

**EXAMINING DEMAND ELASTICITIES IN HANEMANN'S FRAMEWORK: A
THEORETICAL AND EMPIRICAL ANALYSIS**

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ABSTRACT

This paper examines demand elasticities using an integrated framework due to Hanemann, which models the incidence, brand choice, and quantity decisions of a consumer as an outcome of her utility maximization subject to budget constraints. Although this framework has been the mainstay of earlier efforts to examine these decisions jointly, a widely acknowledged and important limitation with it is that it restricts the quantity elasticities to -1 regardless of the brand or category. Our focus is on correctly estimating the quantity elasticities in this framework. In the process, we aim to achieve three objectives. First, we analytically demonstrate how assumptions on the distribution of brand specific econometrician's errors imply certain restrictions that in turn force quantity elasticities to -1. Second, we discuss how these restrictions on the estimates of quantity elasticities can be alleviated by considering a suitable specification of unobserved parameter heterogeneity. Third, we use scanner data to empirically illustrate the impact of the restrictions on quantity elasticities and relative efficacies of different specifications of unobserved heterogeneity in easing those restrictions.

Keywords: *Microeconomic Theory of Demand, Quantity Elasticities, Hanemann's framework, Stochastic Properties of Distribution Functions of Econometrician's Errors, Stochastic Properties of Finite Mixture/Normal Random/Mixture Normal specifications of Heterogeneity, Structural Approach*

1. INTRODUCTION

Ever since the availability of scanner panel data, researchers in marketing have been concerned with jointly modeling the three important decisions facing a consumer of a frequently purchased packaged good: whether to buy in the category (incidence), which brand to buy (choice), and how much of the chosen brand to buy (quantity). Almost all the prior literature that has modeled these three decisions simultaneously from an underlying utility maximizing framework has used Hanemann's framework (Hanemann, 1984); the relevant marketing papers are Chiang (1991), Chintagunta (1993), Arora et al. (1998), Nair et al. (2005), and Song and Chintagunta (2007). Table 1 provides further details on each of these papers.

Unfortunately, these papers suffer from a widely acknowledged and important limitation that restricts their applicability to a number of problems, viz. the price elasticities related to the quantity decision are restricted to be equal to (or very close to) 'one' in magnitude, regardless of brand, consumer, or category. In the context of those categories in which consumers make all three purchase decisions, restricted quantity elasticity estimates of this kind clearly hinder any empirical examination/policy analysis of a firm's decisions related to pricing or price promotions. For instance, consider a much studied research question, viz. deciding which categories/brands a retailer should promote. An accurate estimate of quantity elasticities is clearly central to answering this question, since a common response to promotions is purchase acceleration, the magnitude of which is impossible to assess if one restricts quantity elasticities. Similarly, suppose one wanted to perform counterfactuals, such as examining the change in profits or the change in consumer welfare if a product were deleted from a product line, or if the retailer were to change his pricing policies. Again, one needs an appropriate structural model that models all three purchase decisions, and one needs the model to provide correct estimates of quantity elasticity.

This brings us directly to the focus of this paper, i.e., examining why Hanemann's framework suffers from this restriction, and suggesting ways to overcome it. To do so, we attack the problem in the following three logical steps. In the *first* step, we point out three restrictions imposed by assumptions on the joint distribution of the brand specific econometrician's errors¹ in forcing quantity elasticity to be close to one in magnitude. Specifically, we discuss the nature of these restrictions in the absence of unobserved heterogeneity and show that the three restrictions hold true when the econometrician's errors are IID extreme valued, logistic or GEV distributed (an assumption used in all five prior papers based on Hanemann's framework). In the *second*

¹ Intuitively 'brand specific econometrician's errors' are similar to the usual econometrician's errors in brand utilities in standard discrete choice models. For brevity of expression, we use the term 'econometrician's errors', clarifying the terminology where necessary.

step, we show how the three restrictions on the quantity elasticities can be alleviated at the population level by considering unobserved heterogeneity in the parameters. Specifically, having understood the nature of the restrictions imposed by the econometrician's errors in the first step, we use this knowledge to understand the properties required of the specification of unobserved heterogeneity that would help relax these restrictions. This in turn allows us to hypothesize on the extent to which different specifications of unobserved heterogeneity popularly used in prior literature (viz., normal random, finite mixture and mixture normal heterogeneity) would serve as suitable specifications that would satisfy the properties and thereby relax the three restrictions².

Finally, in the *third* step, we empirically demonstrate the nature of the restrictions imposed by the distributional assumptions of the econometrician's errors on the quantity elasticities, as well as test our hypotheses on the extent to which each restriction gets relaxed at the population level over different specifications of unobserved heterogeneity. We do so using scanner panel data in the yogurt category. Our findings suggest that while a normal random or finite mixture specification of heterogeneity does not do a good job of relaxing the restrictions, a mixture normal specification of heterogeneity does help alleviate the restrictions³.

The rest of the paper is organized as follows. In Section 2, we formally elucidate three restrictions imposed by the choice of the distribution of the errors on the estimates of the quantity elasticities. In section 3, we show how the restrictions can be relaxed at the population level by considering unobserved heterogeneity in the parameters. This is followed by section 4, where we empirically demonstrate the restrictions on the quantity elasticities, and the relative efficacy of different specifications of unobserved heterogeneity in relaxing the restrictions. In section 5, we discuss the substantive implications of our empirical findings. Section 6 concludes with limitations and suggestions for future research.

2. MODEL

In section 2.1, we provide the specifications of the incidence, quantity and brand choice decisions in Hanemann's framework. Using these specifications, in section 2.2 we derive the expressions for the price elasticities related to the quantity decision. With this in place, in section 2.3 we detail the

² Since our analysis rests on the restrictions caused by the distribution of econometrician's errors, and the extent to which different specifications of unobserved heterogeneity satisfy properties needed to relax these restrictions, it is useful to provide some key details on the distributional specifications of the econometrician's errors and the unobserved heterogeneity used in prior papers that have used Hanemann's framework. Table 1 provides these details.

³ Note from Table 1 that two of the five papers referenced used a normal random specification of heterogeneity in the quantity decision, while two did not incorporate unobserved heterogeneity in the quantity decision. All these papers have found quantity elasticities to be close to -1. Only one paper (Song and Chintagunta, 2007), uses the finite mixture specification of heterogeneity and finds quantity elasticities to be close to -1 across all brands in two categories, and between -0.74 and -0.9 across brands in two other categories. This result helps underscore our findings discussed later, i.e., finite mixture heterogeneity succeeds only partially in relaxing the restrictions on quantity elasticity.

restrictions on quantity elasticities implied by the choice of the distribution of the econometrician's errors. Note that the discussion on the restrictions is based on the assumption of *no unobserved heterogeneity* in the parameters. It is important to emphasize, however, that the results we derive are not restricted to the no-heterogeneity case; considering the no-heterogeneity case, however, helps gain a clearer intuition into the nature of the restrictions, and provides a natural segue to section 3, where we discuss ways to relax the restrictions by considering unobserved heterogeneity.

2.1 Specifications of the Incidence, Choice, and Quantity Decisions

We start by outlining the theoretical specification of the incidence, brand choice and purchase quantity model based on the joint utility maximization framework laid out by Hanemann (1984). Since much of this treatment is standard, and has been used in prior marketing papers listed in Table 1, we provide only key details here. Consider the case where there are $m = 1..M$ brands in the product category of interest. Let p_m be the unit price and ψ_m be the consumer's quality index of brand m . Further, let ψ_z be the quality index of the composite commodity and p_z be its unit price, which is taken as a numeraire. Finally, let y be the total basket expenditure of the consumer on the given trip. Given these covariates, the consumer's utility maximization problem can be characterized as:

$$\begin{aligned} \max_{\{q_m\}_{m=1}^M} u \left(\sum_{m=1}^M \psi_m q_m, \psi_z z \right) \\ \text{subject to } \sum_{m=1}^M p_m q_m + p_z z = y \\ q_z > 0, q_m \geq 0 \text{ for } m = 1..M \end{aligned} \quad (1a)$$

where $u(\cdot)$ is the direct utility; and q_m and q_z are the quantities of brand m and of the composite commodity respectively. The linearity of the first argument in the utility in (1a) implies that at most one brand can be purchased in the category, namely the one with the lowest quality adjusted price, p_m/ψ_m , amongst all $m=1..M$ brands in the category. To get the functional form specifications of the purchase decisions, we choose the Homothetic Translog (HTL) specification of the indirect utility (this is the most widely used specification in the five prior papers) that corresponds to the direct utility in (1a) between the composite commodity z and the chosen brand k . This is given as

$$v^{HTL} \equiv \ln y - a \ln \frac{p_k}{\psi_k} - (1-a) \ln \frac{p_z}{\psi_z} + \frac{1}{2} b_{11} \left(\ln \frac{p_k}{\psi_k} \right)^2 + \frac{1}{2} b_{22} \left(\ln \frac{p_z}{\psi_z} \right)^2 + b_{12} \left(\ln \frac{p_k}{\psi_k} \right) \left(\ln \frac{p_z}{\psi_z} \right) \quad (1b)$$

where a , b_{11} , b_{12} and b_{22} are the parameters of the HTL indirect utility, with $b_{11} = -b_{12}$ and $b_{11} = b_{22}$. Further, we specify the quality index of any brand m , ψ_m , and that of the composite commodity, ψ_z as follows

$$\ln(\psi_m) \equiv (\beta_m H_m + \eta_m) / \mu \quad (2a)$$

$$\ln(\psi_z) \equiv \mu_z \eta_z \quad (2b)$$

In (2a), the parameter μ is the inverse of consumer's quality sensitivity in the category and is restricted to be positive, H_m is a vector of explanatory variables that include the brand dummies, presence of promotions, state dependence, and inventory for the category. Finally, β_m is a vector of parameters that represent the consumer's sensitivity towards the explanatory variables, and η_m is a random variable that is IID across all households and purchase occasions. In (2b), η_z is a random aggregate shock that is independent of the errors $\{\eta_m\}$ in (2a), and μ_z is the parameter by which the aggregate shock is scaled.

