\]

which leads to the condition in Inequality (59).
Similar to the variance-increasing effect, in both scenarios with reviews and without reviews, the wholesale prices and retail prices increase in $t$ (i.e., $\frac{\partial w_i}{\partial t} > 0$, $\frac{\partial \hat{w}_i}{\partial t} > 0$, $\frac{\partial p_i}{\partial t} > 0$, and $\frac{\partial \hat{p}_i}{\partial t} > 0$) because an increase in weight $t$ increases the heterogeneity of consumers’ perceived utility differences between the two products. The effects of an increase in $t$ are two-fold. First, the increase makes consumers perceive themselves more toward one product than the other and makes them less price sensitive. Second, the increase leads to more determined consumers toward each product; that is, $y_A$ increases and $y_B$ decreases. However, the magnitude of the effect of $t$ on the extent to which the heterogeneity is increased in a scenario depends on the value of $\beta$ in that scenario. The perceived utility difference for a scenario is affected by $\beta t$ in that scenario, with $\beta = \beta_C$ for the scenario without reviews and $\beta = \beta_R$ with reviews.

When $\beta$ is low, consumers assign a small weight to the fit signals in computing the difference in fit costs between the two products, which means that consumers’ perceived misfit locations are concentrated in the middle of the Hotelling linerange, and therefore an increase in $t$ will have a similar quantitative effect on most consumers. Thus, though the impact of an increase in $t$ on an individual consumer is low, the similar impact is experienced by most consumers and the change in the number of determined consumers would be large. On the other hand, when $\beta$ is high, the impact of an increase in $t$ on $\beta t$ is high. But, in this case, consumers assign a large weight to the fit signals in computing the difference in fit costs between the two products, which means that consumers’ perceived location misfits are distributed more evenly along the Hotelling linerange. In this setting, an increase in $t$ will have dissimilar effect on consumers’ perceived difference in fit cost. Therefore, an increase in $t$ affects some consumers’ valuations more and others’ less. All together, the marginal effect of an increase in $t$ on changing consumers’ price sensitivity and determined consumers would be different when $\beta$ is different. When $\beta_R < \frac{1}{16\alpha^2\beta_C}$, an increase in $t$ enlarges the price increase from the scenario without reviews to the one with reviews because the difference in the effect on changing determined consumers is dominated by the difference in the effect on changing consumers’ price sensitivity, and the decrease in the competition level in the scenario with reviews is more significant than in the scenario without reviews. The results on the manufacturers’ profits and retailer’s profit can be explained similarly.

Proposition 8 presents the impact of weight $t$ when reviews are symmetric. Theoretical and extensive numerical analysis for the general case confirms that Proposition 8 qualitatively holds even when reviews are not symmetric.

In sum, Propositions 6-8 reveal the following key additional insights about the effect of improve-
ments in review precisions on the players in a retail channel. First, an improvement in the precision of quality information does not necessarily benefit the manufacturer with favorable reviews, and an improvement in the precision of fit information need not hurt the manufacturer with unfavorable reviews. Second, the adverse effect of reviews on the retailer may be exacerbated when consumers put more weight on the fit dimension in computing utility, if precision of fit information is low.

The above insights suggest that the perfect review precisions may not necessarily be optimal for all players, and that the optimal precisions for a player will depend critically on the precisions of consumers’ own assessments on the fit dimension, how favorable one product’s reviews are compared to the other in the quality dimension, and the relative weights consumers assign to the quality and fit dimensions.