Based on the HTL specification and the functional forms of the quality indices, we make the transformations $V_m \equiv (\beta_m H_m - \mu \ln p_m)/\mu$, $\varepsilon_m \equiv (\eta_m - \mu_z \eta_z \mu)/\mu$, and $\tau \equiv a/b_{11}$, to get the stochastic specifications of the incidence, brand choice and quantity decisions as follows

Purchase Incidence Decision: A purchase will be made in the category (that is $I=1$) if

$$\max_{m=1..M} (V_m + \varepsilon_m) > \tau \quad (3)$$

Brand Choice Decision: Conditional on purchase in the category (that is $I=1$), brand k will be chosen (that is, $d_k=1$) if

$$V_k + \varepsilon_k \geq V_m + \varepsilon_m \quad \forall m=1..M \quad (4)$$

Budget Share/Purchase Quantity Decision: Conditional on purchase in the category and choice of brand k (that is, $I=1$ and $d_k=1$), the budget share of brand k is given as

$$s = b_{11} (V_k + \varepsilon_k - \tau) \quad (5)$$

where the purchase quantity of brand k is related to its budget share as $q_k \equiv y s / p_k$ (for tractability reasons we generally work with the budget share of a brand instead of its purchase quantity; since one is easily obtained from the other, this choice has no material consequences). Notice that the stochastic specifications of all three decisions in (3), (4) and (5) are summarized in terms of two important elements: ' $V_m + \varepsilon_m$ ', and ' τ '. These are explained as follows.

The first element ' $V_m + \varepsilon_m$ ' can be interpreted as the consumer's sub-utility of brand m , with V_m as the deterministic part and ε_m as the stochastic part. The deterministic part, V_m , is a function of the brand's price, p_m , and the explanatory variables in its quality index, H_m . The stochastic part, ε_m , which we henceforth refer to as the 'econometrician's error' is, in turn, a function of the error η_m and the aggregate shock η_z . The interpretation of ' $V_m + \varepsilon_m$ ' as brand m 's sub-utility follows from the fact that in all three decisions, the impact of the market mix variables of any brand m only comes through its sub-utility. Finally, the second element ' τ ' can be interpreted as the consumer's threshold utility for purchase in the category. This interpretation

follows from the purchase incidence decision in (3) - in order for a purchase to be made in the category, the sub-utility for at least one brand has to be greater than τ .

Given the stochastic specifications of the three decisions, we turn next to the derivation of the price elasticities for the purchase quantity/budget share decision of a brand conditional on its purchase. Note that the only stochastic terms in the three decisions in (3), (4) and (5) are the econometrician's errors $\{\varepsilon_m\}_{m=1}^M$. Thus, the computation of the elasticities requires the specification of the joint distribution of these errors. Since $\varepsilon_m \equiv (\eta_m - \mu_z \eta_z \mu) / \mu$, one obtains various specifications of the distribution for the econometrician's errors, depending on one's assumptions about the distribution of the errors $\{\eta_m\}_{m=1}^M$ and the composite good error η_z . The resulting specifications of the distributions of $\{\varepsilon_m\}_{m=1}^M$, in each of the five papers based on the Hanemann's framework are given in Table 1. Essentially, Chiang (1991), Nair et al. (2005) and Song and Chintagunta (2007) assumed $\mu_z = 1/\mu$, and the errors $\{\eta_m, \eta_z\}$ to be IID EV distributed, which resulted in a Logistic distribution for $\{\varepsilon_m\}_{m=1}^M$. Chintagunta (1993) and Arora et al. (1998) assumed $\mu_z = 0$ and the errors $\{\eta_m\}$ to be IID EV distributed, which resulted in an IID EV distribution for $\{\varepsilon_m\}_{m=1}^M$.

Since our main objective is to show how the choice of the joint distribution of the econometrician's errors can restrict the price elasticities related to the purchase quantity decision (or simply the quantity elasticities), we depart from prior literature and do not derive the elasticities using a *specific functional* form of the joint distribution of these errors. Instead, we start in section 2.2 by deriving the specifications of the quantity elasticities related to the purchase quantity decision using a *general form* for the joint distribution function of the errors, $F(\{\varepsilon_m\}_{m=1}^M)$. Based on that, in section 2.3, we discuss three restrictive properties of the joint distribution function that would restrict the estimates of the quantity elasticities.

2.2 Specifications of Elasticities related to the Purchase Quantity/Budget Share Decision

Using the specifications of the three decisions in (3), (4) and (5), we get the distribution of the budget share of brand k conditional on its choice and purchase in the category as:

$$G_k(s|d_k = 1, I = 1) = \Pr\left(V_k + \varepsilon_k \leq \tau + s/b_{11} \mid V_k + \varepsilon_k \geq V_m + \varepsilon_m \quad \forall m = 1..M, \max_{m=1..M}(V_m + \varepsilon_m) > \tau\right) \quad (6)$$

Making the transformation $\zeta = V_k + \varepsilon_k$ in (6), the distribution of the conditional budget share can be written in terms of the general form of the joint distribution of the errors, $F(\{\varepsilon_m\}_{m=1}^M)$ as:

$$G_k(s|d_k=1, I=1) = 1 - \frac{\int_{\tau}^{\infty} F_k(\{\zeta - V_m\}_{m=1}^M) d\zeta}{\int_{\tau}^{\infty} F_k(\{\zeta - V_m\}_{m=1}^M) d\zeta} \quad \text{where } F_k = \frac{\partial F}{\partial \varepsilon_k} \quad (7)$$

From (7), we get the expected value of the conditional budget share of brand k as:

$$E(s|d_k=1, I=1) = \int_s \frac{\partial G_k(s|d_k=1, I=1)}{\partial s} ds = \frac{1}{b_{11}} \frac{\int_s F_k(\{\tau + s/b_{11} - V_m\}_{m=1}^M) ds}{\int_{\tau}^{\infty} F_k(\{\zeta - V_m\}_{m=1}^M) d\zeta} \quad (8)$$

which yields the price elasticity of the expected conditional budget share (or simply the conditional budget share elasticity) of brand k as:

$$\epsilon_k^{bs} = \frac{\partial \ln E(s|d_k=1, I=1)}{\partial \ln p_k} \quad (9)$$

Since the purchase quantity is related to the budget share as $q_k \equiv y s/p_k$, we can relate the quantity elasticity of brand k to its conditional budget share elasticity in (9) as:

$$\epsilon_k^{pq} = \epsilon_k^{bs} - 1 \quad (10)$$

This completes the specifications of the conditional budget share/quantity elasticities. We now turn to the central focus of this section, viz., a discussion of how the choice of the functional form of the distribution of the errors can restrict the estimates of the quantity elasticities.

Before we do so, there are two important points we would like to make. First, although we discuss the impact of the distributional assumptions of the errors on quantity elasticities for a particular demand system, one based on HTL indirect utility, our results can be generalized to all demand systems (the relevant proofs for a general demand system are given in the Online Technical Appendix). Second, we assume that the purchase quantity is continuous. This is in keeping with most of the prior literature based on Hanemann's framework (Chiang 1991; Chintagunta 1993; Nair et al. 2005). More importantly, Nair et al. (2005) empirically compared the predictions of the continuous quantity assumption with those of the discrete quantity approximation proposed by Arora et al. (1998). They found the results to be very similar across the two, implying that the continuous quantity assumption is fairly robust.

2.3 Restrictions Imposed by the Specification of $F(\{\varepsilon_m\}_{m=1}^M)$ on Quantity Elasticities

We discuss three restrictions that can be imposed by the specification of the distribution function of the econometrician's errors on the quantity elasticities given in (10). We do so in the following two lemmas, where Lemma 1 deals with the first restriction and Lemma 2 deals with the other two restrictions.

2.3.1 The First Restriction

We state the restriction in terms of a lemma and a corollary, and then discuss its implications on the quantity elasticities.

Lemma 1: If the joint distribution function of the errors, $F\left(\{\varepsilon_m\}_{m=1}^M\right)$, is such that for any brand k , the function $L_k(x)$ given as

$$L_k(x) \equiv \Pr\left(V_k + \varepsilon_k \leq x \mid V_k + \varepsilon_k \geq V_m + \varepsilon_m \quad \forall m = 1..M, \max_{m=1..M} (V_m + \varepsilon_m) > \tau\right) \quad (11)$$

is identical across all brands, that is $L_k(x) \equiv L_j(x) \quad \forall j, k \in \{1..M\}$, then the specifications of the expected values of the conditional budget shares (equation 11) will also be identical across all brands, i.e., $E(s \mid d_k=1, I=1) \equiv E(s \mid d_j=1, I=1) \quad \forall j, k \in \{1..M\}$.

Proof: See Online Technical Appendix

An Intuitive Proof of Lemma 1: To intuitively understand Lemma 1, notice that the distribution function of the conditional budget share of brand k , $G_k(s \mid d_k=1, I=1)$, in equation (6) is a simple convolution of $L_k(x)$, given in equation (11). In other words, if we replace ‘ x ’ in the expression for $L_k(x)$ in equation (11) with ‘ $\tau+s/b_{11}$ ’, we get the distribution function of the conditional budget share of brand k as given in equation (6). Now if $L_k(x)$ is identical across all brands $k=1..M$, it implies that the distribution of the conditional budget share in (6) will also be identical across all brands. This would in turn imply that the expected values of the conditional budget shares will be identical across all brands, that is, $E(s \mid d_k=1, I=1) \equiv E(s \mid d_j=1, I=1) \quad \forall j, k \in \{1..M\}$.

Corollary: The condition stated in Lemma 1 holds true when the errors $\{\varepsilon_m\}_{m=1}^M$ follow a Generalized Extreme Value (GEV), an IID EV, or a Logistic distribution.

Proof: See Online Technical Appendix.

Implication of Lemma 1: If the specifications of the expected values of the conditional budget shares of all brands are identical, that is, $E(s \mid d_k=1, I=1) \equiv E(s \mid d_j=1, I=1) \quad \forall j, k \in \{1..M\}$, the conditional budget share elasticities of all brands $j=1..M$ with respect to the price of brand k will also be identical, that is

$$\frac{\partial \ln E(s \mid d_k=1, I=1)}{\partial \ln p_k} \equiv \frac{\partial \ln E(s \mid d_j=1, I=1)}{\partial \ln p_k} \quad \forall j, k \in \{1..M\} \quad (12)$$

The above result follows from the fact that since $E(s \mid d_k=1, I=1) \equiv E(s \mid d_j=1, I=1)$ is an identity, differentiating both sides with respect to $\ln(p_k)$ will preserve the identity. Before discussing the implication of (12), it is useful to clarify what the RHS in (12), which is the

conditional budget share elasticity of brand j with respect to the price of another brand $k \neq j$ really captures. This elasticity captures the impact of brand k 's price on a consumer's budget share decision (i.e., her allocation of the total shopping expenditure between the chosen brand j and the composite commodity, conditional on the fact that she has chosen to purchase brand j over brand k). Thus this elasticity refers *only* to consumers who have decided to purchase brand j and not brand k in the category. Intuitively, we would expect the budget share decision of buyers of brand j to not be affected by changes in the price of brand k because they do not buy brand k . Thus we would expect the RHS in equation (12) to be close to zero. However, note that as per the identity in (12), this implies that the LHS, which represents the conditional budget share elasticity of brand k with respect to its own price, *would be biased towards zero*. Since the quantity elasticity of the chosen brand k with respect to its own price is nothing but the conditional budget share elasticity of the chosen brand k with respect to its own price, minus one (equation 10), this implies that *the quantity elasticity of brand k would be biased towards -1*.

Implication of the Corollary: The corollary implies that if there is no unobserved heterogeneity and if we use a GEV, IID EV, or Logistic distribution for the errors, then the quantity elasticities of all brands will be biased towards -1. Linking this to prior literature, Chiang (1991) employed a Logistic distribution and Chintagunta (1993) employed an IID EV distribution for the econometrician's errors (neither paper considered unobserved parameter heterogeneity in the quantity choice decision), and both reported quantity elasticities close to -1. However, note that if the errors are IID normal, then the condition in Lemma 1 does not hold true. Thus, the quantity elasticities will not suffer from the first restriction when the errors are normally distributed⁴.

This ends our discussion on the first restriction. In the following lemma, we discuss two more ways in which the choice of the distribution of the econometrician's errors can restrict the quantity elasticities.

2.3.2 The Second and Third Restrictions

Lemma 2: The magnitude and sign of the deviation of the quantity elasticity of any brand k from '-1' depend respectively on the magnitude and sign of the slope of the function $M_k(x)$, given as

$$M_k(x) \equiv -\frac{\int_x^\infty F_{kk}(\{\zeta - V_m\}_{m=1}^M) d\zeta}{\int_x^\infty F_k(\{\zeta - V_m\}_{m=1}^M) d\zeta} \quad (13)$$

⁴ We also used a simulation exercise to examine the biases caused by the first restriction, when the econometrician's errors are assumed to be either IID EV or Logistic distributed. Details are in Section 1 of the Online Technical Appendix.

for values of $x > \tau$ (recall that $F_k = \partial F / \partial \varepsilon_k$, $F_{kk} = \partial^2 F / \partial \varepsilon_k^2$, and τ is the threshold utility of the consumer in the category). In particular:

- a) If the joint distribution of the errors, $F(\{\varepsilon_m\}_{m=1}^M)$, is such that $\partial M_k(x) / \partial x > 0$ for all $x > \tau$, then the quantity elasticity of brand k will always be greater than one in magnitude, i.e., $|\varepsilon_k^{pq}| > 1$. Similarly, if $\partial M_k(x) / \partial x < 0$ for all $x > \tau$, then the quantity elasticity of brand k will always be less than one in magnitude, i.e., $|\varepsilon_k^{pq}| < 1$.
- b) If the joint distribution of the errors, $F(\{\varepsilon_m\}_{m=1}^M)$, is such that $\partial M_k(x) / \partial x \approx 0$ for all for $x > \tau$, then the quantity elasticity of brand k will be close to one in magnitude, i.e., $|\varepsilon_k^{pq}| \approx 1$.

Proof: See Online Technical Appendix.

Corollary 1: If $F(\{\varepsilon_m\}_{m=1}^M)$ is distributed either GEV, IID EV, Logistic, or Independent Normal, then $\partial M_k(x) / \partial x > 0 \forall x > \tau$ for all brands $k=1..M$. Thus, as per part (a) of Lemma 2, we will always get the quantity elasticities of all brands to be greater than one in magnitude, i.e., $|\varepsilon_k^{pq}| > 1$.

Proof: See Online Technical Appendix.

Corollary 2: If $F(\{\varepsilon_m\}_{m=1}^M)$ is distributed either GEV, IID EV or Logistic, and if the purchase incidence probability in the category is small, then for all brands $k=1..M$, the function $\partial M_k(x) / \partial x \approx 0 \forall x > \tau$. Thus, as per part (b) of Lemma 2, we will get the quantity elasticities for all brands to be close to one in magnitude, i.e., $|\varepsilon_k^{pq}| \approx 1$.

Proof: See Online Technical Appendix.

An Intuitive Proof of Lemma 2: To intuitively prove Lemma 2, consider the survivor function of the distribution of the conditional budget share of brand k , Ψ_k , which is related to the cdf of the conditional budget share of brand k as follows:

$$\Psi_k \equiv 1 - G_k(s6d_k=1, I=1) \quad (14)$$

Next, consider the case where $\partial \ln \Psi_k / \partial \ln p_k < 0$ for all values of the price of brand k , p_k , and the share of brand k . This inequality implies that $\partial \ln G_k / \partial \ln p_k > 0$, and as a result, the cdf of the share of brand k at a higher value of p_k will be first order stochastically dominated by the cdf of the share of brand k at a lower value of p_k – in other words, as the price of brand k increases, the probability mass in the cdf of the budget share of brand k moves towards zero, which in turn

implies that the expected value of s at a higher value of p_k will be smaller than that at a lower value of p_k . As a result, we will get the conditional budget share elasticity of brand k to be negative, and consequently, the quantity elasticity of brand k to be greater than one in magnitude (since $\epsilon_k^{pq} = \epsilon_k^{bs} - 1$). In a similar vein, we can see that if $\partial \ln \Psi_k / \partial \ln p_k > 0$, we would get the quantity elasticity of brand k to be less than one in magnitude; and if $\partial \ln \Psi_k / \partial \ln p_k \approx 0$, we would get the quantity elasticity of brand k to be close to one in magnitude.

Next, using the specification of $G_k(s|d_k=1, I=1)$ given in equation (7), we get the specification of $\partial \ln \Psi_k / \partial \ln p_k$ in terms of the function M_k as given in equation (13) as

$$\partial \ln \Psi_k / \partial \ln p_k = M_k(\tau) - M_k(\tau + s/b_{11}) \quad (15)$$

Consider the RHS of equation (15), which is $M_k(\tau) - M_k(\tau + s/b_{11})$. Since $b_{11} > 0$, it implies that $\tau + s/b_{11} > \tau$, which in turn implies that

- (i) If $\partial M_k(x)/\partial x > 0$ for values of $x > \tau$, then $M_k(\tau) - M_k(\tau + s/b_{11}) < 0$, and as a result, we will get $\partial \ln \Psi_k / \partial \ln p_k < 0$. Thus, following the line of argument given before, we will get the quantity elasticity to be greater than one in magnitude. Similarly, if $\partial M_k/\partial x < 0$ for values of $x > \tau$, we will get the quantity elasticity to be less than one in magnitude.
- (ii) If $\partial M_k(x)/\partial x \approx 0$ for values of $x > \tau$, then $M_k(\tau) - M_k(\tau + s/b_{11}) \approx 0$ and as a result, $\partial \ln \Psi_k / \partial \ln p_k \approx 0$. Thus, following the line of argument given before, we will get the quantity elasticity of brand k to be close to one in magnitude.

This completes the intuitive proof for Lemma 2. Note that Lemma 2 thus implies two restrictions on the estimates of the conditional budget share/quantity elasticities. The first restriction, implied by part (a) of Lemma 2 (which we henceforth refer to as R2a), relates the *sign* of $\partial M_k(x)/\partial x$ for $x > \tau$ to whether the quantity elasticity of brand k will be less than or greater than one in magnitude. The second restriction, implied by part (b) of Lemma 2 (which we henceforth refer to as R2b), relates the *magnitude* of $\partial M_k/\partial x$ for $x > \tau$ to whether the quantity elasticity of brand k will be close to one in magnitude. Furthermore, the corollaries relate the two restrictions to the specific distributions of econometrician's errors. Specifically, if these errors are IID EV/GEV/Logistic distributed, then the quantity elasticities will suffer from both R2a and R2b; and if the errors are independent normal, then the quantity elasticities will only suffer from R2a.

Given the above intuitive discussion, it is useful to delve deeper into the statistical properties of the distribution of the errors that result in the restrictions implied by Lemma 2. To do this, it helps to examine the term $\partial M_k/\partial x$ in the region $x > \tau$. Consider this for the case when there is only one brand k in the category. (Note that this is a salient difference between the

restrictions implied by Lemma 2 and the restriction implied by Lemma 1. The restriction implied by Lemma 1 only holds when the total number of brands in the category is greater than one. However, the restrictions implied by Lemma 2 hold even if there is just one brand in the category.) In such a case, after making the substitution $\varepsilon_k = x - V_k$, the slope of the function $M_k(x)$ given in (13) simplifies to

$$\frac{\partial M_k(x)}{\partial x} \equiv \frac{\partial}{\partial \varepsilon_k} \left[\frac{F_k(\varepsilon_k)}{1 - F(\varepsilon_k)} \right] \quad (16a)$$

and the space spanned by the condition $x > \tau$ simplifies to

$$\varepsilon_k > \tau - V_k \quad (16b)$$

where F is the distribution function of the econometrician's error in brand k 's sub-utility, ε_k , and F_k is its density. Now notice the RHS of (16a) is simply the slope of the *hazard function* of ε_k . And the space spanned by the condition in (16b) is nothing but the region in which a purchase is made in the category, which can be further couched in terms of the purchase incidence probability in the category as⁵ $\varepsilon_k > F^{-1}(1 - \Pr(I=1))$. Thus, the slope of $M_k(x)$ in the region $x > \tau$ can be intuitively interpreted as '*the slope of the hazard function of the error ε_k in the region in which a purchase is made in the category, i.e., $\varepsilon_k > F^{-1}(1 - \Pr(I=1))$.*'

Based on this interpretation, the restriction R2a implied by part (a) of lemma 2 can be interpreted as follows: '*if the hazard function of the econometrician's error in brand k 's sub-utility is increasing in the region in which the category is purchased, then the quantity elasticity of brand k will be greater than 1 in magnitude.*' Given this interpretation, an intuitive explanation of Corollary 1 follows immediately. Note that if ε_k is either EV, Logistic or normally distributed, then its hazard function is always increasing. Thus, if we assume the errors $\{\varepsilon_m\}_{m=1}^M$ to be EV, Logistic, or normally distributed, we will always restrict the quantity elasticities to be greater than one in magnitude.

The restriction R2b implied by part (b) of lemma 2 can similarly be interpreted as: '*for a given purchase incidence probability in the category, $\Pr(I=1)$, if the magnitude of the hazard function of the econometrician's error, ε_k , for values of $\varepsilon_k > F^{-1}(1 - \Pr(I=1))$ is small, then the quantity elasticity of brand k will be close to one in magnitude.*' Next, we use this interpretation to understand why Corollary 2 holds true for EV and Logistic errors, but not for Normal errors. In Figure 1, we plot the slope of the hazard function for EV, Normal, and Logistic distributions

⁵ For a single brand case, using the stochastic specification of purchase incidence decision in (3), we get $\varepsilon_k > \tau - V_k$ as the region in which a purchase is made in the category. Since F is the distribution function of the error term ε_k , it follows that the purchase incidence probability will be $\Pr(I=1) = 1 - F(\tau - V_k)$. Inverting this purchase incidence probability, we get $\tau - V_k = F^{-1}(1 - \Pr(I=1))$. Substituting $\tau - V_k$ into (16b), we get the condition ' $\varepsilon_k > \tau - V_k$ ' as ' $\varepsilon_k > F^{-1}(1 - \Pr(I=1))$ '.

(after normalizing the scale and location parameters across the three distributions) for the single brand case. Observe in Figure 1 that for high values of ε_k (or low values of the purchase incidence probability), the value of the slope of the hazard function becomes very small for EV and Logistic distributions. For instance, if we consider $\Pr(I = 1) = 0.15$ as in the simulation exercise done before, we can see that for values of $\varepsilon_k > F^{-1}(1 - 0.15)$ (which is approximately '1.0' for all three distributions), the slopes of the hazard functions for EV and Logistic distributions become small. However, that is not the case for the normal random errors. Thus as per the interpretation of part (b) of lemma 2, it follows that if the purchase incidence probability in the category is low (which it typically is for most data sets), then assuming EV or Logistic distributions for the econometrician's errors will yield values of quantity elasticities close to one in magnitude.⁶

This ends our discussion on the two restrictions implied by Lemma 2. We have summarized our discussion so far on the impact of the three restrictions on the quantity elasticities, along with the specific distribution functions of the errors for which each restriction holds true in Table 2. In the next section, we discuss how to relax these restrictions.

3. RELAXING THE THREE RESTRICTIONS ON QUANTITY ELASTICITIES

One can relax the three restrictions R1, R2a and R2b in two possible ways. The first alternative is to use a distribution for the errors that does not suffer from the three restrictions, i.e., a distribution other than IID EV, GEV, Independent Normal, or Logistic. The only distribution for the errors (which has a support from $-\infty$ to $+\infty$) that comes to mind is the multivariate normal distribution. Although an empirical question, it is not clear that a multivariate normal distribution for the errors will relax restriction R2a⁷. The second alternative, which we focus on, is to relax the three restrictions on the quantity elasticities at the population level by considering unobserved heterogeneity in the parameters in the brand sub-utilities.

We start in section 3.1 by explaining how the three restrictions discussed so far in the absence of unobserved heterogeneity can be extended to the case when we have unobserved heterogeneity in the parameters in the sub-utilities of brands. Given this, in sections 3.2 and 3.3 we discuss the properties that are required of a specification of unobserved heterogeneity that will help relax each of the three restrictions. Finally, in section 3.4, we hypothesize on the extent to

⁶ This argument mirrors that used by Nair et al. (2005) who also find that when the purchase incidence probability is low, using a Logistic distribution restricts the purchase quantity elasticities to be close to one in magnitude.

⁷ It is difficult to formally prove the existence of restriction R2a for the general case of $M+1$ alternatives when the econometrician's errors are multivariate normally distributed. However, in the Online Technical Appendix we show a formal proof for a simple case of three alternatives (two brands + the no purchase option) when the errors are bivariate normally distributed. We show that in such a case, the purchase quantity elasticity of at least one brand will always be greater than one in magnitude, implying R2a will not be relaxed.

which different specifications of unobserved heterogeneity used in prior marketing literature would satisfy the properties needed to relax the restrictions.

3.1 Understanding the Restrictions in the Presence of Unobserved Heterogeneity

In order to see how the three restrictions can be extended to the unobserved heterogeneity case, it first helps to note the differences in the constituents of the stochastic terms in the sub-utilities of brands between the ‘no unobserved heterogeneity case’ and the ‘unobserved heterogeneity case’. If there is no heterogeneity, recall that the sub-utility of any brand m is given as ‘ $V_m + \varepsilon_m$ ’, where the deterministic part of brand m ’s sub-utility is V_m , and the error in any brand m ’s sub-utility consists of only the econometrician’s error, ε_m . However, once we consider unobserved heterogeneity in the parameters, $\{\beta_m, \mu\}$, in the sub-utilities of brands, the sub-utility of any brand m will be given as ‘ $\bar{V}_m + \omega_m$ ’, in which the deterministic part of brand m ’s sub-utility is \bar{V}_m (which is the expected value of the deterministic part of the sub-utility of brand m , V_m , after taking out the errors due to unobserved heterogeneity in the parameters), and the error in brand m ’s sub-utility, ω_m , will consist of not only the econometrician’s error ε_m , but also the error resulting from the unobserved heterogeneity (we refer to the composite of these two errors, in the unobserved heterogeneity case, as the *overall stochastic term*, ω_m).

Note that the conditions stated in Lemmas 1 and 2 only deal with the statistical properties of the distribution of the errors in the sub-utilities of the brands. Now, in the ‘no heterogeneity case’, to see if the three restrictions hold true, we only need to check whether the statistical properties of the distribution of the econometrician’s errors, $\{\varepsilon_m\}_{m=1}^M$, satisfy the conditions stated in Lemmas 1 and 2 (which is what we did in section 2). On the other hand, in the presence of heterogeneity, the extent to which the three restrictions hold true at the population level depends on the extent to which the statistical properties of the distribution of the overall stochastic terms, $\{\omega_m\}_{m=1}^M$, satisfy the conditions in Lemmas 1 and 2. Thus in summary, in the presence of heterogeneity, the restrictions discussed in section 2 *remain as they are*, except that the conditions are not on the distribution of the econometrician’s errors, but on the overall stochastic term.⁸

⁸ Note that the proofs of Lemmas 1 and 2 for the ‘heterogeneity case’ follows exactly the same line of reasoning as the proofs of Lemma 1 and 2 given in section 2.3 for the ‘no-heterogeneity case’. As pointed out, the ‘no-heterogeneity case’ and the ‘heterogeneity case’ only differ vis-à-vis the specifications of the sub-utilities of brands. Thus the proofs of Lemmas in the ‘heterogeneity case’ remain as they are in the ‘no heterogeneity case’, except that in the proofs, (i) the deterministic part of the sub-utility of any brand m , V_m , is replaced by \bar{V}_m ; and (ii) the joint distribution of the econometrician’s errors, $\{\varepsilon_m\}$ is replaced by the joint distribution of the overall stochastic terms, $\{\omega_m\}$.

3.2 Relaxing Restriction R1

Based on the above discussion, the restriction R1 implied by Lemma 1 can be extended to the case of unobserved heterogeneity as follows: ‘if the joint distribution function of the overall stochastic terms, $\{\omega_m\}_{m=1}^M$, is such that for any brand k , the function $L_k(x)$ given as

$$L_k(x) \equiv \Pr\left(\bar{V}_k + \omega_k \leq x \mid \bar{V}_k + \omega_k \geq \bar{V}_m + \omega_m \quad \forall m = 1..M, \max_{m=1..M}(\bar{V}_m + \omega_m) > \tau\right) \quad (17)$$

is identical across all brands, that is $L_k(x) \equiv L_j(x) \quad \forall j, k \in \{1..M\}$, then the specifications of the expected values of the conditional budget shares will also be identical across all brands.’

To re-iterate, the restriction in (17) differs from its no-heterogeneity counterpart in (11) in that the sub-utility of brand m was represented as ‘ $V_m + \varepsilon_m$ ’ in the no-heterogeneity case in (11) versus ‘ $\bar{V}_m + \omega_m$ ’ in the heterogeneity case in (17). Given the discussion above, it is easy to see that even if the joint distribution of the econometrician’s errors, $\{\varepsilon_m\}_{m=1}^M$, satisfies the condition in Lemma 1, it does not imply that the joint distribution of the overall stochastic terms, $\{\omega_m\}_{m=1}^M$, will also satisfy the condition in Lemma 1.

Having said that, it is important to note that the extent to which R1 gets relaxed at the population level depends on one crucial property, which is the extent to which the specification of the unobserved heterogeneity imposed by the analyst can adequately recover the true heterogeneity in the data. Thus, if the specification of the unobserved heterogeneity is not able to capture the true heterogeneity in the data (and as a result, underestimates it), then the statistical properties of the overall stochastic term will be dictated more by the properties of the econometrician’s error ε_k than by the error due to heterogeneity; in that case, adding heterogeneity will not help fully relax R1.

3.3 Relaxing Restrictions R2a and R2b

Similar to Lemma 1, the restrictions R2a and R2b as implied by Lemma 2 can be extended to the heterogeneity case as follows: ‘if the slope of the hazard function of the overall stochastic term, ω_m , in the sub-utility of brand m is restricted to be positive (negative), then the quantity elasticity of brand m will be greater than one (less than one) in magnitude; and if the slope of the hazard function of the overall stochastic term, ω_m , is close to zero in magnitude in the region in which a purchase in the category, then the quantity elasticity of brand m will be close to one in magnitude.’ Now given this extension, it is easy to see that the extent to which the two restrictions implied by Lemma 2 get relaxed depends on the extent to which the distribution of the overall stochastic term in the brand sub-utilities is flexible enough to allow for the slope of its hazard function to take both positive and negative values (and thereby relax R2a) over a wide

range of magnitudes (and thereby relax R2b). Clearly, even if the distribution of the econometrician's errors, ε_m , satisfies the condition in Lemma 2, it does not imply that the joint distribution of the overall stochastic term, ω_m , will also satisfy the condition in Lemma 2.

To sum, the extent to which R2a and R2b get relaxed depends on *two* factors. First, as in the case of R1, it is important that the specification of unobserved heterogeneity be able to recover the true heterogeneity in the data adequately. Second, the extent to which R2a and R2b get relaxed depends on how flexible the distribution of the unobserved heterogeneity is, in terms of the slope of its hazard function; the greater the flexibility, the more likely that the slope of the hazard function of the overall stochastic term can take a wide range of positive or negative values (and thereby relax R2a and R2b).

Summarizing the discussion above, the extent to which all three restrictions get relaxed depends on two properties of the specification of unobserved heterogeneity: (i) it should do a good job in adequately capturing the true underlying heterogeneity in the data (to help relax R1, R2a and R2b), and (ii) it should be flexible in terms of the slope of its hazard function, to enable the slope of the hazard function of the overall stochastic term to take a wide range of positive and negative values (to help relax R2a and R2b). Next, we hypothesize on the extent to which the different functional form specifications of unobserved heterogeneity used in prior literature satisfy the two properties.

3.4 Relative Efficacy of Different Specifications of Unobserved Parameter Heterogeneity in Relaxing the Restrictions

Based on prior literature, there are three choices of unobserved heterogeneity distribution that come to mind readily, as candidates for relaxing the three restrictions on the quantity elasticities at the population level. These are a) normal random heterogeneity, b) finite mixture heterogeneity (latent class segmentation), and c) mixture normal random heterogeneity.

Normal random specification: In this specification, we consider the parameters β_m in the deterministic part of the sub-utility of brand m , V_m to be normally distributed, and the parameter μ (which is restricted to be positive as per the model specification) to be log normally distributed across the consumer population. Note that introducing the log normal distribution over μ is a departure from previously used normal random specifications of heterogeneity. This is because the log normal distribution has a flexible hazard function whose slope is neither strictly positive nor strictly negative (Lancaster 1992). This gives more flexibility to the slope of the hazard

function of the overall stochastic term, letting it take positive or negative values and hence potentially overcoming R2a⁹.

Having said that, it remains to be empirically seen whether using such a specification of unobserved heterogeneity would satisfy the two properties needed to relax the restrictions. Pertaining to the first property, if the true heterogeneity in the data is multimodal, this specification may do a poor job of recovering it. Pertaining to the second property, although an empirical question, it is not clear that using a simple fix of assuming log normal heterogeneity over μ will make the distribution of the overall stochastic term in a brand's sub-utility flexible enough to enable the slope of its hazard function to take a wide range of positive or negative values and hence overcome R2a and R2b.

Finite mixture specification (Latent class segmentation): Since this specification is semi-parametric, we would expect it to be flexible enough to enable the slope of the hazard function of the overall stochastic term to take a wide range of positive or negative values. However, as shown by Allenby et al. (1998), the finite mixture specification does not capture heterogeneity as well as the normal random specification, since it does a poor job of describing tail behavior. Thus we would expect this specification to satisfy one of the properties needed to relax the restrictions (viz., flexibility), but it remains to be seen empirically as to whether it does a good job of satisfying the other property (viz., capturing the true heterogeneity in the data adequately).

Mixture normal random specification: In this specification, for a given segment, the parameters β_m are assumed to be normally distributed, while the parameter μ is assumed to be log normally distributed across the consumer population. Prior research (Allenby et al. 1998; Rossi and Allenby 2003) has shown that the mixture normal specification does an excellent job of capturing the underlying heterogeneity since with enough components it can approximate virtually any multivariate density. Further, since the mixture normal specification has a semi-parametric component, it would lend flexibility to the slope of its hazard function, which would in turn enable the slope of the hazard function of the overall stochastic term to take a wide range of positive or negative values and hence overcome R2a and R2b. Thus we would expect this specification to satisfy both the properties needed for alleviating the three restrictions.

⁹ Note that if we do not consider the log normal heterogeneity over μ , then the random error in a brand's sub-utility that results from unobserved heterogeneity on β_m only will be normally distributed. Since a normal distribution has a strictly increasing hazard function, it will not lend flexibility to the slope of the hazard function of the overall stochastic term. Recall from Table 1 that this was the distribution of unobserved heterogeneity used by Nair et al. (2005), who considered the normal heterogeneity on β_m , but no unobserved heterogeneity on μ . As a result, they found quantity elasticities to be close to and greater than one in magnitude across all brands. We have modified their specification by taking log normal heterogeneity over μ .

We now turn to an empirical examination of the three restrictions, their impact on quantity elasticities, and the efficacy of our suggested remedies to relax the restrictions.

4. EMPIRICAL ANALYSIS

4.1 Design of the Empirical Analysis

Our objective in this section is to empirically test the extent of relaxation of the restrictions for a given distribution of econometrician's errors as we move across different specifications of heterogeneity, focusing on an IID EV distribution for the econometrician's errors¹⁰. The four IID EV models that we estimate are: (i) no heterogeneity (Model 1), (ii) normal random heterogeneity (Model 2), (iii) finite mixture heterogeneity (Model 3), and (iv) mixture normal heterogeneity (Model 4). Details of the models estimated are provided in Table 3.

4.2 Data and Variables

Data Description: We use scanner panel data on the yogurt category¹¹. The data are from AC Nielsen and cover the Sioux Falls, South Dakota market from 1986 to 1988. To keep the model tractable, we limit ourselves to the top 5 yogurt brands in the market - Nordica 6 oz, Yoplait 6 oz, Private Label 8 oz, Dannon 8 oz, and WBB 8 oz. Together these brands account for around 70% of the category sales in dollars. Descriptive statistics for all brands are reported in Table 4.

The total sample consists of the purchase activities of 150 households over two years; the households are chosen such that all have purchased at least once in the yogurt category. The sample consists of 19,516 purchase observations, of which yogurt was purchased on 4,769 purchase occasions. We randomly split the total sample into estimation and hold out samples. The estimation sample consists of 100 households with 13,392 purchase observations and the hold out sample consists of 50 households with 6,124 observations.

Variables: For each purchase observation we have three sets of dependent variables: i) an indicator variable representing the purchase incidence in the category, ii) an $M \times 1$ vector of indicator variables (where M is the total number of brands) representing the brand choice decision, conditional on purchase in the category, and iii) the budget share/purchase quantity of the brand, conditional on purchase in the category and brand choice.

¹⁰ We have also estimated the models for each of the four heterogeneity cases when the econometrician's errors are IID normally distributed. We have provided the results and discussion for these four IID normal models in section 7 of the Online Technical Appendix.

¹¹ Bucklin et al. (1998) who used this data set, found the quantity elasticities of brands to be significantly different from 'one' in magnitude. The data thus provide a good test of our method's ability to relax the restrictions.

The explanatory variables for each purchase observation are variables that enter the sub-utilities of the brands. Recall that in the deterministic part of the sub-utility of a brand m , the explanatory variables in the vector H_m impact the preference for brand m , and p_m is the per unit price of brand m . The variables that we include in H_m are: (i) brand dummies, (ii) presence of promotions on the brand (that includes features and displays), (iii) brand loyalty, and (iv) inventory of the category. The choice of these variables is driven by prior literature in marketing (e.g., Chiang 1991; Chintagunta 1993). Note that the inventory variable, unlike the first three variables, is the same across all brands in the category. Thus, it will only impact the purchase incidence and budget share decisions, and not the brand choice decision. Since in the yogurt category, consumers' consumption rates depend on the inventory at hand (Ailawadi and Neslin 1998; Sun 2005), we update the inventories using a flexible consumption rate as proposed by Ailawadi and Neslin (1998).

The parameters for each of the four IID EV models are discussed in the Online Technical Appendix. The number of segments for the finite mixture heterogeneity models and the mixture normal heterogeneity models are chosen such that the Bayesian Information Criterion (BIC) is minimized. Based on that, we end up with 4 segments for the finite mixture heterogeneity models and 2 segments for the mixture normal heterogeneity models. We use the method of simulated maximum likelihood to estimate the following parameters in each model: (i) the parameters in the sub-utilities of brands, $\{\beta_m, \mu\}$, which are homogenous in Model 1, but heterogeneous in Models 2-4; (ii) the parameter b_{11} . Note that in none of the four models can we separately identify the threshold utility τ from the brand dummy parameters (the brand intercepts) that enter the sub-utilities of brands. Thus, we set $\tau = 0$ and instead estimate the brand dummy parameters of all the brands. In what follows, we discuss our estimation methodology briefly; further details are provided in the Online Technical Appendix.

We simultaneously estimate all the parameters by maximizing the joint likelihood of all three purchase decisions. Since our interest lies in accounting for unobserved heterogeneity in all three purchase decisions (especially the quantity decision), we need to integrate out the unobserved heterogeneity over the joint likelihood for all three decisions while estimating the parameters. This let us account for the impact of unobserved heterogeneity in brand sub-utilities in the quantity decision, which in turn allows us to use unobserved heterogeneity as a fix to the quantity elasticity problem¹².

¹² This is in contrast to prior work (e.g., Chintagunta 1993) that has used Heckman's two-stage estimator to estimate the likelihood for the three decisions. In that approach, one estimates the incidence and brand choice decisions along with the unobserved heterogeneity in the parameters in the sub-utilities of brands in the first stage. In the second stage,

Formally, the dependent variables for each consumer i at occasion t are the incidence decision, I_t^i , the brand choice decision if a purchase has been made in the category, $d_t^i \equiv \{d_{mt}^i\}_{m=1}^M$, and the budget share decision $s_t^i \equiv \{s_{mt}^i\}_{j=1}^M$. We represent the consumer specific parameters by θ_{2i} and the population level parameters that capture these consumer specific parameters by θ_2 . The likelihood estimates of the parameters, $\hat{\theta}_2$, are computed by maximizing the joint log likelihood (for the entire sample of N consumers with T_i observations for each consumer i) as follows:

$$\hat{\theta}_2 = \arg \max_{\theta_2} \sum_{i=1}^N \ln \left(\int \left(\prod_{t=1}^{T_i} L(d_t^i, I_t^i = 1, s_t^i | \theta_{2i})^{I_t^i} L(I_t^i = 0 | \theta_{2i})^{1-I_t^i} \right) f(\theta_{2i} / \theta_2) d\theta_{2i} \right) \quad (18)$$

where $f(\theta_{2i} / \theta_2)$ is the joint pdf of θ_{2i} . We integrate out the consumer specific parameters using Monte Carlo simulation with $R=200$ Halton draws. Note that the inner expression in the likelihood in (18) consists of two terms for each consumer i for a specific occasion t , viz., the observational likelihood when a purchase is not made in the category, $L(I_t^i = 0 | \theta_{2i})$, and the observational likelihood when a purchase is made in the category, $L(I_t^i = 1, d_t^i, s_t^i | \theta_{2i})$. When the econometrician's errors are IID EV distributed, these two terms are given as:

$$L(I_t^i = 0 | \theta_{2i}) = \exp(-\exp(W_t^i))$$

$$L(I_t^i = 1, d_t^i, s_t^i | \theta_{2i}) = \prod_{k=1}^M \left[\left(\frac{\mu^i}{b_{11}} \right) \frac{\exp(\mu^i V_{kt}^i)}{\exp(W_t^i)} \exp\left(W_t^i - \frac{\mu^i}{b_{11}} s_{kt}^i\right) \exp\left(-\exp\left(W_t^i - \frac{\mu^i}{b_{11}} s_{kt}^i\right)\right) \right]^{d_{kt}^i} \quad (19)$$

where $W_t^i \equiv \ln\left(\sum_{m=1}^M \exp \mu^i V_{m,t}^i\right)$ and $V_{m,t}^i$ is the sub utility of brand m at occasion t for consumer i .

4.3 Results

We focus our discussion on the quantity elasticities for each of the four IID EV models¹³. The log likelihood and goodness-of-fit of both the estimation and holdout samples for each of the four models are given in Table 5. Further, the estimates of the quantity elasticities of all brands across all four models are reported in Table 6. To aid the reader, we have based the following discussion closely on the information given in Table 2 (which presents a summary of the three restrictions discussed in section 2.3).

however, one estimates the quantity decision conditional on the mean values (over the population) of the heterogeneous parameters in the sub-utilities of brands (which were estimated in the first stage). Thus, for the quantity choice decision, the estimation is done assuming no unobserved heterogeneity in the parameters in the sub-utilities. Clearly then, one cannot take unobserved heterogeneity into account while calculating quantity elasticities, which suggests one cannot relax the restrictions in the manner we suggest.

¹³ We do not report the choice and incidence elasticities for reasons of space – section 8 of the Online Technical Appendix provides full details.

Model 1: No unobserved heterogeneity

As pointed out in Table 2, all the restrictions (R1, R2a, and R2b) on the quantity elasticities would hold in this model. As per R1, we should see expected conditional budget shares to be identical across all brands, which would force the own conditional budget share elasticities to be the same as the cross conditional budget share elasticities. To verify this, we first computed the expected conditional budget shares of all brands at the mean values of the explanatory variables. As expected, they are identical across brands (0.0479 for Nordica, 0.0479 for Yoplait, 0.0479 for Private Label, 0.0479 for Dannon and 0.0479 for WBB). The impact of this can be seen in Table 7, where we report the estimates of the own and cross conditional budget share elasticities for Model 1 - the cross conditional budget share elasticities are identical to the own conditional budget share elasticities.

Next, as per R2a, we should see quantity elasticities greater than 1 in magnitude across all brands. Further, since the purchase incidence probability is low (around 24%), we should also see all quantity elasticities being close to 1 in magnitude, as per R2b. Turning to the results (Table 6), we find these conjectures confirmed – the quantity elasticities of all brands are close to, and greater than, one in magnitude.

Model 2: Normal random coefficients unobserved heterogeneity

The purpose of this model is to see if the introduction of a normal random specification for unobserved heterogeneity (with log normal specification of unobserved heterogeneity over the parameter μ) helps relax restrictions R1, R2a and R2b on the estimates of the quantity elasticities at the population level. Based on the population level elasticities for Model 2 reported in Table 6, we first compare the quantity elasticities predicted by Model 2 with those predicted by Model 1 (no unobserved heterogeneity). There are three points to note here.

First, unlike Model 1, the quantity elasticities predicted by Model 2 are not as close to -1 (they range from -0.7920 to -1.0627), thus implying R2b is more relaxed in Model 2 than in Model 1. Second, unlike Model 1, the quantity elasticities predicted by Model 2 are less than one in magnitude for 4 out of the 5 brands, which implies that restriction R2a is also more relaxed in Model 2 than in Model 1. Third, unlike Model 1, the expected conditional budget shares calculated at the mean values of the explanatory variables in Model 2 are different across all brands (they are 0.0397 for Nordica, 0.0474 for Yoplait, 0.0427 for Private Label, 0.0495 for Dannon and 0.0406 for WBB), thus implying that R1 is relaxed compared to Model 1.

Thus we see that by comparison with the no-heterogeneity case (Model 1), all three restrictions seem to get relaxed for the normal random heterogeneity model (Model 2). The reasons for this follow from our hypotheses in section 3.3 regarding the extent to which the normal random heterogeneity satisfies the two properties needed to relax the restrictions. Pertaining to the first property, the normal random heterogeneity does a good job of capturing the actual heterogeneity in the data, which can be inferred by observing a drastic difference in the BIC of Model 2 (BIC = 1807.2) compared to Model 1 (BIC = 2786.9). And pertaining to the second property, assuming log normal heterogeneity over the parameter μ gives Model 2 some degree of flexibility to allow the slope of the hazard function of the overall stochastic term to take negative values.

Model 3: Finite mixture for unobserved heterogeneity (latent class segmentation)

Based on the population level elasticities for Model 3 reported in Table 6, we first compare the quantity elasticities predicted by Model 3 with those predicted by Model 1 (no unobserved heterogeneity). There are three noteworthy points here.

First, we see that unlike Model 1, the quantity elasticities of all brands are less than 1 in magnitude in Model 3. This suggests a relaxation of restriction R2a to the extent that the slope of the hazard function of the overall stochastic term can take negative values. This also matches with the findings of Bucklin et al. (1998) who report all quantity elasticities less than one in magnitude, using the same data set. Second, unlike Model 1, the quantity elasticities in Model 3 are not as close to -1 (they range from -0.8093 to -0.8849). This implies that R2b gets relaxed for Model 3. Third, unlike Model 1, the expected conditional budget shares calculated at the mean values of the explanatory variables in Model 3 are different across all brands (they are 0.0421 for Nordica, 0.0475 for Yoplait, 0.0423 for Private Label, 0.0479 for Dannon and 0.0428 for WBB), implying that R1 is relaxed. Thus, in summary R1, R2a and R2b seem to get relaxed for the finite mixture heterogeneity model as compared to the no-heterogeneity case (Model 1).

Comparing Model 3 with Model 2 (normal random heterogeneity), we see first that while Model 3 yields quantity elasticities less than one in magnitude across all brands, Model 2 does not. This implies that Model 3 does better than Model 2 in relaxing restriction R2a. Second, we see a smaller dispersion of the quantity elasticities from -1 in Model 3 as compared to Model 2. This implies that Model 3 performs worse than Model 2 in its ability to relax restriction R2b. Third, Model 3 yields a smaller dispersion in the values of the expected conditional budget shares across the five brands as compared to Model 2. This implies that Model 3 performs worse than Model 2 in its ability to relax restriction R1.

The results above hew closely to our hypotheses in section 3.3 on the extent to which the two specifications satisfy the properties needed to relax the restrictions. Since the finite mixture specification (Model 3) is semi-parametric, it is more flexible than normal random heterogeneity (Model 2), which enables model 3 to yield values of the quantity elasticities of all brands less than one in magnitude (and thereby relax R2a more). However, the finite mixture specification does worse than the normal random specification in capturing the underlying heterogeneity (the BIC for Model 2 is 1807.2 versus 2152.3 for Model 3). As a result, the finite mixture heterogeneity model performs worse than the normal random heterogeneity model in its ability to relax restrictions R2b and R1¹⁴.

Model 4: Mixture Normal for unobserved heterogeneity

As pointed out in section 3.3, since the mixture normal is a very flexible specification, we would expect it to do a good job of capturing the actual heterogeneity in the data. This is borne out - the BIC for model 4 is 1792.1 versus 1807.2 for model 2 and 2152.3 for model 3. An added advantage of the flexibility of the mixture normal distribution is that the overall stochastic term will be flexible enough to enable the slope of its hazard function to take a wide range of positive or negative values. Based on the population level elasticities for Model 4 given in Table 6, we next compare the performance of Model 4 with Model 2 (normal random heterogeneity) and Model 3 (finite mixture heterogeneity). There are three points to note here.

First, similar to Model 3, and unlike Model 2, the quantity elasticities of all brands are less than 1 in magnitude in Model 4. This implies that R2a is more relaxed for Model 4 as compared to Model 2. Second, similar to Model 2, and unlike Model 3, the quantity elasticities in Model 4 are not close to one in magnitude (they range from -0.5107 to -0.8530). This implies that R2b is more relaxed for Model 4 as compared to Model 3. Third, similar to Model 2, the expected conditional budget shares calculated at the mean values of the explanatory variables show a wide dispersion across brands (they are 0.0387 for Nordica, 0.0484 for Yoplait, 0.0386 for Private Label, 0.0526 for Dannon and 0.0424 for WBB at the mean values of the explanatory variables). This implies that R1 is relaxed for Model 4.

In summary, Model 4 does a better job in relaxing the three restrictions as compared to both Model 2 (normal random heterogeneity) and Model 3 (finite mixture heterogeneity). This stems from the fact that unlike Models 2 and 3, Model 4 satisfies both the properties needed to

¹⁴ As mentioned earlier, Song and Chintagunta (2007), who use the finite mixture specification of heterogeneity, find quantity elasticities to be close to -1 across all brands in two categories, and between -0.74 to -0.9 across brands in the other two categories. This is similar to our finding that the finite mixture heterogeneity succeeds only partially in relaxing the restrictions on quantity elasticities.

relax the restrictions – similar to the finite mixture specification (and unlike the normal random specification), it provides flexibility, and similar to the normal random specification (and unlike the finite mixture specification), it captures the underlying heterogeneity well.

Summarizing:

For the case where the econometrician's errors are IID EV distributed, as we go from the no heterogeneity case to the most flexible specification of heterogeneity, restrictions R1, R2a and R2b each get relaxed. In particular, we find that for the no heterogeneity case (Model 1), all the three restrictions hold, while for the mixture normal specification of heterogeneity (Model 4), all three restrictions get relaxed the most. For the other two cases, viz., normal random (Model 2) and finite mixture heterogeneity (Model 3), the restrictions only get relaxed partially.

5. IMPLICATIONS

We discuss the implications of using Model 4 and compare it along two dimensions with Models 1–3. First, for each model, we discuss whether the predicted hierarchy across the five brands in terms of their quantity elasticities makes sense based on prior research. Following that, we compare the models in terms of their predictions on primary vs. secondary demand elasticities.

5.1 Hierarchy in Quantity Elasticities across Brands: We rank order the brands in terms of their quantity elasticities for Model 4 (mixture normal heterogeneity) which relaxed the restrictions the most. The ranking is: (i) -0.8530 (Private Label), (ii) -0.6668 (Yoplait), (iii) -0.6008 (Dannon), (iv) -0.5715 (Nordica)¹⁵; and (v) -0.5107 (WBB).

A natural question that follows is whether this hierarchy is consistent with prior research. Bell et al. (1999) found that the two factors that differentiate brands on their quantity elasticities are (i) the extent of brand loyalty (greater the loyalty, greater the magnitude of the quantity elasticity), and (ii) price variability (greater the price variability of a brand, greater the quantity elasticity). In Table 8, we report the brand loyalty and price variability (as percentages) across all five brands¹⁶. Note that the Private Label ranks the highest along both factors, which implies its quantity elasticity should be the highest, which is what we get. Although Yoplait's loyalty is almost the same as the Private Label's, its price variability is low - thus its quantity elasticity would be expected to be lower than the Private Label, as our results show. Dannon and Nordica

¹⁵ Note that Dannon and Nordica are not statistically different from each other.

¹⁶ Following Bell et al. (1999), we construct the brand loyalty variable as the average number of purchases of a brand by all consumers who purchase the brand, normalized by the total number of purchases by the consumers who have purchased that brand. Price variability is the coefficient of variation of the brands price.

rank lower than Yoplait on both factors and thus have lower quantity elasticities than Yoplait. Finally, WBB ranks the lowest on both factors, and thus has the lowest quantity elasticity.

Next, we compare the hierarchy across brands in terms of their quantity elasticities as predicted by Models 1-3. In Model 1 (no unobserved heterogeneity), since the quantity elasticities of all brands are restricted to be very close to one in magnitude, we do not observe any hierarchy across brands. Similarly, for Model 3 (finite mixture heterogeneity), we do not observe a clear hierarchy across brands since the quantity elasticities are not statistically different from each other. Finally for Model 2 (normal random heterogeneity), we see that the Private Label has the highest quantity elasticity (-1.0627) and WBB has the lowest (-0.7920), which is reasonable based on the two factors given in Table 9. However, the quantity elasticities for the other three brands do not follow the hierarchy based on the two factors.

5.2 Primary versus Secondary Demand Effects: We follow Bell et al. (1999) and decompose the total elasticity into primary and secondary demand components. The primary demand elasticity component is computed as incidence elasticity plus quantity elasticity, as a percentage of the total elasticity. We report the primary demand elasticity components across all five brands for Models 1-4 in Table 9. Observe that across all brands, Model 4 predicts the lowest primary demand elasticity percentage. Further, averaged across all brands, we see that for Model 1, the primary demand component is 52.5%, for Model 2 it is 52.1%, for Model 3 it is 50.4%, and for Model 4 it is 43.6%. There are two points to note here. First, the average primary demand elasticity component for model 4 is very similar to that obtained by Bucklin et al. (1998), who used the same data set. Second, this decomposition tells us that one would overestimate the primary demand component if one did not correct for the restrictions properly.

6. CONCLUSIONS

The principal objective of this paper was to formally examine the reasons behind the restricted quantity elasticity estimates obtained in the Hanemann's framework and suggest solutions for these restrictions. Our analytical results and empirical examination suggest the following recommendations for future researchers attempting to use Hanemann's framework to examine consumer behavior: it is important to use a specification of heterogeneity that i) adequately captures the true underlying heterogeneity in the data, and ii) is flexible in terms of the slope of its hazard function taking a wide range of positive and negative values. In that regard, we show that the mixture normal unobserved heterogeneity does a better job in relaxing restrictions on quantity elasticities than the normal or finite mixture specifications of heterogeneity. This

suggests that empirical modelers building integrated models of the incidence, choice and quantity decisions are best served by using a mixture normal specification of unobserved heterogeneity.

We view our work as providing a foundation for a number of empirical industrial organization questions pertaining to those categories where consumers buy multiple units of an alternative on a purchase occasion. As pointed out by Chintagunta et al. (2006), existing work based on disaggregate or aggregate models used to address supply side questions has generally ignored the quantity choice aspect of demand. For instance, suppose one wishes to examine the change in consumer welfare and firm profits over different pricing policies, e.g., quantity discounts or bundling. To address this appropriately, one would have to model demand carefully, which would entail modeling a consumer's incidence, choice, and quantity decisions as arising from a joint utility maximization framework. Since the policies under consideration would be expected to affect the consumer's quantity choice decision, it would be crucial to assess quantity response accurately. Hanemann's framework with our proposed solution to ease the restriction on quantity elasticities, is precisely suited to such an exercise. Coupled with a suitable supply-side specification, researchers can obtain a number of useful recommendations on pricing and product line policies. For instance, is a 'buy one, get one' deal profitable in the short run? Would it be profitable to prune the assortment of products offered in a particular category?

Our work suffers from some limitations that could provide opportunities for further research. Perhaps most important, one needs to conduct an analysis similar to ours on a number of additional data sets across diverse product categories. While the restrictions we have shown formally in the lemmas are independent of data, the relative efficacies of the remedies we suggest rely on the extent of the true underlying heterogeneity in the data, suggesting the need for analysis across a variety of data sets.

Table 1: Related Empirical Literature in Marketing based on Hanemann's Framework

Paper	Product Category	Treatment of Unobserved Heterogeneity	Distribution of the Econometrician's Errors	Estimated Population Level Quantity Elasticities
Chiang (1991)	Ground Caffeinated Coffee	Did not consider unobserved heterogeneity	Logistic	Constrained to -1 across all brands
Chintagunta (1993)	Yogurt	Considered unobserved heterogeneity in brand choice and incidence decisions, but not in the quantity decision	IID EV	Constrained to -1 across all brands
Arora et al. (1998)	Canned Vegetable Soup	Considered normal random unobserved heterogeneity in all three decisions	IID EV	Not reported
Nair et al. (2005)	Refrigerated Orange Juice	Considered normal random unobserved heterogeneity in all three decisions	Logistic	Constrained to -1 across all brands
Song and Chintagunta (2007)	Paper Towels, Toilet Tissues, Laundry Detergents and Softeners	Considered finite mixture heterogeneity in all three decisions	Logistic	Constrained to -1 across all brands in Paper Towels and Toilet Tissues; between -0.8 to -0.9 for brands in detergents and between -0.74 to -0.89 for brands in softeners

Note: The three decisions referred to above are purchase incidence, brand choice, and quantity choice.

Table 2: Summary of Restrictions on the Quantity Elasticities, Assuming no Unobserved Parameter Heterogeneity

Restriction	Implication of the Restriction	Distribution of Econometrician's Errors for which the restriction holds true
R1: Restriction implied by Lemma 1	The specification of the expected conditional budget share is identical across all brands, which biases the quantity elasticities of all brands towards -1.	IID EV, GEV, Logistic
R2a: Restriction implied by part (a) of Lemma 2	Since the slope of the hazard function is positive, the quantity elasticities of all brands are restricted to be greater than one in magnitude	IID EV, GEV, Logistic, Independent Normal
R2b: Restriction implied by part (b) of Lemma 2	If the purchase incidence probability is low, the slope of the hazard function for these distributions is small, which implies that the quantity elasticities of all brands will be close to one in magnitude	IID EV, GEV, Logistic

Table 3: Details of Models Estimated on Yogurt Data

Model Number	Model Description: Econometrician's Errors	Model Description: Unobserved heterogeneity	Number of Parameters
1	IID EV	None	11
2	IID EV	Normal random with log normal heterogeneity on μ	18
3	IID EV	4 segment finite mixture	35
4	IID EV	2 segment mixture normal	33

Table 4: Descriptive Statistics of Yogurt Data

Brand	Mean Price in cents/unit (std dev)	Promotion frequency	Share in the category
Nordica 6 oz	41.87 (6.10)	14.22 %	19.3 %
Yoplait 6oz	61.89 (6.66)	2.92 %	34.0 %
Private Label 8 oz	38.43 (10.65)	16.31 %	25.3 %
Dannon 8oz	65.22 (5.26)	4.99 %	13.6 %
WBB 8oz	43.71 (7.44)	4.45 %	7.8 %

Table 5: Goodness of Fit for the Four IID EV Models

Model Number	Model Description: Unobserved heterogeneity	-Log Likelihood: Estimation sample	BIC Estimation sample	-Log Likelihood: Hold out sample	BIC Hold out sample
1	None	2734.7	2786.9	1260.8	1308.8
2	Normal/log normal	1721.7	1807.2	884.4	962.9
3	4 segment finite mixture	1986.0	2152.3	1014.6	1167.2
4	2 segment mixture normal	1635.3	1792.1	817.0	960.9

Note: The Bayesian Information Criterion (BIC) is given as $-L + k \ln(n)/2$, where L is the log likelihood value, k is the number of parameters, and n is the number of observations. A lower BIC is preferred.

**Table 6: Population Level Quantity elasticities for Models 1-4:
IID EV distribution for econometrician's errors**

	Model 1: No unobserved heterogeneity (std. dev)	Model 2: Normal random unobserved heterogeneity (std. dev)	Model 3: 4 segment finite mixture unobserved heterogeneity (std. dev)	Model 4: 2 segment mixture normal unobserved heterogeneity (std. dev)
Nordica 6 oz	-1.0206 (0.0019)	-0.8373 (0.0166)	-0.8696 (0.0403)	-0.5715 (0.0332)
Yoplait 6oz	-1.0362 (0.0028)	-0.8643 (0.0140)	-0.8804 (0.0289)	-0.6668 (0.0185)
Private Label 8	-1.0270 (0.0015)	-1.0627 (0.0462)	-0.8849 (0.0452)	-0.8530 (0.0403)
Dannon 8oz	-1.0145 (0.0010)	-0.9217 (0.0314)	-0.8486 (0.0415)	-0.6008 (0.0263)
WBB 8oz	-1.0084 (0.0003)	-0.7920 (0.0193)	-0.8093 (0.0378)	-0.5107 (0.0226)

**Table 7: Model 1, Own and Cross Conditional Budget Share Elasticities:
IID EV distribution for econometrician's errors, No unobserved heterogeneity**

Price	Conditional Expected Budget Share				
	Nordica (std. dev)	Yoplait (std. dev)	P. Label (std. dev)	Dannon (std. dev)	WBB (std. dev)
Nordica	-0.0206 (0.0019)	-0.0206 (0.0019)	-0.0206 (0.0019)	-0.0206 (0.0019)	-0.0206 (0.0019)
Yoplait	-0.0362 (0.0028)	-0.0362 (0.0028)	-0.0362 (0.0028)	-0.0362 (0.0028)	-0.0362 (0.0028)
P. Label	-0.0270 (0.0015)	-0.0270 (0.0015)	-0.0270 (0.0015)	-0.0270 (0.0015)	-0.0270 (0.0015)
Dannon	-0.0145 (0.0010)	-0.0145 (0.0010)	-0.0145 (0.0010)	-0.0145 (0.0010)	-0.0145 (0.0010)
WBB	-0.0084 (0.0003)	-0.0084 (0.0003)	-0.0084 (0.0003)	-0.0084 (0.0003)	-0.0084 (0.0003)

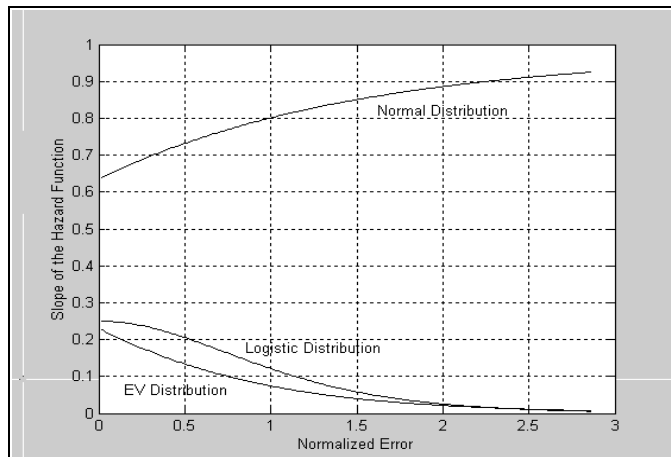
Table 8: Brand Loyalty and Price Variability of Brands in Yogurt Data

Brand	Brand Loyalty Measure	Price Variability Construct
Nordica 6 oz	23.33 %	14.56 %
Yoplait 6oz	42.15 %	10.76 %
Private Label 8 oz	42.88 %	27.70 %
Dannon 8oz	24.40 %	8.07 %
WBB 8oz	14.24 %	17.05 %

Table 9: Elasticity Based Decomposition across brands as predicted by Models 1–4

Brand	Primary Demand Elasticity (% of the total demand elasticity) as predicted by			
	Model 1	Model 2	Model 3	Model 4
Nordica 6 oz	49.69 %	47.40 %	47.59%	38.34 %
Yoplait 6oz	57.83 %	56.17 %	56.88 %	48.82 %
P. Label 8 oz	53.25 %	57.54 %	51.15 %	49.07 %
Dannon 8oz	46.12 %	50.95 %	43.30 %	36.41 %
WBB 8oz	42.65 %	38.46 %	39.98 %	28.67 %

Figure 1: Slope of the Hazard Function for Normal, Logistic, and EV Distributions



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